

**Fundamentals of Combustion (Part 2)**  
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**Lecture – 63**  
**Flame Height Estimation and Smoke point in Diffusion Flames**

Let us start this lecture with the thought process from Charles R Swindoll, the difference between something good and something great is attention to detail. And of course, the attention is very important for doing something good and not only the good, but also the great. So, in the last lecture we basically looked at how to handle the flame height of a two-dimensional jet diffusion flame.

And today we will be looking at how we can, how it has been extended by roper the analysis of the Burke Schumann, but I would not be getting into the detail of the analysis, which will a quite lengthy in nature. And what way I will be doing we will be looking at you know giving the final result.

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Roper (1977) extended the Burke-Schumann model by varying the velocity to vary along the axial direction.

**Circular Port:**

$v$ : molar stoichiometric oxidizer-fuel ratio  
 $D_{\infty}$ : mean diffusion coefficient evaluated for oxidizer at  $T_{\infty}$   
 $T_f$ : fuel stream temperature  
 $T_f$ : mean flame temperature

Theoretical: 
$$h_F = \frac{Q_F \left( \frac{T_{\infty}}{T_f} \right)}{4\pi D_{\infty} \ln \left( 1 + \frac{1}{v} \right)} \left( \frac{T_{\infty}}{T_f} \right)^{0.67}$$

Experimental: 
$$h_F = 1330 \frac{Q_F \left( \frac{T_{\infty}}{T_f} \right)}{\ln \left( 1 + \frac{1}{v} \right)}$$

**Square Port:**

Inverf: inverse error function

Theoretical: 
$$h_F = \frac{Q_F \left( \frac{T_{\infty}}{T_f} \right)}{16 D_{\infty} \left[ \text{inverf} \left( (1+v)^{-0.5} \right) \right]^2} \left( \frac{T_{\infty}}{T_f} \right)^{0.67}$$

Experimental: 
$$h_F = 1045 \frac{Q_F \left( \frac{T_{\infty}}{T_f} \right)}{\left[ \text{inverf} \left( (1+v)^{-0.5} \right) \right]^2}$$

So, and then which we can use it roper in 1977, extended the Burke Schumann model which I had discussed earlier, by varying the velocity to vary along the axial direction. In actually in the Burke Schumann we use that velocity would not change along the axial direction means y direction in that which is not true right. So, therefore, that was a (Refer

Time: 01:38) in that, so, he has extended that and he has he extended this work also for the circular port and then square port and other things right.

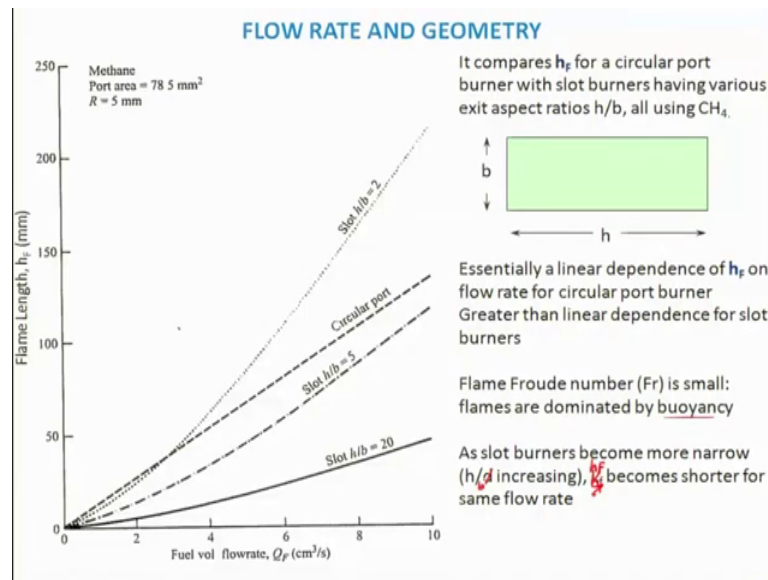
So, let us circular port, if you look at this is the analysis what he did right the final relation if we got that is this one right. Which is having  $Q_F$  divide by  $T_F$  and  $T_F$  right and this is of course, the  $4\pi D_\infty$  this is your diffusivity. And  $\ln 1 + 1/\nu$  and  $T_\infty$  divided by  $T_F$ ;  $T_F$  is your mean flame temperature right and the  $T_F$  if you look at is the fuel stream temperature.

Because, this fuel temperature may change right and  $T_\infty$  is the ambient temperature whatever it will there right at the oxidizer side right. And  $D_\infty$  is the mean diffusion coefficient evaluate generally for the oxidizer side, this is the diffusivity right and this is of course, theoretical by roper right. And they also looked at this expression and tried to couch the experimental data and come off with a experimental relationship like that  $h_F$ ,  $h_F$  is the flame length or flame height whatever you call it is having a constant.

This is basically like constant right and  $Q_F$  and  $T_\infty$  by  $T_F$  and  $T_F$  is the fuel mean stream temperature  $\ln 1 + 1/\nu$ . So, they have retained the similar this thing but however, they match a constant to match the experimental data you know that is all I can say, right and then in the passes they have eliminate this diffusivity and then this term the coefficients also this is the simplification they have done..

So, for the square port; however, they have looked at in a little you know similar they have arrived at this relationship flame height similar only thing they have put it something inverse error function here and this is the similar in nature. And if you look at the coefficient looks to be same for both the circular in the square they have written. And the constant is very different right from the experimental data and they have also extended this for the various fuel kind of things right.

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Which is they have plotted with the flame height what say the volumetric flow rate in centimeter cube and you can see that, very interesting thing you can see this is the circular port right. And when you talk about this slot kind of burner this slot this is  $b$  by  $h$  it can be 2 right and it can be 5, it can be 20 it can be 1; if it is 1 it is a square right kind of thing. And you can see that the as this aspect ratio or the slot  $h$  by  $b$  increases the flame length becoming very small as compared to circular when it will be twice you know kind of thing  $h$  is twice of that  $b$ . You will get particularly at the higher flow rate the flame might increase right.

And actually what they have found that is a linear dependence on the flow rate for a circular port and linearity of course, depends on the slot burners right and they say that this flames are basically dominated by buoyancy because, the Froude number is small and this is basically  $h_f$  becomes shorter for same flow rate when the  $h$  by  $b$  for  $h$  by  $b$  is decreasing this is basically  $b$  right.

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**FACTORS AFFECTING STOICHIOMETRY**

- Recall that stoichiometric ratio,  $\nu$ , used in correlations is defined in terms of nozzle fluid and surrounding reservoir
  - $\nu = (\text{moles ambient fluid} / \text{moles nozzle fluid})_{\text{stoic}}$
  - $\nu$  depends on chemical composition of nozzle and surrounding fluid
  - For example,  $\nu$  would be different for pure fuel burning in air as compared with a nitrogen diluted fuel burning in air
- Influence of fuel types, general HC:  $C_nH_m$ 
  - $CH_4: n=1, m=4, \text{ Ethane } n=2, m=6$

Fuel	$\nu = \left(\frac{\text{mole air}}{\text{mole fuel}}\right)_{\phi=1}$	Relative Flame Length, $h_f / h_{f, CH_4}$
Hydrogen or carbon monoxide	~0.5	~0.5
Methane	1	1
Ethane	~1.5	~1.8
Propane	~2.2	~2.5
Butane	~3.2	~3.2

$\nu = \frac{n + \frac{m}{4}}{X_{O_2}}$

Plot of flame lengths relative to  $CH_4$

Circular port geometry

$$h_f = 1330 \frac{Q_f \left( \frac{T_x}{T_f} \right)}{\ln \left( 1 + \frac{1}{\nu} \right)}$$

Flame length increases as H/C ratio of fuel decreases

**Example:** Propane ( $C_3H_8$ ; H/C=2.66) flame is about 2.5 times as long as methane ( $CH_4$ ; H/C=4) flame

So, what I was telling, they have also looked at the factors that affecting the stoichiometric ratio and this stoichiometric basically mole of ambient a fluid by the moles of number fluid at the stoichiometric ratio. It depends on the chemical composition and also the surrounding fluid, surrounding fluid generally it will be air, but you can have some different thing as well right you can because, whenever you are using co axial jets you can use that. For example,  $\nu$  would be different for pure burning in air as compared to nitrogen dilute and fuel burning in air or some other like helium, some other thing you can also dilute right.

And the various fuel one can think of hydrocarbons let us say like methane, ethane, propane, butane, a hydrogen and carbon dioxide comes over here and this is the relative flame height with respect to the methane right and versus the  $\nu$  mole fraction they have. It looks to be very linear kind of things and they are getting and because the fact that what they are attributing that this is  $\nu$  which will be depend on that what is the n and m.

Because, n if you look at per methane basically n is equal to 1 m is equal to 4 right and for propane butane it will be you know as it goes on ethane it will be 2 and m will be 8 2 4 not, 6 kind of things right, for ethane n is equal to 2 and m is equal to 6, it will be go on increasing. So, therefore,  $\nu$  will be it will be changing right, 3 H 8 H C ratio that is the which will be effecting 2.6 and then flame is about 2.5 times as long as the methane, like if you look at propane right.

So, as it is changing then this thing will be changing. So, therefore, the flame height will be goes on increasing when the hydrocarbon become higher hydrocarbons being used. One can also look at it is the it will be diffusivity also will be playing a role because it is a heavier fuel right, it will take more time to pass go through and mix that so, that is also another way of looking, but right. So, flame length basically increases with H C ratio of as the ratio of the fuel decreases that is the thing what they have found out.

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• **Example** : A laminar butane gas jet issued from a tube into air has a flame height of 10 cm. Determine volumetric fuel flow rate and heat release rate. If the fuel tube diameter is increased by 25 % and velocity is decreased by 25 % what will be the flame height? Take heat of combustion of butane gas = 45000 kJ/kg,  $T_{ad} = 2300$  K,  $T_c = 298$  K.

To find:  $\dot{V}$ , HRR: Heat release rate.  $h_f$

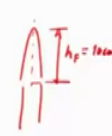
Solution: By using Roper formula (Eqn 1)

$$h_f = 1330 \frac{\dot{V}_F \left( \frac{T_c}{T_{ad}} \right)}{\ln \left( 1 + \frac{1}{\phi} \right)}$$

For stoichiometric  $C_4H_{10}$ -air mixture,  
 $C_4H_{10} + 6.5 \left( O_2 + \frac{79}{21} N_2 \right) \rightarrow 4 CO_2 + 5 H_2O + 6.5 \times 2.76 N_2$

$$v = \frac{\dot{m}_a}{\dot{m}_F} = \frac{6.5 \left( 32 + \frac{79}{21} 28 \right)}{58} = 15.28$$

The volumetric fuel flow rate can be determined from Eq 1 as

$$\dot{V}_F = \frac{h_f \ln \left( 1 + \frac{1}{\phi} \right)}{1330 \left( \frac{T_c}{T_{ad}} \right)} = \frac{10 \times 10^{-2} \ln \left( 1 + \frac{1}{15.28} \right)}{1330 \left( \frac{298}{2300} \right)} = 2.7 \times 10^{-6} \text{ m}^3/\text{s}$$


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So, we will taken an example and will be using in this basically how to use the ropers formula for calculating the flame height laminar butane gas jet is issued from a tube into air right, has a flame height of 10 centimeter right.

So, let us say this is a is there and their flame is given and this height  $h_f$  is given as basically 10 centimeter right. And we will have to find out to find volumetric flow rate I can say  $\dot{V}$  dot basically and we will have to find out heat release rate, heat release rate I can say H R heat release rate right and that is the first part. But then what happened we will have to also find out  $h_f$  when the diameter of the tube is increased by 25 percent and velocity decrease by the 25 percent what will be the flame height right? And take the heat of combustion of the butane we have taken 45000 kilo joule per kg T adiabatic we are taking 2300 Kelvin.

So, if you look at we are basically will be using the Roper's formula solution by using Roper's formula right and keep in mind that this is experimental right we are not using

theoretical value. We will get  $h_f$  is equal to  $1330 \dot{V} T_{\infty} T_f$  divided by  $\ln 1 + \nu$  right. In this case  $T_f$  is given right and actually this is given if you look at this is given,  $h_f$  is given right and then we will have to find out basically  $T_f$  is given and we will have to find out  $\nu$  right and then we will have to find out basically the  $\dot{V}$ . So, if you look at what will be doing we will be looking at the stoichiometric mixtures right, the stoichiometric mixtures for the butane, this is the butane gas right.

So,  $C_4H_{10}$  plus I can write down plus 79 by 21  $N_2$  right is going to the carbon dioxide and it will be getting to the water and will be getting also the 3.76  $N_2$ . So, if you just do that balance I will be having 4 here and if you look at H then I will have to do this 5 here and if I do this will be 6.5 I can say 6.5 right and this is for the stoichiometric  $C_4H_{10}$  air mixture. So, then  $\nu$  I can find out will be  $m_{\text{air}} / m_{\text{fuel}}$  is nothing, but your  $6.5 \times 32 + 79$  divided by  $21 \times 28$  into 58 is equal to 15.38 right and which is roughly because the hydrocarbon generally it will be coming something around you know around 15 right.

So, then like a we know  $T_{\infty}$ ,  $T_{\infty}$  is not given here you can take basically  $T_{\infty}$  as something 298 Kelvin right I can take  $T_{\infty}$ . So, therefore, if I say this is a question 1 I can find from these, I can find out the volumetric fuel flow rate can be determined from equation 1 as  $\dot{V}$  is nothing, but your  $h_f \ln 1 + \nu 1330 T_{\infty}$  by  $T_f$  right.

So,  $h_f$  already given that is basically 10 centimeter, 10 centimeter means we will have to say it is meter right it will be  $10^{-2}$  right into  $\ln$  this is  $1 + \nu$  divided by 15.38 divided by 1330 into 298 divided by 2300 you will get something  $3.7 \times 10^{-5}$  meter cube per second. So, we will have to now find out basically heat release rate right. So, how we will find out heat release rate?

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$$\begin{aligned}
 HRR &= \rho_F \dot{V}_F \Delta H_c = \frac{P \cdot MW_F}{R_u T} \dot{V}_F \Delta H_c \\
 \Rightarrow HRR &= \frac{101325 \times 58}{8314 \times 298} \times 3.7 \times 10^{-5} \times 45000 = 46.95 \text{ kW} \\
 d_f \text{ is increased by } 25\% \text{ \& } \dot{V}_F \text{ is decreased by } 25\% \\
 \dot{V}_F &= A_F V_F = \frac{\pi}{4} d_f^2 V_F \Rightarrow \frac{\dot{V}_{F, \text{new}}}{\dot{V}_{F, \text{old}}} = \left( \frac{d_{f, \text{new}}}{d_{f, \text{old}}} \right)^2 \left( \frac{V_{F, \text{new}}}{V_{F, \text{old}}} \right) \\
 \dot{V}_{F, \text{new}} &= (1.25)^2 (0.75) \dot{V}_{F, \text{old}} = 4.28 \times 10^{-5} \text{ m}^3/\text{s} \\
 \text{We can determine } h_F \text{ by using Eq. (1)} \\
 h_F &= \frac{1830 \dot{V}_{F, \text{new}} \left( \frac{h_F}{L} \right)}{\ln(1 + \frac{h_F}{L})} = \frac{1830 \times 4.28 \times 10^{-5} \left( \frac{204}{254} \right)}{\ln(1 + \frac{h_F}{254})} = 118 \text{ cm} \\
 h_F \text{ is increased due to increase } \dot{V}_F.
 \end{aligned}$$

Heat of combustion right, heat release rate HRR, I can write down rho of F right density into volumetric flow rate right into delta H of combustion right. So, the rho F I can find out of course, the pressure right P by R u T into molecular weight of fuel right into this is your V F delta H c.

So, therefore, I will just substitute these values I am taking atmospheric pressure I think that is not given in the question, but you will assume that it is a ambient atmosphere pressure. So, therefore, pressure will be 101325 and the molecular weight of that; of the butane right will be what 58 right into 8314 into 298 because this will be T infinity right. Ambient temperature we will have to say that wherever it is coming and already V dot F we know that is 3.7 into 10 power to minus 5 into 45000 right.

Student: Is (Refer Time: 17:37)

Is equal to 46.95 kilo watt because this is already 45000.

Student: Kilo joule.

Kilo watt right kilo joule per.

Student: Kg.

Kg of that so, therefore, you will get in kilowatt. So, now, basically as I told that there will be tube diameter is increased right and then tube diameter is increased by.

Student: (Refer Time: 18:05)

D is increased by 25 percent and velocity  $V_F$  is decreased by 25 percent right. Then you will have to look at basically how much the  $nu$  flame height right, for that you will have to evaluate basically new volumetric flow rate and if you look at the volumetric flow rate of fuel is nothing, but your.

Student: A.

A.

Student: (Refer Time: 18:41)

Of fuel port into this is nothing, but your  $\pi$  by  $4 d_F$  square right I can say this right into  $V_F$  right. So, if I look at this  $V_F nu$  will be equal to basically 1.25 right, it will be increased by 25 percent diameter. So, square into it will be decreased by the velocity, so, I can write down 0.75 into  $V_F$  is not it or right the I can say may be old right old values. So, implies I can write down  $V_F$  new divided by  $V_F$  old is nothing, but your  $d_F$  new by  $d_F$  old right square and  $V_F$  new by  $V_F$  old right that is alright and then this is basically 1.25 right I mean is the one you can divide that and then this will be like that right.

So, if it is coming this is something coming as  $4.3 \times 10^{-5}$  meter cube per second then when you put that thing what you will get is.

Student: (Refer Time: 20:30)

This will be 38 so, therefore,  $h_F$  we can determine the  $h_F$  by using equation 1 right,  $h_F$  is equal to  $1330 V \cdot F$  of course, this will be new one will be putting into  $T$  increased by  $T_F$  into  $\ln 1 + 1$  by  $nu$  right and I know all those values I will be putting that values into  $4.38 \times 10^{-5} \times 298$  divide by  $2300 \ln 1 + 15.38$  you will get you will get something around 11 point I think it will be little different 11 point maybe.

Student: 7.

8 centimeter you will get right. So, I mean what it indicates? It indicates that the flame height is increased due to increase in, basically what?

Student: Volumetric.



Volumetric flow rate right, it is basically  $h F$  is and of course, there is a increase in diameter right and whereas, the you when you increase the diameter then what will happen, the  $V F$  has to also decrease right of course, a provided the all the thing, but here the flow rate is changing. So, therefore,  $h F$  is basically is increased due to increase in volumetric flow rate of fuel ok.

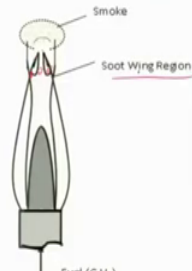
So, with the using this simple formula one can get that of course, it is similar in nature to the phenomenological analysis what we are also seen, but when you do that this result determination of flame height by this method and their phonology there might be changed in the actual number. But; however, the train looks to be similar that is the thing you should keep in mind right.

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**Smoke Point :** It is one of the criteria of ascertaining the tendency of any fuel to produce soot.

As to ASME standard, Smoke point is the height of flame in mm in a standard burner without causing any smoke.

Another way is to measure the minimum flow rate of fuel in which no smoke is produced.



The diagram shows a burner with a fuel inlet labeled 'Fuel (C<sub>2</sub>H<sub>6</sub>)'. The flame is shown with a 'Smoke' region at the top and a 'Soot Wing Region' below it.



Alkanes	mg/s	Alkenes	mg/s	Alkynes	mg/s	Aliphatic	mg/s
C <sub>3</sub> H <sub>8</sub>	7.87	Ethylene	3.84	Acetylene	0.51	Toluene	0.27
C <sub>4</sub> H <sub>10</sub>	7.0	Propylene	1.12	1-Heptene	0.65	Styrene	0.22
n-Heptane	5.13	1-Octene	1.73	1-Decyne	0.8	O-Xylene	0.28
i-Octane	1.57	1-Decene	1.77	<b>Alkanes &gt; Alkenes &gt; Alkynes &gt; Aliphatic</b>			

And what will be doing now? Now we will be looking at about some points which is known as smoke point right and what do you mean by smoke point? Particularly it will very important for the jet diffusion flame and also with the increase concern for the emission this is very important. Smoke point is basically is used to ascertain the tendency of any fuel produce soot right.

Suppose a fuel is sooty or not right one can determine of course, there is a various ways of doing it, one way that you just take some certain amount of liquid fuel and go on heating it and see that like what condition what temperature that is giving producing a

soot right that is one, but in our case we are talking about the jet diffusion flame. So, therefore, we will be concerning about the for the jet diffusion flame right.

So, according to the ASME standard, the smoke point is the height of flame in mm in a standard burner means certain diameters you know is required without causing any smoke right. If I consider a burner a tube I will take and in this case I will be passing through certain amount of fuel flow rate and it will be there. So, then there will be a flame will be coming right and if I goes on increasing this flow rate of fuel, then what will happen? Flame height will be goes on increasing right.

If I will increase this thing and then flame this is basically the flame which will be increasing and then as find out then it will be producing a large amount of soot. Because, this is in quiescent atmosphere these are basically air which we are quiescent atmosphere right and then the it will be start smoking right. Smoking means lot of soot will be produce that means we will have to find out what is the minimum flow rate of the fuel in which there is no smoke.

In other words that is a critical flow rate at which the smoke will start you know producing right so that we call it as a basically smoke point. So, if you look at a figure here of course, this is taken under the zero gravity condition right use by the this photograph is basically by the produced by NASA right and this is your jet flame as you goes on increasing the fuel flow rate I mean as you that as with increasing this the mass flow rate of fuel is increasing right. And as you go on you will see these are the soot is being form start forming you know like these are the soot is open right soot being I mean formed and then smoke it is very much smoky kind of thing.

So, generally that height what is called as basically a; that means, when it will start you know opening of this tips right and then because of soot is there would not be any flame. You know it will be cooling the flame right because soot is lot of soot's are been produce here there will be soot and these are the soot wing region large amount of soot will be formed here.

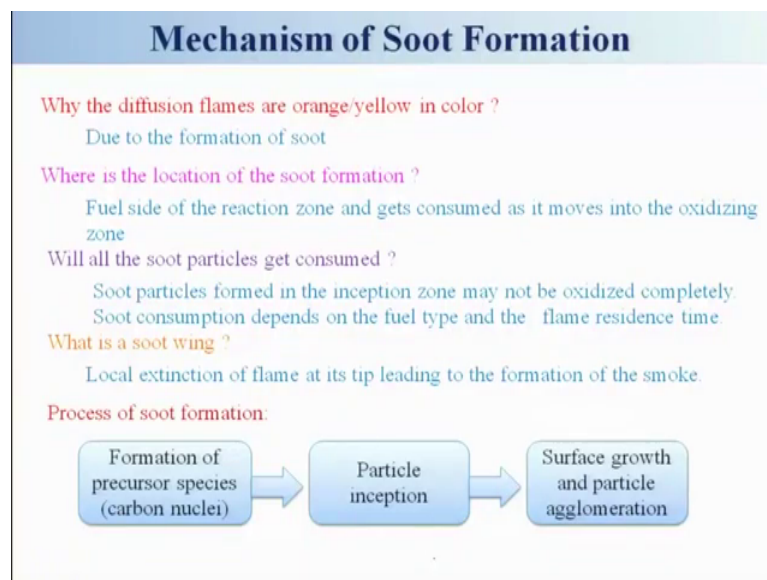
As a result that flame is not there it is quenching right and then it will be start all the soot will be going instead of oxidizing it is going up and that point where it will be coming this is basically known as smoke point right. And this depends on the kind of fuel you are using, it is basically alkanes and propane, butane, heptene, octane you can see that this is

for the critical mass flow rate at which the smoke started producing ok; smoke means a large amount soot right opening of this steep and I think.

So, if you look at the propane is becomes propane's smoke point is higher as compared to octane, what is the meaning of that? That means, that octane iso octane is more prone to the formation of the soot as compared to propane. Are you getting lower the value means it is at very small flow rate is started smoking right; that means, this fuel is a not good from the smoke point of point of view are you getting? And similarly the alkynes right is the again the similar situations for ethylene, propylene, you know decyne you can look at it is of course, that is a little bit change here, but it is in similar number right kind of things.

So, similarly alkynes these are the kind of things the flow rate is being done aliphatic is having very low; that means, aliphatic is more prone to the smoke as compared to alkynes. And alkynes is more prone to soot than the alkenes and alkenes is more prone to soot than alkynes that is the general conclusion you can draw from this data right that you should keep in mind.

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And so, let us now look at about the soot formation how does because the smoke point is coming due to the soot formation. Now, we will be looking at how does the soot basically been formed in the jet diffusion flame or any other flame right. So, and we had had a question why diffusion flames are orange or yellow in color we told due to the

formation of soot. And where is the location of the soot formation right, where the soot being formed is it at the tip or is it the beginning or these are the questions we need to ask our self and whether it is the because for that fuel side of reaction zone and which will be there and gets consumed at it is move into oxidizing zone right.

And we will all the soot particle get consumed a in a oxidizing zone or not that is a one question has to be ask and if it is consumed then you would not really find the soot particles right. Soot particles form in the inception zone may not be oxidize complete inception zone means where it will be initiated and soot consumption depends on the type of fuel and also the residence time the fuel remains in the flame height using the flame height how much it will be remaining.

So, that is also dictate the soot, the life of the soot particle and what is the soot wing? Is basically the local extinction of flame at it is tip leading to the formation of smoke. Keep in mind the smoke is form that does not mean smoke has not been formed in jet diffusion does not mean soot's are formed soot's are formed. but it is good enough to extin the flame locally and then soot will be getting going out and flame extinction, the we will have to look at process of soot formation basically formation of precursor species or carbon nuclear will be there and then there will be a particle inception, particle will be formed and then there will be the surface growth and particle agglomeration.

Student: (Refer Time: 31:20)

Will be there.

Student: (Refer Time: 31:22)

So, with this I will stop over in the next class will be discussing about mechanism of soot formation.

Thank you very much.