Fundamentals of Combustion (Part 2)
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Lecture – 42
Analysis of One Dimensional Combustion Wave

Let us start this lecture with the thought process. We have taken birth in this earth to use the inner fire for the benefits of mankind without getting burned by it. So, if you recall in the last lecture, we initiates the discussion about the one dimensional combustion wave and where we have discussed, found out the difference between the detonation and deflagration. And, today we will continue how to analyze the one dimensional combustion wave.

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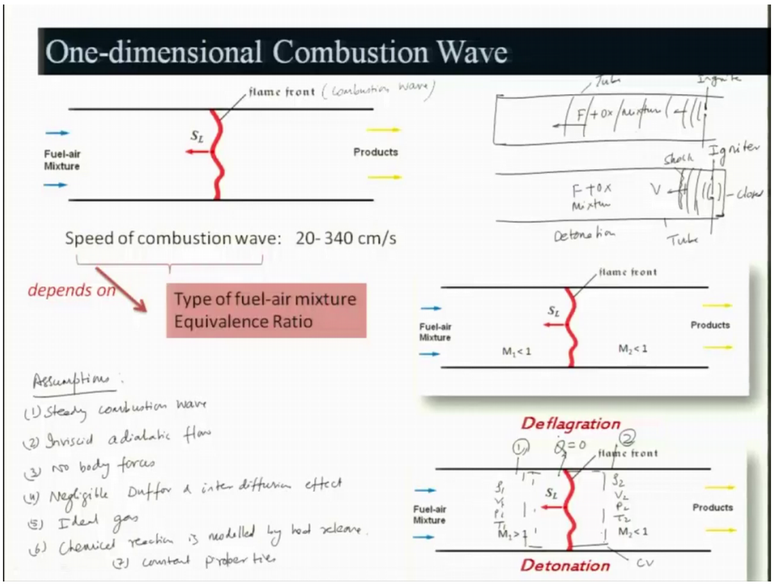

So, as I told that combustion wave will be having certain speed and with which it will be travelling towards the fuel air mixtures and it depends on the type of fuel air mixtures and also the equivalence ratio. And, in case of deflagration we have seen that if it is the speed of the combustion wave is subsonic we call it as a deflagration and if it is greater than the sonic speed then we call it as a detonation.

And, question arises practically when there will be deflagration when there will be detonation. For example, if I take a tube mixed with the fuel air mixtures right, this is fuel and oxidiser mixtures right, it is being filled. If I ignite it right as I told layer that if this is the igniters what will happen this kernel, the ignition kernel will be moving towards that then will it be a detonation or will it be a deflagration right, because it will be moving with a subsonic speed.

So, therefore, it will be basically a deflagration. Now, what will happen if I close this portion, this is the closed, both are open tube, this is a tube right, tube is open both the side. If I close and if this a filled with fuel oxidizer and I will ignite it what will happen? We leave the flame will be moving with the same speed, like combustion wave will be have a certain velocity or certain speed, and it will be always moving towards the fuel and oxidizer. Therefore, we are calling it as a speed, because the direction is known right. So, we can call it as a combustion wave speed right, will it be in the same right.

Student: (Refer Time: 03:24).

It will be.

Student: (Refer Time: 03:27).

Changing, but will it be, why it will be changing?

Student: (Refer Time: 03:31).

Student: Fuel air mixture (Refer Time: 03:36).

Fuel air mix this is in the beginning, it is a 0 and there is no movement as such. So, it would not be changing unless it reaches here nearby, it may change. Because, the pressure will be build up in this region when the flame propagates, it will be propagating [FL] will be moving right and then towards that there will be a some pressure will develop.

Right will it pressure will develop? No, [FL] why it will develop because I have told.

Student: (Refer Time: 04:10).

Compression, where it will be, it is a moving, it will not really compression will be taking place, but; however, I will do other way around. What I will do? let us say I will close this side and this is fuel and oxidiser, keep in mind that this is stationary mixed there is no movement as such I am not talking about that of the movement of the a proper of the movement of the fuel oxidizer mixture. It is just a stationary mixture then what will happen I will ignite here. This is your igniter right, what will happen? This is the closed one closed, tube is closed this tube is closed right, what will happen because this kernel will come.

And this also will go this side there might be some mixture right and then what will happen, this will be and this will be burnt and this will be unburnt right. So that means, the temperature will be very higher as this is closed the mix, the vertical expansion will be taking place of the gas because temperature is higher, if the temperature is higher and expanding, but it is closed. So, what will happen? It will be trying to get compressed, yes or no right.

So, if it is there then what will happen? It will be trying to basically get compressed, see you know the adiabatic temperature you have seen, the mixture let us say 300 if it is fuel air methane air what will be the temperature 2200 Kelvin into divided by 300 something around more than 6 times, it is changing right. So, therefore, there will be change in the pressure and then pressure will be acting like a piston as if there is a piston which is move in the web. So, it will be moving at a higher velocity and it happens to be the higher than the speed of sound right what will happen, detonation will occur.

That means, there will be deflagration to detonation right will be, which will be taking place, but this will occur, why does it occur? It occurs because what will happen there will be a compression. If there is a compression, right there will be shock formation and once this is the shock will be fed by what by the chemical reactions right, the shock will be here somewhere and the heat is taking place here right, there will be shock once it is detonation right then that will be feeding the shock, are you getting the velocity will be very high and then it will be moving it will lead to the detonation.

So, therefore, of course, it is a quite a complex process, but keep in mind that whenever the detonation is there, there will be shock and it will be shock is fed by what fed by the combustion or the heat release right, which will be much higher speed than that of the whatever the natural or the shock will be there right. So, now we will be considering and as a one dimensional combustion wave, but actually in real situation it need not to be one dimensional and let us say that we will say this is as a station 1 and I am considering as a this is station two kind of things which will be discussing and we will make some assumption for analyzing this thing. What are those, one consider that we are considering as a steady combustion wave, but in real situation it need not to be because it is coming from the deflagration to detonation.

So, therefore, it not, but we are considering it is already developed only it is moving with a detonation velocity or the speed. So, therefore, we are considering as a steady. The assumption are steady combustion wave keep in mind that when we are analyzing it, we are not considering basically whether it is a deflagration or detonation, we are considering it as a combustion wave; that means, it may be a detonation or it may be deflagration that you keep in mind.

And we are also considering that a inviscid adiabatic flow; that means, no heat although heat being released no heat is going out from here; that means, Q is 0. But however, heat being generated inside right, due to what due to chemical reaction right that we are, but no heat is going from the system if I consider this as a system you know, if I consider this as a control volume from this no heat is going out.

And it is inviscid; that means, there is no viscous effect, viscous is not playing an important role and three is the no body forces either you look to magnetive or gravitational no forces are there right. But however, you know when the deflagration to detonation, which will be occurring in the supernova or some other places it will be quite complex you know supernova, cosmic phenomena right, there also people are saying it is occurring because of deflagration to detonation right. So, but here we are not considering those body forces and the four is negligible Duffor and inter diffusion effect we are not considering, we may not consider the basically expressive equation to it.

And the ideal gas right, we are considering the ideal gas, the gas need not to be idea right and 6 is the chemical reaction is being modelled by the just heat release right and then you know we are not considering the species equation because it just you were adding some heat, are you getting instead of chemical reactions and another thing we are also assuming as a constant properties, which is not true; that means, you know c p is a function of temperature right. So, also the other properties which we are saying is the remaining constants.

So, we are not rather we are using the same you know gamma values c p by c we will be using, but which is not true because the across the combustion wave there will be rise in temperature, change in density, change in pressure, all those things will be taking place we will be not considering those into account ok. So, with this assumption now what we will have to do we will have to basically you know invoke the conservation equation like continuity.

What it would be continuity; that means, for this one dimensional what we call combustion wave between the station 1 and station 2 if you look at see this is your station 1 and this station 2, we are considering in between right. So, what it would be if I say the rho 1 if I say this is basically rho 1 V 1 P 1 T 1 right rho 2 V 2 P 2 T 2 right then we can relate those properties. Rho 1 is the density, V is the velocity, P is the pressure, T is the temperature right.

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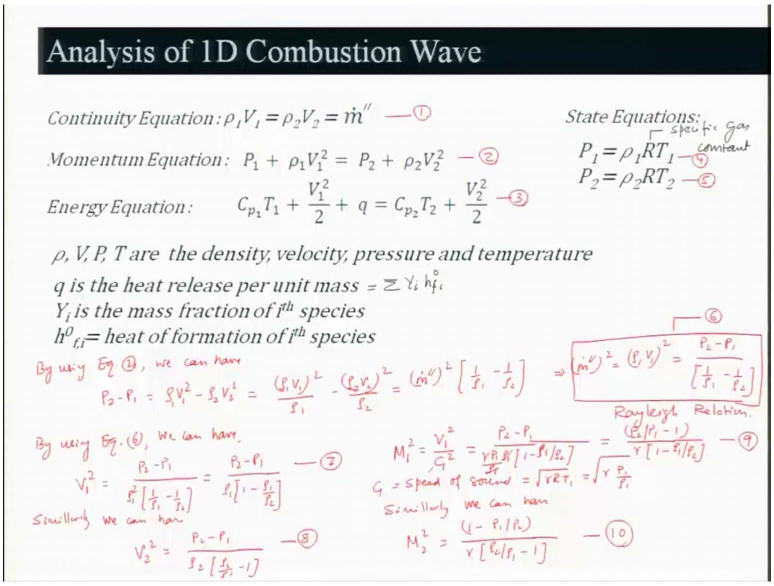

I can write down then as the rho 1 V 1 is equal to rho 2 V 2 is equal to mass flux right rate, this is mass flux rate right from the momentum equation if you look at what it would be that will be nothing, but P 1 plus rho 1 V 1 square is equal to P 2 plus rho 2 V 2 square because we are not considering the body forces and it is inviscid. So, therefore, those terms are not coming, which is a very simple and this is similar to what you might have what you call use for analyzing a normal software keep in mind that here we are considering as a combustion wave right, which is one dimensional in nature. So, this type of equation you might have used there. So, energy equation what it would be it will be for ideal gas law.

So, therefore, C p 1 T 1 plus V 1 square divided by 2 plus q, q is nothing, but your heat release rate and which is due to the combustion; however, in this case we are saying that it is substituted by the heat. But however, we can use also some for calculation purposes we can use that as a from the enthalpy balance kind of thing is equal to C p 2 T 2 plus V 2 square divided by 2 right, this is a kinetic term. So, beside this as it is a compressible flow, so we will have to consider equation of states also for station 1 that will be p one is equal to rho 1 RT 1 and for station 2 P 2 is equal to rho 2 RT 2 right, R is your specific gas constant right, this is your specific gas constant.

So, q is the heat release rate per unit mass and I can write down this as Y I h I f I naught right and where Y I is the mass fraction of I species and h f I naught is basically heat of formation of higher species right. I can say this is as a equation 1, this I can consider as a equation 2, this is equation 3, I can consider this as a 4 and this is equation 5. So, from equation 2, I can write down or by using equation 2, we can have P 2 minus P 1 is equal to rho V 1 square minus rho 2 V 2 square. I can write down as rho 1 this will be 1 rho 1 V 1 whole square divided by rho 1 minus rho 2 V 2 whole square divided by rho 2, which is nothing, but I can write down by using continuity equation, this is the equation rho 1 V 1 is equal to rho 2 V 2 then I can write down as mass flux square equal to 1 by rho 1 minus 1 by rho 2 and this equation is basically you can consider this as a Rayleigh equation right.

I can write down this as, I can write down rho 1 V 1 square this is also equal to rho 2 V 2 square is nothing, but P 2 minus P 1 divided by 1 by rho 1 minus rho 2 and this equation is basically known as a Rayleigh relation. I will consider this as let me write down here as a 6 equation number 6 and from by using this equation 6, I can basically derive the relationship for the velocity at station 1 and also the velocity of station 2, similarly I can also find out mach number at station 1 and mach number at station 2. So, let us that by using equation 1 6 V 1 square is nothing, but P 2 minus P 1 rho 1 square 1 by rho 1 minus rho 2 P 2 minus P 1 rho 1 I can write down as similarly we can have V 2 square as P 2 minus P 1 rho 2 rho 2 by rho 1 minus 1.

Now, we can express these as basically in terms of mach number, we know the mach number is nothing, but your M 1 square is nothing, but your V 1 square by C 1 square and C 1 square is the speed of sound result of C 1, which is nothing, but your root over gamma RT 1 is equal to root over gamma P 1 by rho 1 right. And I can write down substitute here I can find out this will be P 2 by P 1 divided by rho 1 and this will be I am you know let me write down here 1 minus rho 1 rho 1 divided by rho 2 into what it will be gamma P 1 divided by rho 1. So, this rho 1 will be cancel it out.

So, I will get as P 2 minus P 1 divided by it became because if I take P 1 here this will be P 2 by P 1 minus 1 gamma right into 1 minus rho 1 divided by rho 2. Similarly, I can this is basically equation 9, similarly we can have M 2 square mach number at station 2 square is nothing, but your 1 minus P 1 by P 2 gamma rho 2 by rho 1 minus 1.

So, now we know you know we can relate this pressure ratio and density ratio and gamma to the mach number right in other words mach number at the both the station can be expressed in terms of pressure ratio, density ratio, and gamma. Now we will be basically deriving expression for the heat release using the energy equation by using equation 3 we can have is nothing, but your C p T 2 minus T 1 plus V 2 square V 1 square divided by 2 right.

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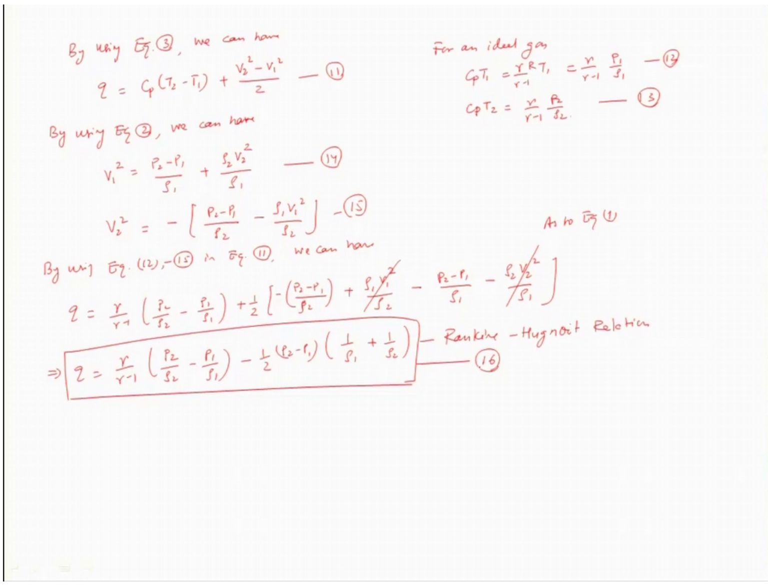

And if you look at the C p T 2 right what it would be can I not express in terms of pressure for an ideal gas we know that C p T 1 is nothing, but your what is C p gamma gamma minus 1 r T 1 right is nothing, but a gamma gamma minus 1 P 1 by rho 1 right yes or not similarly I can write down C p T T 2 is gamma gamma minus 1 P 2 by rho 2. And from equation let me say this is basically equation maybe ten it would be 11. So, this is 11 by using equation 2 we can have expression for V 2 right and V 1. We can have

V 1 square is nothing, but your P 2 minus P 1 rho 1 plus rho 2 V 2 square divided by rho 1 and similarly I can write down this as a basically 11 12 this is 13, this is 14.

So, V 2 square is nothing, but your minus V 2 minus P 1 rho 2 minus rho 1 V 1 square divided by rho 2, this is 15. By using equation 12 12 to 15 in equation 11, we can have q is equal to what you call gamma gamma minus 1 into P 2 by rho 2 minus P 1 by rho 1 then it will be plus half that is V 2 square right V 2 square will be nothing, but your P 2 minus P 1 by P 2 right minus plus rho 1 V 1 square divided by rho 2, and minus P 2 minus P 1 rho 1 minus rho 2 V 2 square by rho V 1 ok, keep in mind that this will be cancel it out, this will be cancel it, why as two equation one that is continuity right because that will cancel it out.

So, then finally, what you will get you will get basically q is equal to gamma gamma minus 1 P 2 by rho 2 minus P 1 by rho 1 it will be minus half I will take P 2 minus P 1 as a common P 2 minus P 1 I will get 1 by rho 1 plus 1 by rho 2. This is basically known as the Rankine-Hugoniot relation, which is one of the most important relationships in the explosion and the flame right because flame can lead to explosion right.

So, this is a very important relationship you should we will be using it and this is 16 ah, with this we will stop over here and we will discuss in the next lecture.