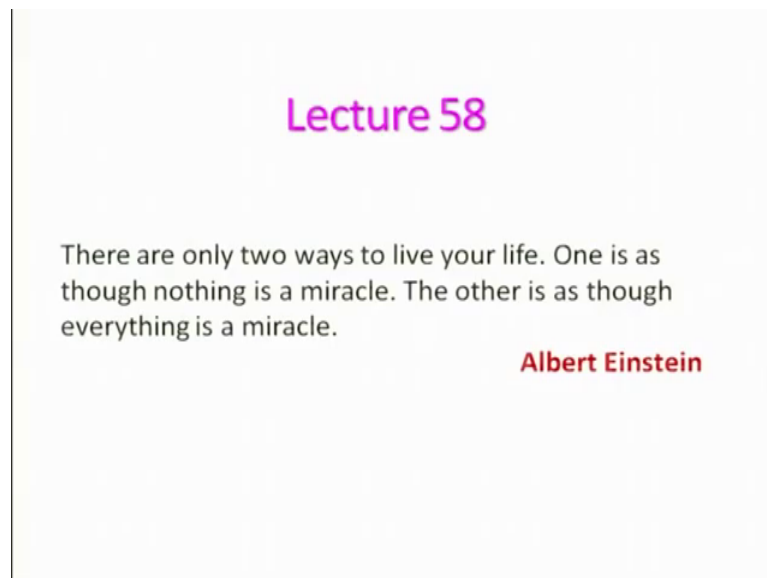


**Fundamentals of Combustion (Part 2)**  
**Dr. D. P. Mishra**  
**Department of Aerospace Engineering**  
**Indian Institute of Technology, Kanpur**

**Lecture – 58**  
**Turbulent Burning Velocity and Premixed Flame Regimes**

Let us start this lecture with a thought process from Albert Einstein.

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Who says there are only two ways to live your life: one is as though nothing is a miracle that most of us think. Like other is as though everything is a miracle, that you enjoy the life right. And that is our scripture always talk about that right, but we do not follow it unfortunately and getting into depression and other problems right in modern time.

So, let us look at what we learnt in the last lecture. Basically, if you look at we looked at the various Regimes of Turbulent Flame Premix Flame. One is of course, the weak turbulence flame right, other is wrinkle laminar flame. The flame let in eddies and distributed reaction kind of things right we looked at the various kind of non dimensional parameters right Reynolds number and then Damkohler number and then root mean square velocity ratio and although all scale ratios right,  $l_{naught}$  by  $\Delta l$  k by  $\Delta l$  all those things we have looked at and looked at a Borghi diagram. And if you look at we basically discussed about the distributed reaction zone which occurs in the low Reynolds number and low Damkohler number.

But that is very difficult to establish; however, one can establish using a stirred reactor. Stirred means you will have to give some jets like various directions and put together and you do that. Now we will be looking at the turbulent burning velocity because, as I told earlier that laminar burning velocity is very fundamental to the premix plane why, because, the all other parameter like minimum ignition energy, quenching distance.

And then your flammability limits and other things will be related to the laminar burning including flame thickness. So, similarly turbulent burning velocity is very important.

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**Turbulent Burning Velocity** *108 - Turbulent Burning Velocity (10)*

- Depends on characteristics of fluid flow
- Velocity at which unburnt mixture enters the flame zone normal to flame.
- Difficult to measure velocity of unburnt gas near the turbulent flame

**Turbulent burning Velocity ( $S_T$ )**

**How to measure ( $S_T$ )?**  $S_T = \frac{\dot{m}}{\bar{A}\rho_u}$

$\dot{m}$  is the reactant flow rate  
 $\bar{A}$  is the time average flame surface area  
 $\rho_u$  is the density of unburnt gas

But it is not that easy how to define it, because of fact that turbulent burning velocity it depends on the characteristics of the fluid flame right. And velocity at which unburned mixture entered the flame zone normal to the flame, because the surface is so turbulated that which one you will take as surface each zone it will be corrugated or a different.

So, that is also very difficult and that is why it is very difficult to measure the velocity of unburned gas near the turbulent flame. And then we will be looking at some average sense right. That is why it is difficult to measure the way we did in the laminar burning velocity.

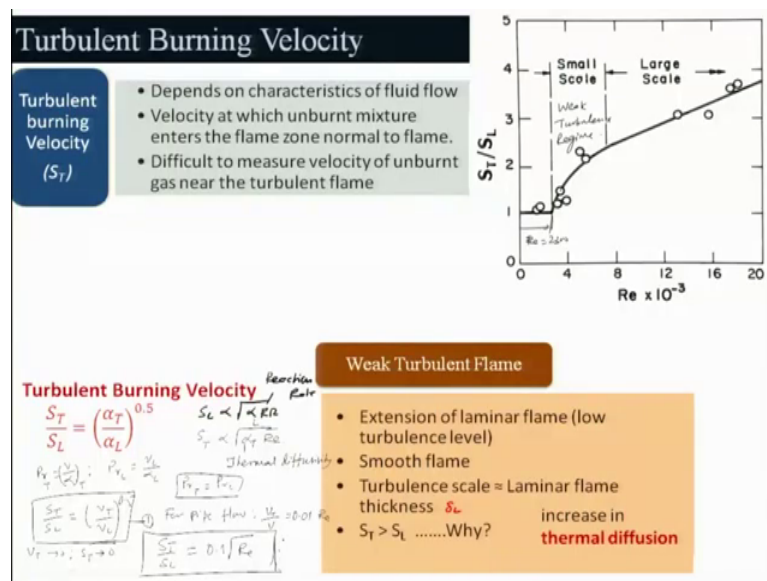
So, that is  $S_T$  is equal to what will be talking about is  $S_T$  is equal to  $\dot{m}$  is nothing, but reactant flow rate into  $\rho_u$  is the density of unburned mixture. And  $\bar{A}$  is the time average of flame surface right the flame surface will be changing right with respect

to time. So, if you look at the flame surface will be changing it need not to be so regular right, it will be this is your flame surface right, if I take one dimensional flame right frame surface, which is with respect to time right.

And this is at one instant of time there might be another instant of time which you can be like another one, this is with respect to time right.

So, at different instant of sorry this is; I am sorry this is with respect to x, but this is the two instant of time T is equal to T 1 and this is corresponding to T is equal to T 2 2 instant have that will be changing we have already seen. So, then you will have take average over the time and then you will consider that A dash right and which is need not to be really one dimensional in nature also.

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So therefore, we will define that way and we will be trying to get expression for that for that matter what will be doing will be considering the weak turbulent flame regimes right. Where the extension of laminar flame and that means, the flame will be basically similar to laminar, because that low turbulence will be there. The flame surface will be looking to be smooth it would not be affecting that much you know. And turbulent scale is order of laminar flame thickness that is delta L, right.

Of course the S T will be greater than S L, which is expected I am like we will see how it is right. And this is because of fact that with increase in thermal diffusion that we will

see, we have seen basically we know that  $S_L$  this is derived from very simple relationship. We have seen that  $S_L$  is basically proportional to  $\sqrt{\alpha R}$  what is that  $R$  is reaction rate right. You can say  $\dot{m}'''$  you know  $f_{avg}$  right. And similarly we can define right  $S_T$  is sorry  $S_T$  is proportional to  $\alpha T R$  right. If you will assume that the reaction rate is not affected by the turbulence ok, that means, reaction rate would not be will be remaining same for both the laminar and turbulent I can arrive at that  $S_T$  by  $S_L \alpha T$  divided by  $\alpha L$  ok. I am saying  $\alpha L$  means thermal  $\alpha$  is the thermal diffusivity, this is thermal diffusivity right.

And now, if you look at for the; if I take consider the Prandtl number as same for laminar and turbulent right which you need not to, but if I will consider that we know this Prandtl number is equal to  $\nu$  by  $\alpha$  right. And if I say it is for turbulent and this will be for turbulent and similarly, if I say this is laminar plate  $L$  by  $\alpha L$ .

And if I consider that Prandtl number is basically same; that means, Prandtl number  $T$  is equal to Prandtl number of laminar. Then what will happen? I can write down the  $S_T$  by  $S_L$  is basically about  $S_T$  by  $S_L$  is equal to I can write down  $\nu$  by  $T$  by  $\nu$  of  $\alpha^{0.5}$ . For a laminar for a  $\pi$  flow we know for  $\pi$  flow we know that is  $\nu T$  by  $\nu$  is equal to  $0.01 Re$  right.

So, then if I will say this is equation 1 and then substitute these values in equation 1. And I will get  $S_T$  by  $S_L$  is equal to  $0.1 \sqrt{R}$ . So, this is the relationship you can get basically from the phenomenological analysis. This is that turbulent burning velocity is proportional to the laminar burning velocity and depends on the Reynolds number right.

But keep in mind that this is having limitation right, what is the limitation? That is if this  $\alpha T$  or the  $\nu T$  is 0, there is no turbulence right. Then what will happen? That turbulent velocity will be 0 right, if Reynolds number is not there is a very very small right it would not or in other words if I look at this expression there will be some kind of a limitation right.

And that is a  $\nu T$  right tendencies toward 0  $S_T$  will be tending toward 0, right. Of course, when you put this Reynolds number, this would not be  $\rho V D$  by  $\nu$  like  $\nu$  till 0. Then this will be very very what you call Reynolds numbers very high it will be very kind of having no meaning. So, that has to be looked at it and that is the limitation of this model right. So, let us look at some experimental data  $S_T$  by  $S_L$  versus the Reynolds number.

You can see that we certain Reynolds number let us say that around may be 2300 right this is  $Re$  is equal to 2300.

That  $S_T$  and  $S_L$  is not changing. It is remaining almost constant, but afterwards it changes right with the power to the 0.5 we can say or around that it need not be 0.5 it may be something. So, it is a kind of thing, but once it is in this regime that we call it as a weak turbulence regime right this is weak turbulence regime right.

But here of course, it is different these are the experimental means it become almost linearly increasing this is a parabolic in say kind of thing right. So, this is a simple relation we say state that from this one can say that- this is the relationship what we have derived is basically is matching or similar to the experimental data, these are all experimental data right kind of things. So, by this one can really you know determine the laminar a turbulent burning velocity.

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### Wrinkled Laminar Flame

- Flamelets in flame surface propagate at laminar burning velocity
- Turbulence only causes wrinkling of flame

Turbulent burning velocity is given by,

$$\dot{m} = \rho_u \bar{A} S_T = \rho_u A_W S_L \Rightarrow \frac{S_T}{S_L} = \frac{A_W}{\bar{A}}$$

According to Damköhler, for constant laminar burning velocity


$$\frac{A_{Fl}}{\bar{A}} = \frac{V_u}{S_L}$$

Similarly for turbulent flame,

$$\frac{A_W}{\bar{A}} = \frac{V'_{rms}}{S_L}, \quad A_{Fl} = \bar{A} + A_W$$

$$\frac{S_T}{S_L} = \frac{\bar{A} + A_W}{\bar{A}} = \left(1 + \frac{V'_{rms}}{S_L}\right)$$

Damköhler Relation



According to Klimov,

$$\frac{S_T}{S_L} = 3.5 \left(\frac{V'_{rms}}{S_L}\right)^{0.7} \quad \text{when } \frac{V'_{rms}}{S_L} \gg 1$$

According to Calvin and William,

$$\frac{S_T}{S_L} = \left[0.5 \left\{1 + (1 + 8C V'_{rms})^2\right\}^{0.5}\right]^{1/2}$$

$$\frac{S_T}{S_L} = 1 + C \left(\frac{V'_{rms}}{S_L}\right)^2$$

For small values of  $V'_{rms}/S_L$

There is another regime where the wrinkle laminar flame which will be occurring. So, in this case, if your fuel plus oxidizer mixture is moving a certain velocity which is having a some fluctuating velocities. And it may affect the flame keep in mind that it will be distorting the flame right. It would not be changing in the; it will be not affecting the inside the flame thickness kind of thing so, and that we call it as a wrinkle laminar flame.

The flame let us in the flame surface propagates at a some laminar burning velocity. So, if you look at locally if you look at it will be moving with a laminar burning velocity. And turbulence causes only the wrinkling of the vessel corrugated make it corrugated by wrinkle right. The surface and that is of course, which will occur when the Reynolds number and Damkohler root over is very large values right. That means, the length scale is much larger than the laminar burning thickness flame thickness.

So, turbulent burning velocity if you look at is given by we can do a mass you know balance that is a  $\rho u A_{star}$  is the average velocity. I mean if you take all those things with respect to time and average you may get something which I have shown here like into  $S T$ ,  $S T$  with which this will be moving right. This is a just a hypothetical thing it is moving with  $S T$  right. And into  $\rho u$  and at the  $A W S L A W$  is the wrinkled area surface area right into  $S L$  which will be moving right. So, then from this I can cancel it out right and this is basically  $S T$  by  $S L$  is equal to  $A W$  by  $A_{average}$ . And according to Damkohler the constant laminar burning velocities right, can be as the  $A F l$  right.

It is the flame let surface area divided by  $A_{average}$  if I take this is average velocity right  $V u$  by  $S L V u$  is the velocity with which it will be the fluid will be moving right in this case. And in the similar way I can also find out turbulent velocities like a related into  $A W$  that is a wrinkled laminar divided by  $A_{average}$  is nothing, but because the wrinkling is occurring due to  $V_{dash rms}$  values right into  $SL$ .

So, we know that  $A W A$  this is  $A F l$  right I can write down here,  $A F l$  the flame surface is equal to basically  $A_{average}$  plus  $A W$ . This is due to this is wrinkling surface that will be there and this is the average which will be taking place right. At that will be at any instant of kind of things will be given.

So, therefore, I can write down  $ST$  by  $S L$  is nothing, but your  $A F l$  divided by  $A A_{dash}$  and this if you look at I can write down as is equal to  $A F l$  by  $A$  is equal to that. And  $A F l$  is nothing, but  $A_{average}$  plus  $A W$  divided  $A W$  is nothing but your  $1$  plus  $A W$  by  $A_{dash}$  and  $A W$  by  $A_{average}$  is nothing, but you have  $V_{dash rms}$  by  $S L$  right.

And this is relationship is basically known as the Damkohler relationship right. Which is and keep in mind that that when you know it will give you a relation linear relationship be there is  $rms$  is 0,  $ST$  divided by  $S L$  is equal to 1 right that is the thing because if it is there is no turbulence then naturally  $ST S L$  will be equal to 1 right.

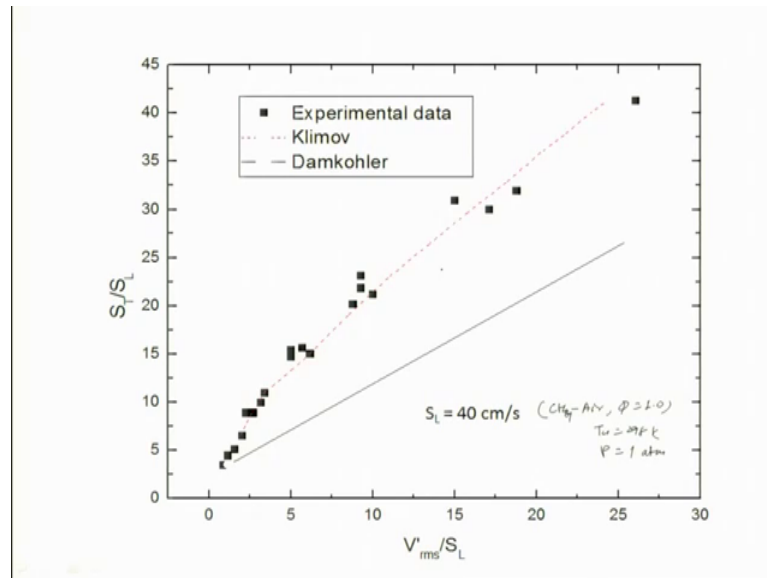
But however, this does not really predict the experimental data then Klimov as talked about taken the experimental data and find out  $S_T$  by  $S_L$  is equal  $3.5 V_{rms}$  does  $rms$  divided by  $S_L$  power to the 0.7.

Keep in mind that this is having a problem; problem is that when  $V_{rms}$  is 0; that means  $S_T$  by  $S_L$  will be 0. And which is not true because, if turbulence is not there right that does not mean the laminar burning velocity would not be there right. So, therefore, this is the limitation of this model right. So, that is the limitation of this model and then the Calvin and William has given another model right, which is  $S_T$  by  $S_L$  is will  $0.5 \sqrt{1 + 8 C V_{rms}^2}$ .

And then the whole square of root over then again root over of the entire thing right. And if you expand this thing by in the Taylor series you will get basically  $S_T$  by  $S_L$  right is equal to  $1 + C V_{rms}^2$  divided by  $S_L^2$  whole square right. That you will get and keep in mind that this is possible only when that for small values of  $V_{rms}$  divided by  $S_L$  this is the valid 1 right.

So, this; however, it is giving a some consistent value  $V_{rms}$  is 0, this  $S_T$  by  $S_L$  is nothing, but your equal to 1; that means, that if turbulence is not there laminar volume velocity and turbulent velocity is equal to same right that is a consistent. And keep in mind these are all little phenomenological analysis you may find several relationship for the ever linker laminar burning velocities. You can get several relationship for turbulent velocity and laminar velocity ratio in this regime in literature. However, I am discussing it is three of them.

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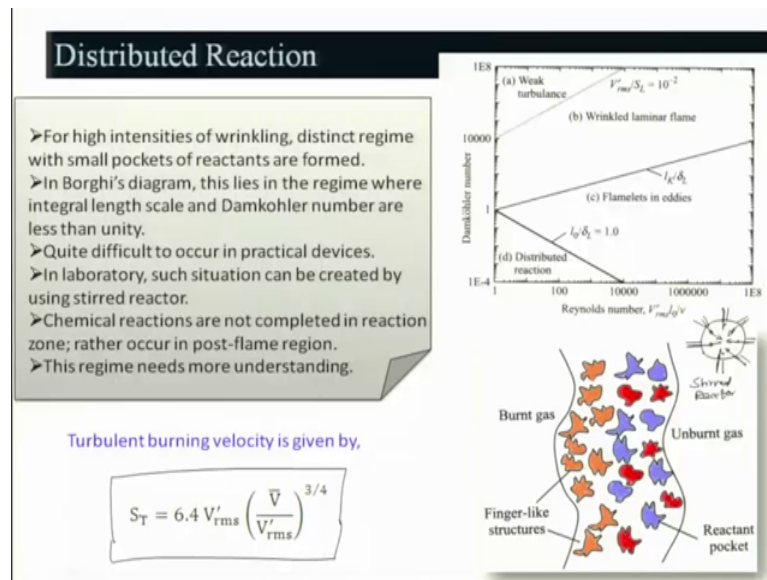


And if you look at the experimental data you will find for  $S_L$  is equal to 40 centimeter, which is corresponding to methane air and  $\phi$  is equal to 1 right  $T_u$  is 298 Kelvin  $P$  is equal to one atmospheric pressure right. You will find that this is changing according to Damkohler relationship changing with respect to  $V'$  dash divided by  $S_L$  linearly right.

And these are all experimental data's and a Klimov actually is data is matching well with the experimental data right. And unfortunately the Calvin and William does not match I have not included does not match with the experimental it may be little bit better than the Damkohler, but he still away from that right. And this is about you know how to handle the turbulent burning velocity for wrinkled laminar flame.



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Let us look at distributed reaction zone right kind of regime, where what will be happening that this is eddies which will be coming over here right. And this regime as I had told you earlier that it is happening where like a intensity high intensities are of the wrinkling is there. And distinct regime for with a small pockets of reactants are formed right. And you will get some kind of finger like structures and you will get where this is a very very turbulence label will be very high.

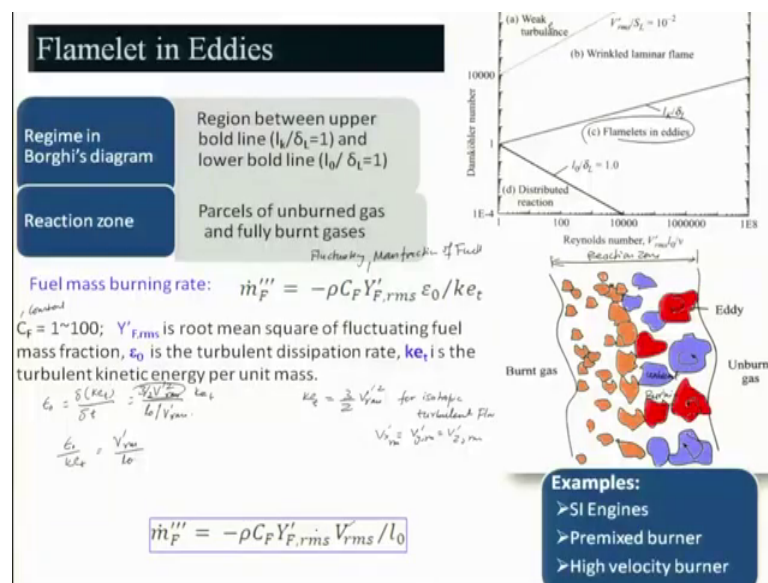
As I told that it will be occurring in it would not be occurring in the practical devices kind of things till now nobody has designed such a devices that where the turbulence level will be with the smaller Reynolds number the turbulence will be higher. But however, eddies will be smaller in size right. This is your it is possible of course, however, in laboratory scale the stirred reactor you can think of a reactor here right these are the jets right where the fuel can be coming over here, these are the jets with which the mixture will be coming right and these are impinging into a surface right. So, that what happened the intense mixing will be taking place and this is known as stirred reactor it will be mixing. You can think of putting a fan and then moving it at a very high velocity and be you are mixing you know that way are you getting? It is a very intense mixing is taking place.

So; that means, scale will be very very small right and this is a stirred reactor. Which will be taking place, but that is quite difficult and I have already told you that this is in the

Borghi diagram where Damkohler number less than unity. And Reynolds number also very very small right, and quite difficult to occur in practical device in laboratory such situation are created using a stirred reactor I have already discussed.

And chemical reactions are not completed in reaction zone; rather occur in the post flame region because the mixing is so high. That reaction would not be really taking place you know that is another problem with this kind of things right regime needs more understanding. However, people have given relationship  $St$  is equal to  $6.4 V_{rms}$  that  $V$  average divide  $V_{rms}$  3 powered way this is a semi empirical relation what is being used kind of things right, which is of course, meant only for the turbulent you know velocity and when it will be 0, this  $V_{rms}$ . Of course, this will be their own when a turbulent velocity kind of thing.

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So, let us look at flame let in eddies right and according to the Borghi's diagram, this region between the upper bold line  $l_0$  by  $\delta_L$  equal to 1 and  $l_0$  by  $\delta_L$  right. And this is the flame let eddies which will be taking place. And keep in mind that is here what will be happening that large eddies, large parcel of unburned gases which will be entering into this is your reaction zone, right this is your reaction zone.

And when it is entering into that it will be undergoing kind of a changing it is shape and size. It will be there will be like breaking down eddies and when its break down it will be trying to mix with the hot gases and a surface area will be increasing right. And as a

result the interfacial contact between the burnt and unburnt gases will be taking place. In some places it may extinguish; that means, there will be some kind of a flame which will be for example, there is a burning is taking place right here burning right.

And this is unburnt right, unburnt if it is unburnt then what will happen there will be some extinction region will be there. So, as a result the combustion is determined by the rate at which right the parcel of unburnt gases are broken into smaller right a smaller one caused by the eddies. And, that means the burning rate is governed by the turbulent mixing rate rather than the chemical reaction rate. So, that you will have to keep in mind and this is basically as I told that parcels of unburnt gases and fully burnt gases you will be getting right. And a flame mass burning will be dependent on  $\rho C_F$  and that is basically concentration of fuel right. Now sorry  $C_F$  is the constant right  $C_F$  is the constant  $Y_F$  rms dash is basically mass fraction this is mass fraction of fuel right this is fluctuating mass fraction right, mass fraction of fuel.

And epsilon is your dissipation rate and  $K_e$  is the kinetic energy right. And if you look at the  $C_F$  which is a constant right this is a constant, it may vary one to you know 100 generally people above 10 that it is a little lower site. And as I told that, the epsilon is basically the turbulent dissipation energy.

If you look at turbulent dissipation energy by definition we know this is nothing, but your kinetic energy. Change in the time and this kinetic energy we know that kinetic energy turbulent kinetic energy is basically  $\frac{3}{2} V_{rms}^2$ . And this is possible when this is for the isotropic turbulent flow right.

What do you mean by isotropic turbulent flow? That is  $V_x$  dash is equal to you know of course,  $V_y$  rms dash is equal to  $V_z$  rms dash it will be uniform. But that is not the case in most cases, but we have taken for this right. And then, I can say change in that is nothing but by  $\frac{3}{2} V_{rms}^2$  right divided by change in basically, if you look at this nothing but integral scale by  $V_{rms}$  right. And now, if you look at this I want to find out what is this epsilon by  $k_e t$  is nothing but your  $V_{rms}$  by  $l_{naught}$ .

Is not, I am taking this is nothing, but your  $k_e t$  right you can say initially it is 0 and then that thing and then you are doing that. So, this is proportional that then I can write down this  $m \cdot \text{triple dash}$  is basically  $\rho F_y F_{dash}$  rms is equal to nothing, but  $V_{rms}$  dash divided by  $l_{naught}$ .

That means, this mass fraction rate of the fuel right you can say that is the consumption per unit volume will be dependent on what dependent on the fluctuating velocity of the component of rms velocity divided by the integral scale. So, that will dictate and also the fluctuating mass fraction of the fuel right. So, it will be the reaction rate will be basically not calculated from the Iranians from the turbulence right. Wherever there will be mixing so it will be getting consumed because, the reaction is very fast here as compared to the as compared to your mixing time right. So, this is the thing which is being used for eddies and we can really use this for finding out the burning velocities and other things.

So, with this I will stop over and in the let us look at this kind of flame let in eddies occurs in spark ignition engine and premix burner and also the high voltage burner high velocity burners. And with this, we will stop over. In the next lecture we will be discussing about basically diffusion flame right.

Thank you very much.