Fundamentals of Combustion (Part 2)

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Lecture – 62
Theoretical Analysis of a Two -Dimensional Diffusion Flame (Contd.)

Let us start this lecture with a thought process from Albert Einstein.

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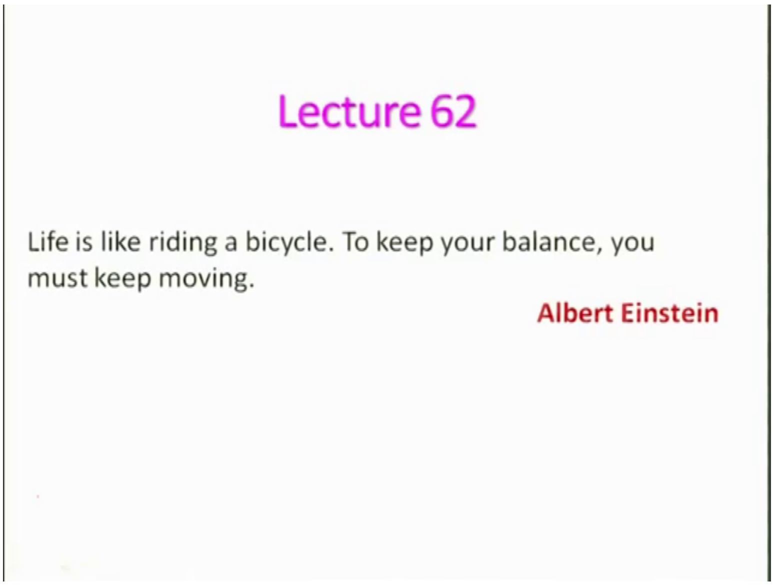

Life is like riding a bicycle. To keep your balance, you must keep moving. Actually this is very important even though you may have some problems, and then some success and failure, but one has to move on. So, in the last lecture if you recall that we basically looked at how to analyze a 2 dimensional diffusion flame. And we invoke the continuity equation, momentum equation, spaces equation, energy equation all those things right. In the last lecture we basically derived relationship involving the universal variable; in which you can express the fuel and oxidizer, because both for the fuel and oxidizer you can solve one equation right.

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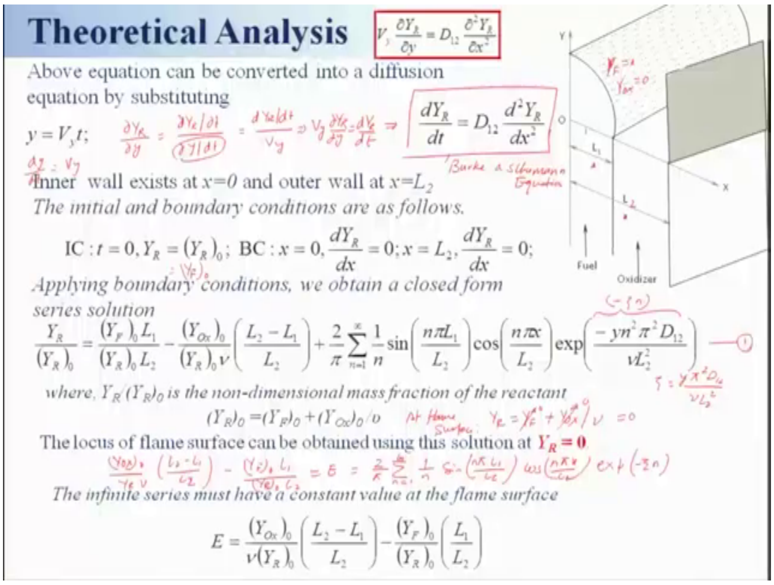

If you look at we have basically looked at this expression and which is Y R is basically it can be either fuel or it can be oxidizer right. So, the equation form will be similar. Keep in mind that this is the partial PD, partial derivative partial differential equations. And now this is second order right. We need to solve we can also do little more simplification take considering the assumption we are made, and this is basically we can convert into a diffusion equation right by considering y is equal to V y t right.

And if I want to look at basically dou y by y I by dou y, I can write down this as basically Y R by dou t. And I can write down dou Y by dou t right. And this is term if I take it that, this is nothing but your what?

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Basically V y right, if I use this one and this is dy by dt is nothing but V y. So, therefore, this will be basically and dou or I can say this is a ordinary differential dY R by dt, because the function of t only right. Now and then V y. So, therefore, I can write down that is basically dy by I can write down this is, I can write down V y dY R by dY is nothing but your dY R by dt.

So, I can derive this expression, keep in mind that these are basically a differential equation of second order, and it has to be solved right. And this equation is known as the Burkes Schumann equation right. This equation is known as because the a Burkes

Schumann equation. He derived these thing both the guys so therefore, it is known as that and we will have to now apply the boundary conditions and other thing.

Keep in mind that these equation is valid where all the region except on the flame surface right. And therefore, the source term has been neglected in both the equation. So, now if you look at inner wall exists at x is equal to 0 right. And outer wall at x is equal to L 2 right. This is basically L 1 ok, and L 2.

So, the initial and bound initial conditions and the boundary condition are given as a; initial condition will be t is equal to 0 right. T is equal to 0 means Y is basically 0. Y R will be what? Y R will be whatever at the initial conditions. In this case this will be fuel right, in this zone it is a fuel.

So, that is equal to you can say is equal to Y F right of at this point right. And boundary condition x is equal to 0 right, at this point right. This is x is 0, then gradient dY by R by d x at the 0 x is 0 in this point, this is 0. And that what it says it is basically symmetric condition right, flame is that means, other half will be also there right, you can see. And similarly L 2 by at x is equal to L 2 at this point dY by R d 0; that means, again it is gradient is 0 at the far away air side you know if you look at there will be no gain, because your flame is here. So, it is far away from the plane. So, you can expect the gradient will be 0.

Now, applying this boundary condition we can obtain a close as series solutions right. And that you can get in this form that is Y R divided by Y R naught is equal to Y F naught L 1 by L 2 right. And Y R naught is coming over here, because you are just dividing right. And Y a o x naught divided by Y R, and this nu is your stoichiometric ratio right. And L 2 minus L 1 divided by L 2 and this is your series solution. I would suggest that you can derive this because due to paucity of time, I am not derived it. And this is a series solution, you might have done in your, you know math course right.

This is n can be one to infinity thing, but; however, when you compute you will have to do certain number of these things as that it will be good enough to give you a result, you should not take one or 2. But at least something maybe 20-30 kind of things and keep in mind that, this term is this is the zeta I can say, zeta is y pi square D 1 2 by nu L 2 square.

So, this expression you can write down as basically minus zeta n right. This will be exponential terms right. And keep in mind that the we are interested to find out the flame locus, that is the our objective right. And once I get the flame locus I can find out what will my flame length. So, keep in mind that this Y R by Y R naught at the initial conditions is the non-dimensional mass fraction of the reactant. It v it can be both fuel it can be both oxidizer.

And Y R naught is nothing but your Y F o plus y oxidizer o divided by nu. That we have already seen right, this thing. And at the locus of flame, at the flame surface what will happen? Y F will be 0 right. And so, also why oxidizer will be 0 right; that means, Y F will be 0 at the flame surface, why oxidizer will be 0 because it will be consumed instantly right, as soon as it will come in contact with that due to the first chemistry. This term will be 0 here, this term will be 0.

So, therefore, this is 0 at flame surface. So, you can find out this is basically in this equation, you can say this is 0, right. Because Y R is 0 right and then at the surface and this is of course, the initial condition Y R will be 0. But if I want to say that then like this is will be I can write down Y R will be Y F plus y oxidizer into nu at the flame surface.

So, therefore, this will be 0 and this is 0. So, this will be 0 at the flame surface. Now if this is 0 in this equation, if I say this is let us say equation 1 I am taking for the time being ok. So, this is 0, then what will happen? There is a basically one is the series term in the right hand side of equation 1. And this term right, and this thing I can say is equal to basically is E right. E means, I can take out this I can say this is Y oxidizer by Y R nu L 2 minus L 1 divided by L 1 L 2 minus Y F L 1 divided by Y R, the initial condition L 2 is equal to e; is equal to what? 2 by pi n 1 to infinity 1 by n sine pi n L 1 by L 2 cos n pi x by L 2, exponential zeta minus n right, and that is itself is E right.

Because if it is you know it will be having some finite value of course, in certain situation you may get it this series part will be 0. But here, it will be, it will be some finite value on the left hand side. So, this infinities must have a constant value at the flame surface. At the flame surface, it will be have some constant. It can be positive, it can be negative right, but it will be having some finite value at the flame surface right.

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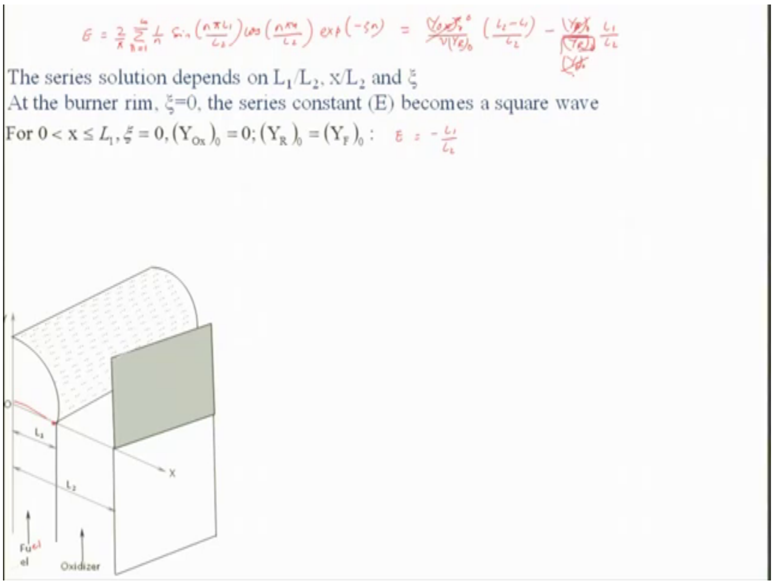

Now so, the series solution if you look at, it is basically depend on if you look at x by L 2 right, it is depend upon x by L 2 and it is depend on zeta right, and it is dependent by L 1 L 2, right. These are the values on which this has to be dependent. I will write down that will be better.

So, this series solution depends on L 1 by L 2. Why I am saying so? Because what happened like a if you look at E is equal to 2 by phi right, 1 to infinity 1 by n sine n pi L 1 L 2 cos n pi x by L 2 exponential zeta n right. So, therefore, it will be dependent on L 1 by L 2 right, it will depend on x by L 2 and dependence on zeta.

At the burner rim; that means, here right, this is your burner rim right, at this point right the series constant I can call this as a constant value right; becomes a square wave right. And for x when x varies from 0 to L 1, here in this region, right it will be zeta will be 0 right. And Y oxidizer will be what in this region because the fuel so, naturally it will be 0 right is not is it? Because here no fuel can penetrate into the fuel stream. Sorry, no oxidizer will penetrate into the fuel stream. And x 2 when x between 0 and L 1, this is the fuel stream right, this is your fuel stream right.

So, it will not really oxidizer cannot penetrate, therefore. Therefore, this oxidizer will be 0 ok. And Y R is nothing but your fuel at the inlet. So, if that is the case, then now, what will be E, if you look at, this is equal to we have already seen right. That is E is y oxidizer by nu Y R L 2 minus L 1 divided by L 2 minus y f oxidizer Y R oxidizer L 1 by

L 2 right. So, what is there? If it is 0 to x lies between 0 to L 1 in this region right. So, what is happening? Y ox is 0; that means, this term is 0 right ok.

So, if that is 0 so, what I will get? E is basically equal to minus L 1 by L 2 and Y R naught is nothing but your this portion is what? Is nothing but here Y F naught. So, this is cancel it out, are you getting? This Y R naught is nothing but your Y F. So, therefore, you will get this term right.

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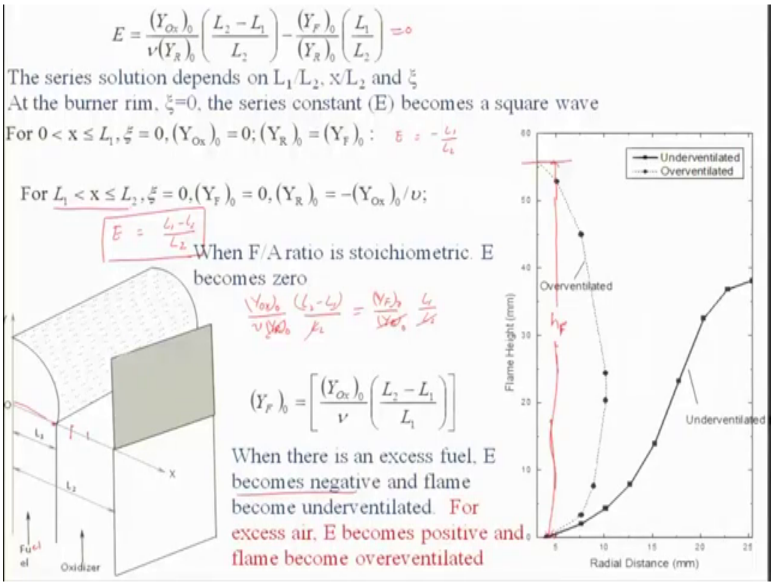

So, therefore, E is coming that way right. Let us now look at therefore, between your oxidizer side the L 2 is between L 1 and L 2 right.

In this case, Y F will be what? Basically 0, yes or no? In the oxidizer side there will not be any fuel that is the assumption. That means, as soon as fuel or the oxidizer will come in contact or it will reach the flame surface, it will just consumed their mass fraction of the fuel and oxidizer on the surface of the flame will be 0. It cannot cross each other across the plane. So, therefore, whereas, Y R naught we know that Y x by nu naught.

So, if I look at that in this term particularly, if I look at in this region right, and this is your oxidizer region right, in this region I am considering in the surface. So, that will be Y F will be 0, this term will be 0. And Y R naught is nothing but your in this place, what I will be writing? I will be writing Y oxidizer by nu.

So, this will of this and this will cancel it out, this will cancel it out. So, what you will get is basically, you will get E is equal to L 1 L 2 divided by L 2, no sorry, and so, by that you can really get very easily in terms of this dimensions and then E evaluating E. So, if you look at when the fuel air ratio is stoichiometric E becomes 0 right. If you look at the other terms, and if it is E 0 right, what you will get? If E this is equal to if you look at this is the equal to 0 for because fuel air stoichiometric so, this is equal to 0.

So, if it is equal to 0 what I will write? I will write down this Y ox o by nu Y R L 2 minus L 1; L 2 minus is equal to Y F by Y R naught L 1 by L 2. So, if you look at this will cancel it out, this will cancel it out, and you will get basically Y F naught is equal to Y ox right nu that is L 2 minus L 1 divided by L 1 right. So, this is the expression one can get. And which is basically you are looking at how it will be the flame surface. You can solve these equations right using this various condition, and you are basically working on this side on the oxidize side on the flame side, then for a particular because you will be knowing this L 2, you will be knowing this L 1, you will be know this nu and all these things all those things you will be knowing right.

So, by using this series, you one can get a various points you will be getting at different radial directions, you will see and then zeta will be changing like that x and that series solution has to be obtained right. And then you will get a control like that and this is a condition which is over ventilated right. And there is a condition where it will be under ventilated. Under ventilated means, basically when there is an excess well right, what will happen E? E becomes negative right, E becomes negative and flame becomes under ventilated. And for excess air E becomes positive and the flame becomes under ventilated.

Once you calculate this thing you can find out what will be the flame height. Because flame height will be in this case, this is the reason what you can say as a flame height h f right. So, this is the reason what you can get, and by this you can really find out flame line. But limitation of this model is that it is a very, very rudimentary, and it requires a mathematical this thing and then thin flame approximation need not to be right radiation is not taken care.

However, it gives some experimental data train, but not exact value. And there are several people who have tried to modify it getting analytical solution. And that is a one what we will be discussing is basically the roper later on modified this thing added some terms, and which is quite elaborate I will not be discussing.

But however, this is about the 2D flame. But there will be some axisymmetric flame which generally we use for that if you want to do, you will have to get basically the similar equations. But it will be the against second order equation, you will be getting using the same similar analysis. But you will have to use basically Bessel functions, instead of series solution you will get that.

So, if you want to see it is given in the book of a principle of combustion by KK Co you can see that if want to see. But I will not be covering in the lecture. I think with this we will stop over. In the next lecture we will be discussing about the ropers analysis basically, and also some experimental result, and how to estimate the flame length or the flame height of a jet diffusion flame.

Thank you very much.