Fundamentals of Combustion (Part 2)
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Lecture - 66
Liquid Droplet Combustion

Let us start this lecture with a thought process from Winston Churchill.

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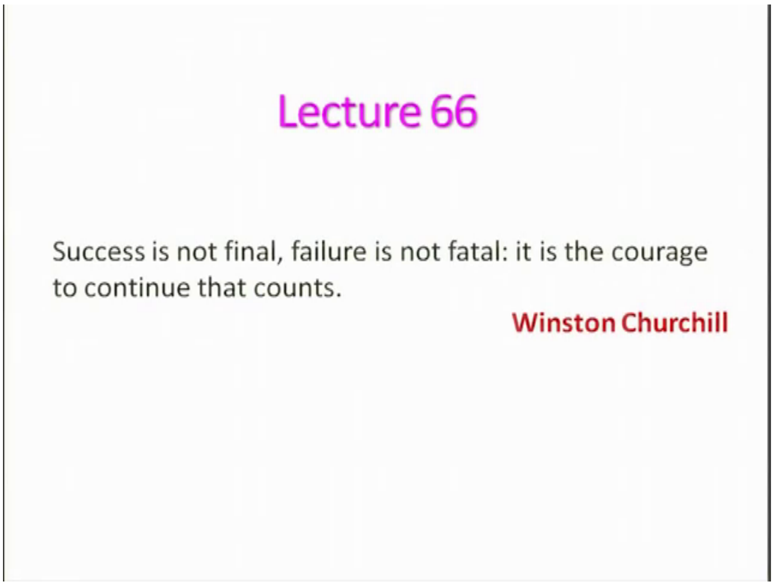

"Success is not final, failure is not fatal: it is the courage to continue that counts". So, in the last lecture, basically we had initiated discussion of how to analyze a single droplet under questioned atmosphere. And of course, under zero gravity condition considering the flow to be one dimensional in which flow and other assumption we have already discussed. And what we will be doing now? We will be basically looking at all this conservation equation one by one and try to do some approximation and let us see how we can solve it analytically.

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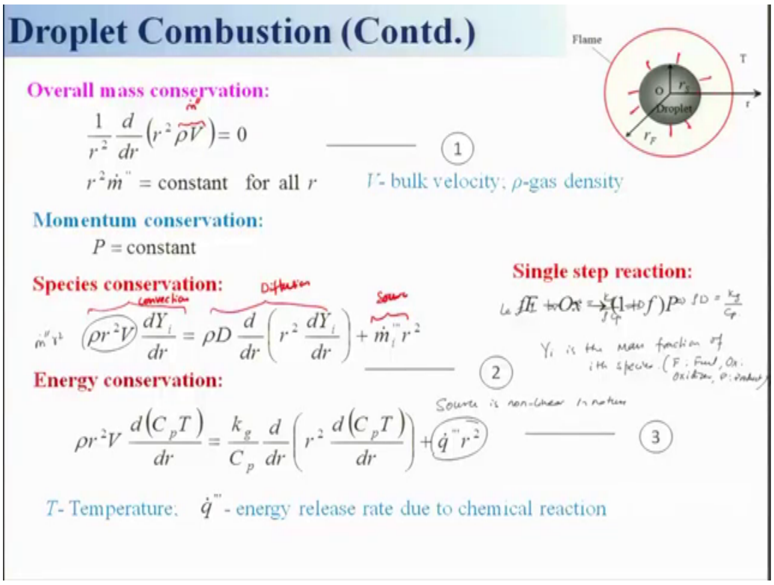

And overall mass conservation equation, if you look at as we are taking is a steady process. So, therefore, this will be 0 and then we will get this 1 over r squared by dr r square rho V. This is basically if you look at, what is this term? Can anybody tell me? This is nothing, but you mass flux right, how much mass is moving out per unit area. You know, these are the mass which is going out well mass right and that is saying basically mass flux into area is basically mass is conserved for all that all radial location. If you look at this is the all radial location, it is basically remaining constant mass right.

But how our mass flux will be changing and this is the our equation one keep in mind that V is the bulk velocity, rho is the gas density right. And momentum conservation of course, we have seen one dimensional situation actually there is no need because pressure remain constant. We have seen pre mixed flame and then you know kind of things we have already seen that. So, there is no need to discuss anything.

So, species conservation if you look at, this is your basically the convection term rho r square V dY i by dr, Y is the mass fraction of i's species and rho d; d is the diffusivity rho is the density into d by dr in the bracket r square dY i by dr into m dot triple dash i r square. This is basically source term and this is diffusion term, this is your convection term right.

So, if you look at this term is basically I can write down as m dot triple double dash r square because rho V is your mass flux and I can write down and keep in mind that your

Louis number is equal to 1 right; Louis number is equal to 1 right. If it is Louis number equal to 1, I can of course, substitute this thing rho D as if Louis number is equal to 1 that is basically means alpha by diffusivity K g by rho C P into D. So, I can write down rho D is equal to K g by C p. We will use that later on.

And as I told earlier that Y i is basically is the mass fraction of i’th species i'th species means it can be fuel, it can be oxidizer, it can be part of right kind of things right i'th species means F will be fuel like we will be writing oxygen is oxidizer and P is product.

So, now let us look at energy equation. It is similar term this is basically convection terms and this is conduction term and this is your source term. The heat release is being done here. So, what we will do is basically we will have to solve these equations right if you look at this is your source term which is non-linear in nature which is quite difficult to solve. So, you keep in mind T is the temperature q dash is the energy release rate due to chemical reaction.

So, let us look at the single step chemistry as usual we do take F moles of fuel react with one mole of oxidize going to 1 plus f moles of product and from these we can actually like look at how we can connect this species and energy conservation or in other words how we can solve them.

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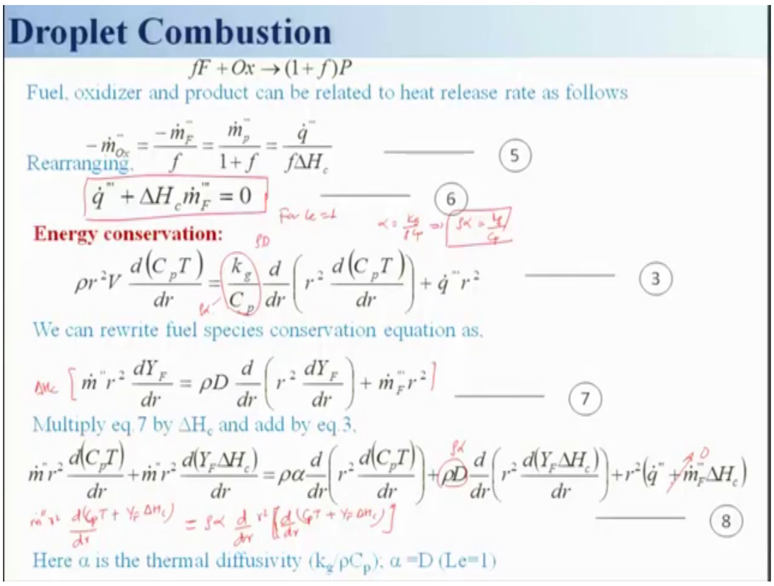

Looking at basically single step reactions and we need to relate this thing to the heat release rate as follows. We can do that is basically mass conjunction of oxidizer per unit volume is equal to m dot triple dash F divided by f is equal to product P divided by 1 plus f and q dot triple dash by f delta s right. So, this is we can do that thing and from these if you look at you can find out very easily that is q dot triple dash is equal to delta H c m dot triple dash F is equal to 0.

So, if you look at this thing, now we can use to simplify our equations basically to overcoming the source term. So, look at let us see energy conservation equation and we have already discussed this and let us consider the fuel species conservation equation right. And what we can do now, what we will be doing we will be basically multiplying this equation seven with delta H c right. I can multiply with the delta H c right and then add that equation to the 3; that means, we can add equation 3 with the 7.

So, if I will do that, what is happening? This term is equal to 0 because already as per the equation 6; we have already seen right. So, if you look at these things, these are all terms which is has to be changed because I can club this together because this looks to be similar terms and the diffusion on the right hand side of this equation. This is the diffusion terms from the energy equation and this is from the species equation which is similar in nature right, provided will invoke the Louis number equal to 1 right.

And already we have seen that this K g by rho C P right if you can look at this one right I can write down for Louis number equal to 1 is nothing, but here rho by D right, I can write down this one. Already we have done that this is right or I can use that as a rho alpha because this is this itself is equal to basically rho alpha right because, what is alpha? Alpha is equal to K by K g by rho C P so; that means, rho alpha is equal to K g by C p. So, I can use any one of them and then. So, that both are same right.

And so, if you look at this I can write down basically here itself this is m dot r square I can write down as d by d r C P T plus Y F delta H c right. I can take that on the left hand side, in the right hand side I can take this as basically rho alpha right I can take this as a rho alpha then rho alpha d by d r, I can write down this r square into d by d r C p T right plus Y F delta H c this term that is all, isn't?

So, this is the equation, we can get right and by these basically we call you know we have eliminated the source term and this expression is basically known as Shvab

Zeldovich transformation. We are basically doing that and it will be only possible if the Louis number equal to one otherwise it is not possible right.

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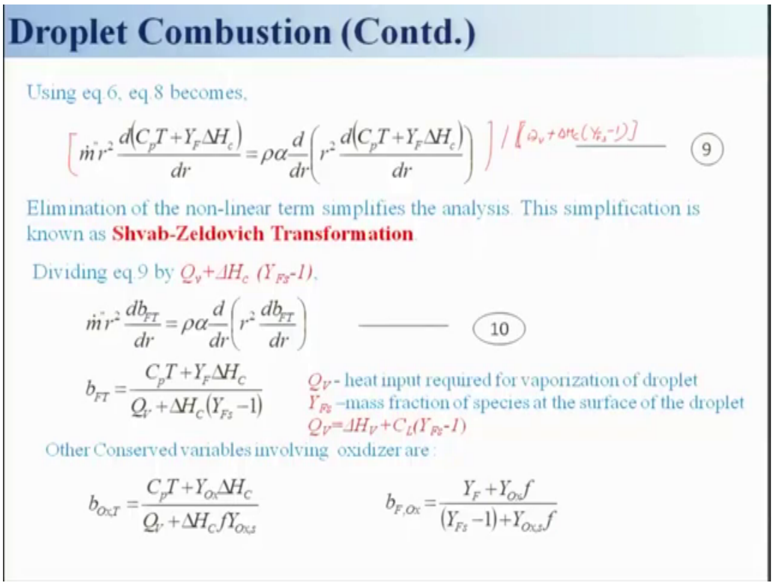

So, I have already written this expression earlier right and as I told that we have eliminated non-linear source term in this analysis and this is known as Shvab Zeldovich transformation. And what we will be doing basically we will be non-dimensionalizing it and trying to express this equation 9 in terms of non-dimensional variables right.

What we will do? We will basically divide this equation by q V plus delta H c Y F as minus 1 right. I can divide this entire equation right and that will get into the term because already we have seen this is the heat of vaporization. This is due to the combustion right all is coming and we will see that why we have taken this term little later on right. And then if we do that then we can express this m dot r square d b FT; FT means what? We have consider fuel and temperature right, two equation we have considered. So, therefore, I am writing FT, I can also consider fuel and oxidizer I can also consider the oxidizer and temperature any two combination I can overcome the and this is the row alpha r square d b FT.

What is this b FT? B FT is basically this term C P T plus Y F delta H c divided by q V plus delta H c y F s minus 1 in the bracket right entire thing has to divided. So, this b FT, I have already shown here and similar way if you derive you can get other relations right keep in mind that this q V is equal to in heat input required for the vaporization of droplet. YF s is the mass fraction of species at the surface of droplet right and C L is the specific heat of the liquid and delta H V is basically heat of vaporization right. And other conserved variable involving oxidizer will be like this, you can get right you can derive that as well right by considering the similar procedure, you can get. And that is a another one is b F o x T is a basically Y F plus y o x f divided by Y F s minus one plus Y o x at the surface this s transfer surface into F right; F is just basically in the stoichiometry.

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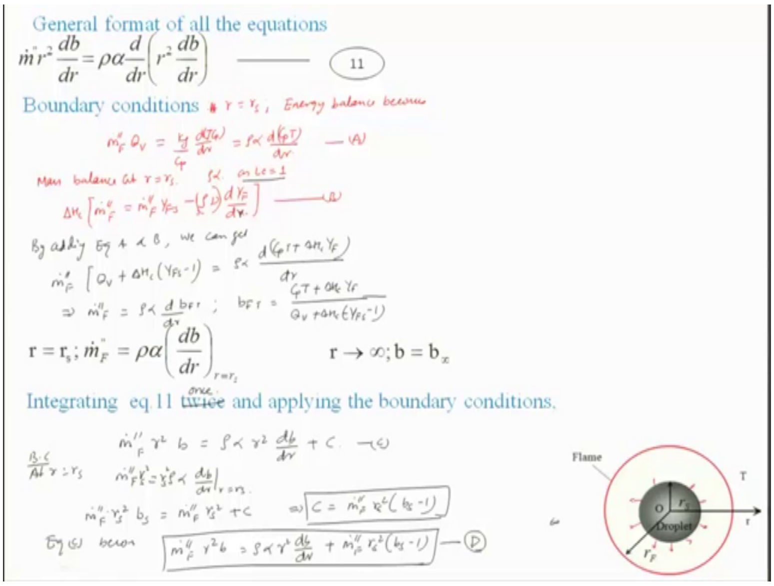

So, let us consider that how this we can solve because this is the equation we have derived right, we will be now looking at how we can solve it. So, this is the equation which will be solving and I have not considered now F T right. It can be anything it will be similar form whether for FT or for your F o x or ox T right it will be similar. So, that is why I have uses b and what we will have to do now basically we will have to use the boundary condition right.

The boundary condition if you look at, what are the boundary condition here? At r is equal to r s from the energy point of view, what is happening right? There we will have to carry out energy balance because this is not very easy. This you will have to use basically a energy balance, how much heat is coming in, how much is being it is getting vaporized. So, we can say that this at that I can write down energy balance right will be m dot F into Q V. This vaporizes the amount of heat which will be coming into for vaporization is this is the total amount of which will be reaching the surface right.

How it will be coming is basically is equal to K g d r by d r because of gradient you know along the direction. So, heat conduction is coming right due to heat conduction there will be the transfer from the flame to that right. And I can write down this as K g by C P and I can write down C P here right and keep in mind that this K g by C P is nothing, but your rho alpha and d C P T by d r right.

And now we will have to look at, this is one we have done the energy balance at the surface of the droplet. Now we will have to do also mass balance right, because the vapour is going out it is getting vaporized and getting out right, it will be moving out the vapours right. So, therefore, we will have to look at that also mass balance right.

So, mass balance will have to look at, surface at r is equal to r s. So, what it would be? It will basically again this m dot F right will be nothing, but your m dot F Y F s minus rho D d Y F by dx. This already we have done; this is the bulk due to the bulk and this is your diffusion right and let say this is I will say this is something let say A because I will be using some different number A and this is your B equation.

Now, what we do? We will have to basically multiply this equation by delta H c right delta H c. And then add this equation to with the a keeping in mind that we are using the Louis number equal to 1 and therefore, instead of these I can write down this is nothing, but your alpha as Louis number equal to 1 right. This is as Louis number equal to 1. And so, what we will do? We will be basically getting now if I will do this; this will be by adding equation A and B right, after multiplication of delta H c. We can get m dot F I can say this is basically q V plus, what it would be? Delta H c right Y F s minus 1 is equal to rho alpha d C P T plus delta H c Y F divided by d d r; this will be d r this is dr.

What I will do? I will divide this equation I can get basically u is equal to rho alpha I can take is d by d r b F T, I can do right how it is because b FT is nothing, but your C P plus C P T H c Y F divided by Q V delta H c Y F s minus 1 right. So, it is coming similar way right. So, I can write down then my boundary condition right at I am combining these two at r is equal to r s, r is equal to rs is nothing, but m dot F rho alpha d V by dr r is equal to r s. This is the boundary condition I am getting right.

And now you have understood, why we have we are divided earlier this q b delta H c Y F s minus 1 right because of this there is the balance, then we are because this is the amount of heat which will be you know taken care for this mass to move around. And when r is equal to infinity, what will b be? B will be b infinity is far away from the flame, far away from the droplet far away; it will not here it is far away.

So, with this we can what we will do? We will integrate this equation 11 right twice and apply the boundary condition right. So, if you look at if I will do, first let me do that is basically the first once, if I integrate equation 1, I will get m dot F r square b is equal to rho alpha r square db by d r plus C right so; that means, now I will have to find out the constant C by integrating equation once first we are doing. So, what we will do? At boundary condition we know that is at this is your boundary condition at r is equal to r s, what you are getting? You are getting basically m dot F is nothing, but your rho alpha db by d r r is equal to r s right, is not it?

So, if I say this is r s square and I can say this is r s square also right from this equation I can say, this is basically C and putting this boundary condition in equation C, I can get m dot triple dot F r s square b s is nothing, but your m dot triple dash F r s square into plus C. So, therefore, c becomes m dot triple dot F r s square b s minus 1 right this is my C right. So, then equation C becomes right, let me write down equation C becomes m dot F r square b is equal to rho alpha r square db by d r, then plus m dot F r square b s minus 1. And we will simplify further and we will discuss this and how to we can do let us say that this is equation you know D and we will be discussing the next lecture fine.

Thank you very much.