

**Fundamentals of Combustion (Part 2)**  
**Dr. D. P. Mishra**  
**Department of Aerospace Engineering**  
**Indian Institute of Technology, Kanpur**

**Lecture – 60**  
**Phenomenological Analysis of a Laminar Jet Diffusion Flame**

Let us start this lecture with a third process from Henry Ford. When I cannot handle events I left them handle themselves. So, if you look at in the last lecture we basically discussed about the diffusion flame various kinds of diffusion flame, then we initiated a discussion on the jet diffusion flame. In the process we looked at the various structures of the jet diffusion flame. And today what will be doing we will be basically find a curve and expression for flame height using the phenomenological analysis.

So, we are making this following assumption for this that we are saying this is the flow is laminar steady in V c flow, right and that is the things we are talking about.

(Refer Slide Time: 01:07)

### Phenomenological Analysis

**Assumptions:**

- (i) Laminar steady inviscid flow
- (ii) Flame is diffusion controlled
- (iii)  $V_z$  remains constant along  $h_F$
- (iv) Buoyancy effect is neglected
- (v) Constant thermo-physical properties

The **flame height** is defined as the point along the axis on which inter diffusion of the fuel and oxidizer is reached for the first time.

The time,  $t$  required for oxidizer element to reach axis of the jet is given by

$$t = h_F / V_z \quad \text{--- (1)}$$

By the Einstein diffusion equation, we can have

Molecular Diffusivity between Fuel and oxidizer

$$\bar{x}^2 = 2 D_{12} t$$

Average Square Displacement due to molecular Diffusion

The expression for flame height is given by

$$h_F = \frac{V_z R^2}{2 D_{12}} = \frac{V_z \pi R^2}{2 D_{12} \pi} = \frac{\dot{V}}{2 D_{12} \pi}$$

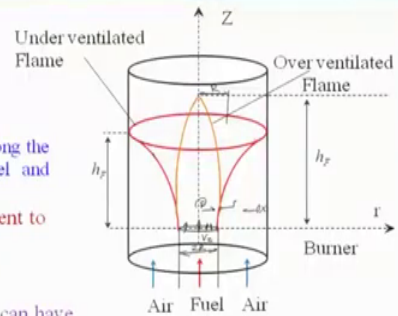
Volume flow rate

$$h_F = \frac{\beta_F \dot{V}}{2 \pi \rho_F D_{12}} = \frac{m_F}{2 \pi \rho_F D_{12}} = \frac{\dot{m}_F}{2 \pi (\rho_F D_{12})}$$

$\rho_F / c_p$  would not change much.  $\Rightarrow \rho_F / c_p = \text{const}$

$Le = 1$   
 $\Rightarrow \rho_F = \rho$   
 $\Rightarrow \rho_F / c_p = \rho / c_p = \text{const}$

Flame height is independent of burner diameter for a particular volume flow rate



And then the flame is diffusion controlled, that means, the flame will be basically governed by the how fast the fuel is diffuse into the Vincent atmosphere in the oxidizer and that will govern not the reaction rate. And the  $V_z$  remains constant along the  $h_F$ , which is not true that means, the actual velocity remains constant along the flame height which will not be true, but however we are doing it for the simplicity reason.

And Buoyancy effect is neglected because if we look at this is the quotient atmosphere and the some amount of fuel is being burnt and lot of amount of heat being released, as the result there will be buoyancy effect apart from the momentum which will be coming due to the jet.

That is being neglected, and constant thermo physical properties which will be using most of our analysis not only for this otherwise we cannot really handle. Keep in mind that in combustion process generally the thermo physical properties do change with temperature, because the temperature change in case of a combustion is quite high. You cannot neglect it, but we are doing for a simplicity reason.

So, we will have to now define a flame height right, though flame height can be defined as a point along the axis right on which inter diffusion of fuel and oxidizer a reach on the flame surface for the first time. Of course, this definition is very limited. Let us say this is the I have taken a little different kind of thing configuration and if you look at this definition what is says if there is a only fuel  $z$ , and you will get a flame this is known as over ventilated flame right and this flame is basically what is saying, it is the height  $h_F$  from here this distance to this distance right. That is known as flame height.

And what is saying? It is because if this is the fuel, fuel will be diffusing into that fuel and oxidizer will be coming that and it is not at the centre line number 1. And then the flame will take place it will go as on at the centre line it will be reach such that the fuel and oxidizer will be at this height will be coming contact on this axis, right, and that is known as flame height.

And this is a valid statement only for the over ventilated flame. If there is a under ventilated flame this is the flame you know this flame is the under ventilated flame right, where the air is very very less amount right and then fuel is much higher such that flame will be getting into that, but that is very very rarely you will get, unless you control natural flame will be always over ventilated flames are you getting natural jet diffusion flame.

So, therefore, you will be using this definition  $h_F$ . And that what we need to know is basically the time required for oxidizer to reach the axis of the  $Z$ . If you look at this is my  $Z$  axis this is my  $r$  axis the oxidizer is coming over here right oxidizer and fuel is coming over this due to diffusion. And it will be coming such that it will be reaching the

axis of the Z because the time, what is the time required that will have to look at. And this is coming due to what? Due to only the diffusion only the diffusion that means, the momentum we are not considering right is only it is as if it is diffusing and then it is coming.

So, then we can say that this time whatever is required is nothing, but your  $h_F$  divided by  $V_z$  that is the one way of looking at because it is having certain momentum, right, and which is having a velocity  $V_z$  in at this point this we are saying it is a uniform velocity and that is  $V_z$ , right. And the if I already assume that  $V_z$  is not changing along the Z direction along the flame. So, therefore, we can say that time required for the fuel to reach this height will be basically governed by the  $V_z$  that means,  $h_F$  divide by  $V_z$  is nothing but your  $t$  time required. And that is the fuel to reach, but the oxidizer to reach oxidizer to reach will be by the diffusion, right.

So, by this Einstein diffusion equation we can have this  $\overline{r^2}$  is equal to  $2 D t$ ,  $t$  is the again time required for that is and  $D$  is the diffusivity. This is the molecular diffusivity rate and  $\overline{r^2}$  is basically average and this is average square displacement, due to molecular diffusion. And I will tell you this is basically diffusive between fuel and oxidizer right, one to one can save fuel.

So, then I can write down if I will just use maybe this is equation 1 and this is equation 2 if I combined this together what I can write down is  $h_F$  is nothing, but your  $t$  into  $V_z$ , and  $t$  is nothing but your  $\overline{r^2}$  divided by  $2 D$  and this  $\overline{r^2}$  the distance, which is covered if you look at is basically this is your  $2 r$ , right. So that means, from here to this distance is nothing, but your  $r$ . So, this  $\overline{r^2}$  is nothing your capital R square right in this case. So, therefore, you will get  $h_F$  is equal to  $V_z r^2$  by  $2 D$ .

And if you look at that I can say that there is a  $h_F$  is basically depends on one it depends on the  $V_z$  that means, if  $V_z$  is higher then flame height will be higher and it will be dependent also the diameter. If diameter is more right or the radius or the diameter like it will be also dependent on that and diffusivity is higher the  $h_F$  will be smaller right, if it is a lower then lam length will be lower. So, that you can think of, but let me look at it some other aspect of that I can write down that  $V_z \pi R^2$  right I can write down divided by  $2 D$  by  $\pi$ . I can this is portion this is nothing, but your volumetric flow

rate,  $V_z$  is a velocity into area that will give the volumetric flow rate by  $2 D \dot{V}$  right, and this is basically volumetric flow rate.

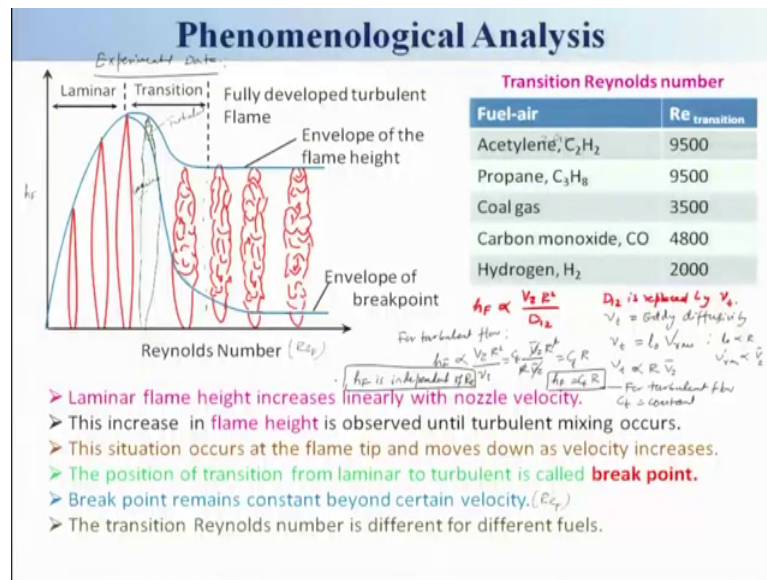
And if I write down this one is basically in terms of density you know like you can say mass flow rate, I can say write down  $h_F$  is basically I can say a density of the fuel, I can say this one because this is the fuel flow rate right  $\dot{m}_f$  into  $\dot{V}$  divided by  $2 \pi \rho D \dot{V}$ . And this is nothing, but your mass flow rate of fuel divided by  $2 \pi \rho \dot{V}$ .

What is saying is that basically it is independent of the diameter rather the  $h_F$  is a function of it is a function of flow rate and it is independent of burner diameter for a particular volumetric flow rate or the mass flow rate right. And keep in mind that if I will take  $Le$  is equal to 1, that means,  $\alpha$  is equal to  $D K_g / \rho c_p$  is equal to  $D$ .

So, I can write down basically  $K_g / \rho c_p$  is equal to  $D$  in place of this I can write down  $\dot{m}_f / 2 \pi K_g$  I can say this is basically  $K_g$  I can write down  $K_g$  gas by  $c_p$ , right. So, this you can also see that these are the properties which is basically do not change much this  $K_g$ , this  $K_g / \rho c_p$  will not change much with at the various temperature kind of things will not change much. So, therefore,  $h_F$  is basically proportional to the flow rate of the fuel.

And keep in mind that we are not talking about the coaxial  $Z$  we are talking about the single  $Z$  in this case right. And the  $h_F$  is as I told for a fixed value of the mass flow rate and  $h_F$  is independent of the pressure right ok, because if I say this thing because  $K_g$  and  $c_p$  will not be depending much on with pressure right. Because whereas, the diffusivity is changing so therefore, you will find  $h_F$  is not really very much depending too much on the pressure flame length.

(Refer Slide Time: 12:45)



So, let us look at how this will be having this is the  $h_f$  this is from the experimental data and you can see that I am in Reynolds number it is being put because of fact that you can see this  $Re$  is equal to  $\rho V z D$  by  $\mu$ .  $D$  in this case is  $2R$ , diameter of the diameter of tube. And in this case this is basically fuel and this is fuel. Are you getting? This is the basically I can save fuel number of  $Re F$ .

What you can see as  $h_f$  is basically increases with increase in Reynolds number right, that we have already seen that means, a  $h_f$  we have already seen it is basically depends on flow rate that is  $h_f$  is equal to  $V z$  by  $R^2$  to  $D^{1/2}$ . So,  $h_f$  by the goes by the  $V z$  so, it is increasing, right. And that is true for of course, for laminar flame height the laminar raise in it is true. But in the transition regime what is happening here you can observe that the flame length is basically will be start decreasing, right.

But however, the this portion this line is indicating is basically the onset of turbulence. Like if I draw here a, which is not there but if I draw a flame length here, something like this. Then what happened? This region is turbulent may little bit transition means in a combination of laminar turbulent kind of a or onset of turbulent right. And here then what is happened the flame length is remaining almost constant with the increase a laminar particularly for beyond this transition regime, right, beyond this transition regime it is a remaining almost constant.

But however, there will be also the envelope breaking point the breaking point means where this the flame will be turbulent in nature right, there will be the flame brass will becoming you know as I shown you in the premise flame it will be randomly moving the flame surfaces right, and that will be coming here.

As you increase this Reynolds number it is a goes on decreasing this point like for example, it started doing here you only in the tips right, but in the transition, but as it is goes then what will happen it will be goes on decreasing which I have not shown another flame may be here is a very very less, and it will be goes on changing and it is a remaining constant almost because only is a smaller portion nearby the Z inlet of the rfs of the Z, right. That there will be laminar rest of the thing will be turbulent.

So, the this increase in flame height is observed until the turbulent mixing occurs, and this situation occurs at the flame tip because this will start the turbulent you know will be start occurring to a dominant a play a role particular in flame t moves downward as the velocity increases that is moving downwards, right. And the position of transition from laminar turbulent is called the break points right, where the laminar the because this is laminar right laminar and these are the turbulent right. And that position will like this position, this is the one position this is the point these are the break point.

And break point remain constant beyond certain velocity or beyond certain Reynolds number right that is basically Reynolds number right kind of things. So, and transition Reynolds number is different for different fuels that you should keep in mind it is not same for all. Like your pi flow if you look at, pi flow generally turbulent means are the that is Reynolds number will be what?

Student: (Refer Time: 17:45).

What will be Reynolds number?

Student: (Refer Time: 17:50) 2000.

2200, right. But whereas, in this case the transition this thing will be will be changing, right. So, if you look at this fuel air if you look at transition Reynolds number for acetylene is basically 9500 and propane it is 9500 and whereas, the coal gas 3500. Basically if you look at coal gas sometimes it is known as a its a town gas earlier days

people who were using like using the coal convert the solid coal into gas and then using in the town particular in the England, other places not in India 3500. Carbon monoxide is for 4800, hydrogen is 2000 you know it is a near to a pi flow. So, this is the kind of things what it would be everything.

And keep in mind this fully developed turbulent flame is basically very noisy, and also less suited as compared to the laminar flames. Laminar flames are more suitable. But if you look at let us look at little analysis and to show that whether these you can capture this phenomena of the flame height is remaining constant. So, if you look at the  $h_F$  right, is we have already seen the  $h_F$  is basically  $V_z r^2$  by diffusivity, and the diffusivity  $D$  like if you it can be use, replace. Because we have seen that  $h_F$  in case of laminar flame we have seen is proportional to  $V_z R^2$  by  $2$  diffusivity right,  $D$ . This is case of laminar right laminar flame.

But we will be using similar thing and then trying to replace this diffusivity, diffusivity is replaced by  $\nu_t$ ,  $\nu_t$  is the turbulent flow right.  $\nu_t$  means the it is basically eddy diffusivity and this is known as the eddy diffusivity, right, basically eddy diffusivity. And this eddy diffusivity we know is basically is equal to length scale like integral length scale and  $V_{rms}$ , right.

Student: (Refer Time: 21:02).

And which is proportional to you can say that this length scale will be proportional to what  $r$  right, I can say the else is proportional to  $r$  right and  $V_{rms}$  is proportional to  $V_z$  average, right. Now, if you look at I can write down this as basically that means, is  $V_t$  is proportional to  $r$  and  $V_z$ . So,  $h_F$  I can write down for turbulent flow,  $h_F$  can write down as  $V_z R^2$  by  $\nu_t$  is equal to I can say this is constant right is equal to this is by  $r V_z$  and this is I can say this is a constant  $c_t$  and  $V_z$  by  $R^2$ , right.

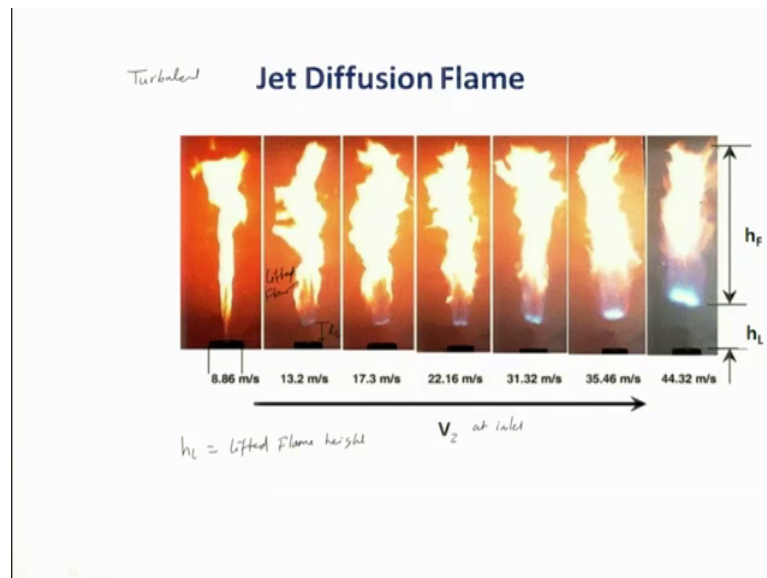
So, this will cancel it out I can say this is average, this is cancel it out and you can find out that this is nothing but  $R$  that means,  $h_F$  I can write down  $h_F$  is equal to  $c_t$  by  $R$  for turbulent flow right for length flow. That means,  $h_F$  is not really depending on the velocity it is because if the diameter is remaining constant for the experiment you can see that  $h_F$  is remaining constant that means, it is not dependent on the Reynolds number. What I am trying to say these are the experimental data right there all data experimental data, right and by using a very simple phenomenological analysis you can get a you

know you can explain this variation very using very simple phenomenological analysis that is the thing you must appreciation, ok, fine.

Student: Then what is  $c_t$ ?

$C_t$  is a constant right, because this is a, I have taking the proportional right. So, naturally you will have to take a constant  $c_t$  is basically a constant. It can be any constant which will be from you want match experimental value and put some constant and then try to match him that way. What I am trying to say that  $h_F$  is independent of Reynolds number that is the point I am wring  $h_F$  is independent of Reynolds number that is the point what I am trying to say.

(Refer Slide Time: 24:23)



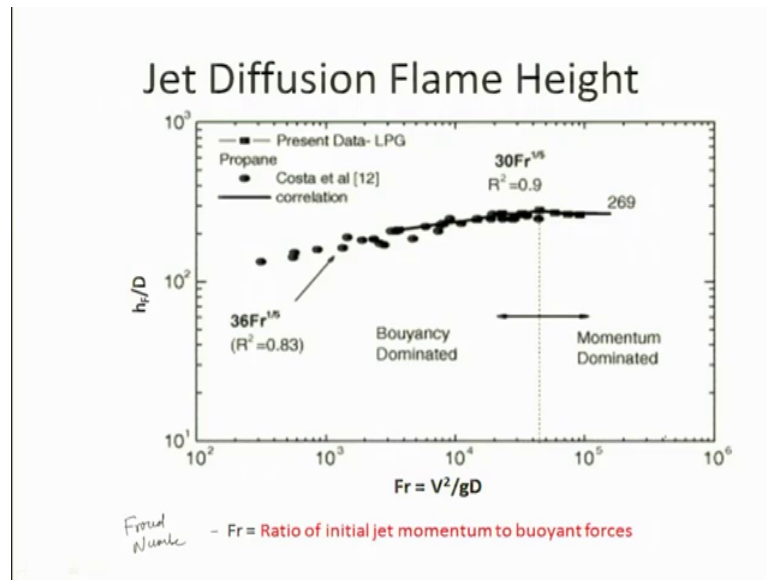
So, let us look at like what are the jet diffusion flame what are the things we can see, like if it is the flame here which is a turbulent of person jet diffusion flame. This is turbulent rigid this is the  $V_z$  velocity; I can say  $V_z$  which is increasing, right.  $V_z$  means at inlet at inlet.

So, you will find a flame, and when you increase this flame what will happen? Flame is lifted this is the lifted flame, and this height is known as  $h_L$  lifted height right, this is lifted  $h_L$  lifted height, flame height and this is the flame height, ok. As your increasing this may oscillate go up and then you know the blue color regions are an increasing and when you go beyond certain velocity what will happen? Flame will be very very small it



will go up and then after that glow out will occur ok. That means, this portion will be reduced and some kind of thing it is I have not shown we have conducted some experiment earlier. So, that is the things what I can say.

(Refer Slide Time: 25:47)



Now, if you put this  $h_f/D$  and put this data in terms of Froude number Froude number is basically the ratio of initial Z moment and to buoyancy forces. And this is Froude number which is non dimensional number, you can see these are the experimental data is being plot and which is varying  $h_f/D$  is varying with the Froude number and after that it is remaining constant.

And these portion where its remaining constant is known as momentum dominated that means, here it is not affected by the buoyancy. And this will occur only when the  $V$  is very high right, and rather the Froude number will be very high and  $V$  will be very high and  $D$  will be small right kind of things that, where momentum dominated will be flame will be thing that flame height will not be changing much even though you are increasing the Froude number or the velocity, right.

So, with these will stop over and in the next lecture will be discussing about basically how to analyze the jet diffusion flame then little more rigorous way, ok. Well.

Thank you very much.