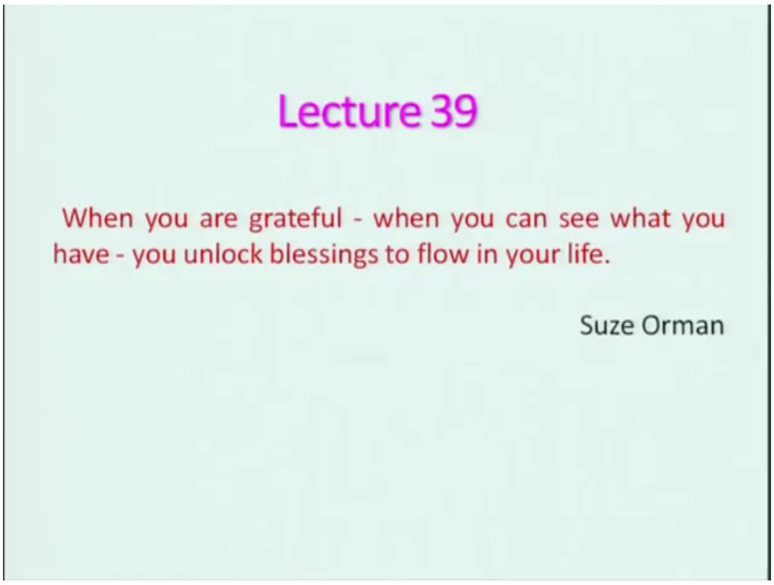
Fundamentals Of Combustion (Part 1)
Dr. D. P. Mishra
Department of Aerospace Engineering
Indian Institute of Technology, Kanpur

Lecture – 39
Conserved Scalar Approach For One Dimensional Flows

So, let us start this lecture with a thought process from Suze Orman; when you are grateful; that means, when you can see what you have, you unlock the blessings to flow into your life, right?

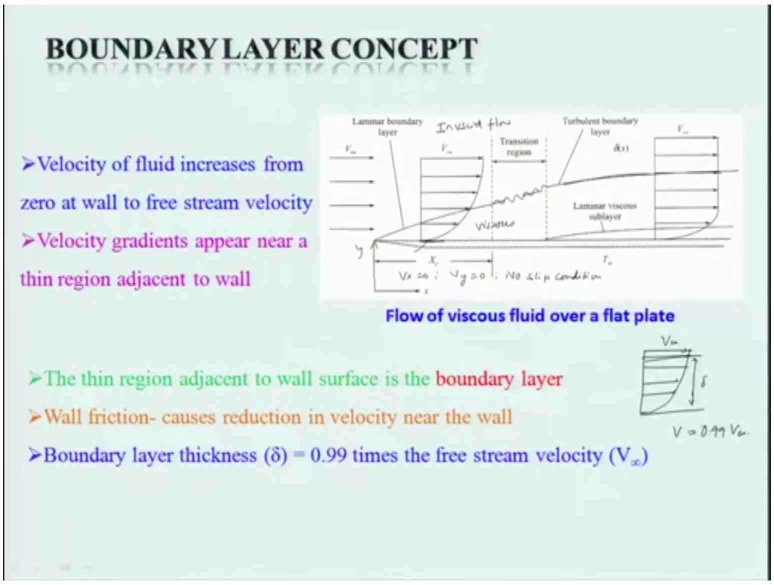
(Refer Slide Time: 00:20)



That means, basically what is trying to say an attitude of gratitude, you know, helps you to get the blessings flowing into your life. So, that is the things what he is saying, and which is there in our scriptures as well. So, let us look at in the last lecture we basically derived the equation for energy or the heat.

And then later on I summarized those equation for steady purposes and then club this continuity equation into the momentum species. And energy all I have done I didn’t tell in the last lecture, but we have done are you getting; that means, continuity is already being taken care of by that, and in the sense using the continuity we have neglected those terms, and then simplify the equation. And today I will be just touching upon little bit about boundary layer.

(Refer Slide Time: 01:37)



We are considering a f and if you look at low over a flat plate, right. You might be wondering that where the combustion will be taking place flow over a flat plate is it possible, no, certainly no.

But however, whenever you will be looking at flow over a solid propellant, or you know combustion over a solid propellant, or a wood combustion, right. We can consider the flow over a solid surface, and that you can consider as a not a flat plate, but; however, if I will take a wood let us say there is a fly board, you a nowadays we are using fly board, right. In our door and other things. So, there is a combustion there is a fire, what will happen? Is the flow will be taking place you know something like that or let us say your bench your now there is a come, you know, start burning, then some flow will be taking place of course, which will be natural convection and keep in mind that natural convection will be using the gravity. The gravity what we consider na that is very important in the flow due to natural combustion a natural convection right.

And what is happening here? That if you look at in this which already you might have studied these are the laminar flow that is a transition regime like there will be some disturbances there will be also turbulent flow, and there will be laminar sub layers, and keep in mind that velocity profile for the laminar and turbulent is different. So, what do you mean by turbulent, right? We will be seeing little bit later on, and how we can define can you define the turbulent flow or not, let us you can define, right. Ok we will discuss that whether you can define or not. So, what is happening here? The velocity profile you know it is increasing, because at this solid surface at this solid surface if you look at what will be the condition here? The V x will be 0, right. In this place V x will be V x will be 0. And V along the y direction if I say this is y at the solid surface V x and V y will be 0, that is known as no slip condition, and that will be valid for what?

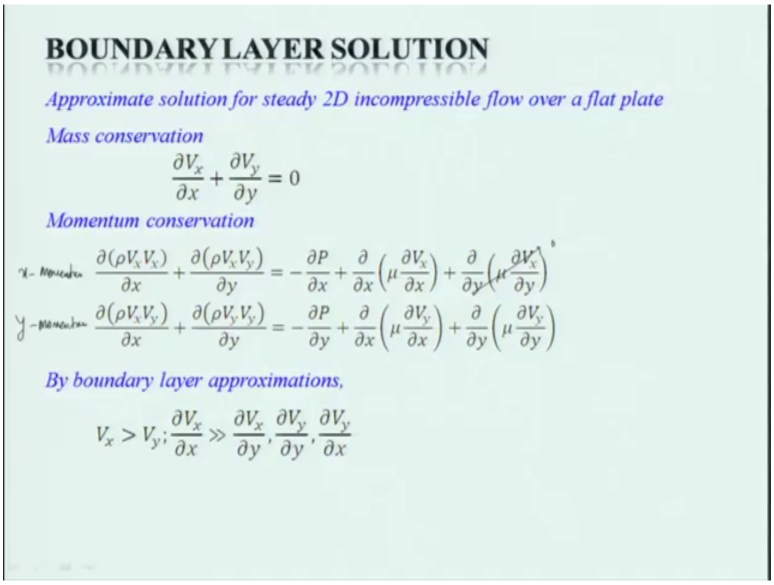
Student: Newtonian.

Not only Newtonian fluid, but also for continuum, when non-Newtonian fluid little bit true also. But continuum concept you know Knudsen number, right if I go rarefied gas that is not true ok.

So, therefore, most of the flow what we will be considering is basically coming under continuum. So, therefore, will be using no slip conditions and keep in mind that this region is basically known as viscous region, right. And this region will be inviscid flow, right flow, right there the viscosity term will be neglected. And this portion is basically known as you look at this portion is boundary layer. And boundary layer thickness you can define as what? The wherever this is the velocity V infinity, wherever it will get into 99 point of the stream velocity, right and keep in mind this is little exaggerated velocity profile it is not run properly.

So, in principle it should be like that if I draw a velocity profile, you know, like these, then my boundary layer will be if it is V infinity boundary layer will be somewhere here this is my boundary layer you can say because, where, thickness this is boundary layer where the V is equal to 0.99 of V infinity. Keep in mind that boundary layer thickness can be defined different way like momentum boundary layer thickness, right. And this is known as the velocity boundary layer thickness. So, wall friction is very important here, and viscous viscosity terms has to be used.

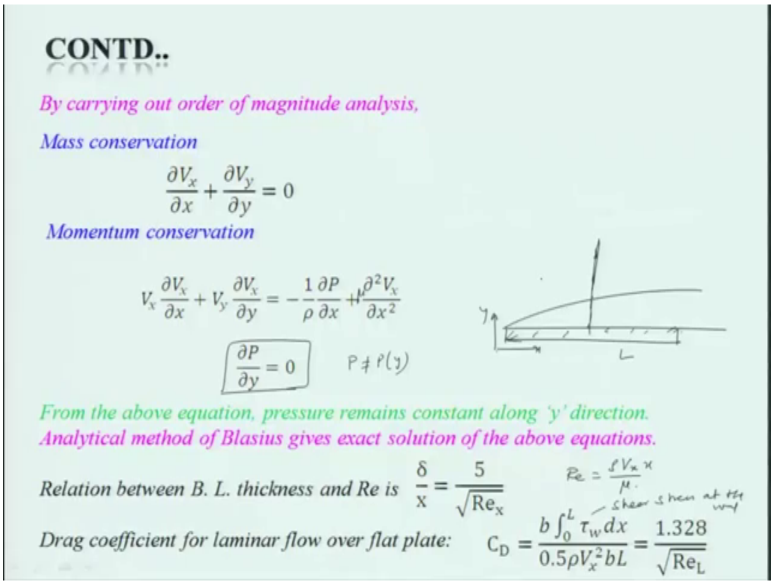
(Refer Slide Time: 06:23)



And if I look at incompressible fluid of course, we have need not to consider that, but keep in mind that, this mass conservation like you know dv x by dx dv y dy is 0, and this is the equation what we have already talked about it. And we are not considering the gravitational effect here.

And for the boundary layer application approximation, V x is very, very greater than the V y. V y along the y direction is very, very small, right. And the gradient dv x by d dx will be far greater than the dv x by d y and dv y by d y and dvy d x. So, therefore, these terms you know will be neglected; like, these terms can be neglected here. And what will say this is the basically x momentum, and this is y momentum, right? And y momentum there will be some approximation it will be turns out to be.

(Refer Slide Time: 07:41)



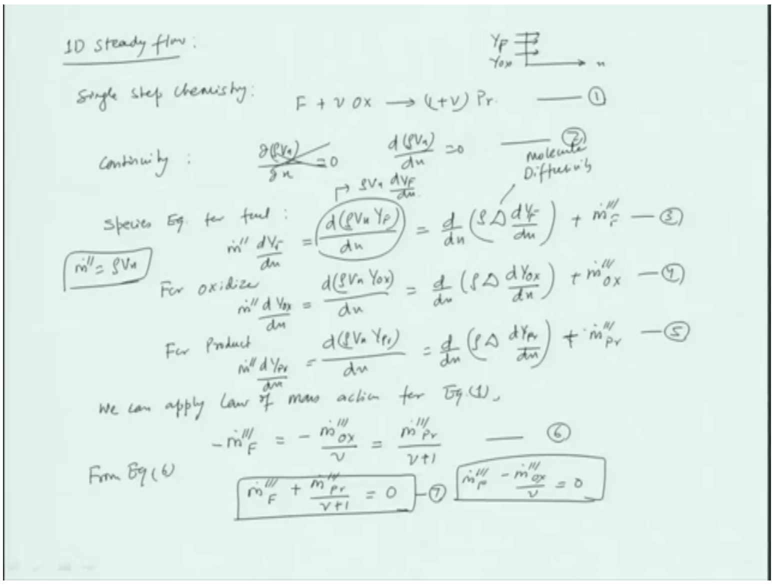
So, with this approximation you will get the momentum equation basically, dou p by dou y is equal to 0. What it says that along the boundary layer, right. If you look at this is my boundary layer, and we are considering laminar flow only, right. What is saying? The pressure along with this direction will be if I say this is my x this is y, that is remaining constant, it is not changing along the y direction at any point of x, right. That is the meaning of this; that means, pressure is not a function of what? Pressure is remaining constant.

So, with these you know you need you can approximate this thing, and this will be mu, right. Mu we are taking out is not a function of that I am taking out. So, this will be the equation which you need to solve, and then you can get these solutions, and also you can derive a relation of a delta by x 5 by root over Re x. And Re is basically rho V x by what you call? Rho x by mu, right x is your distance at which you are considering the Reynolds returns number, right. Reynolds number will be changing along with the x direction, if you are considering a boundary layer.

Similarly, drag coefficient you can find out using basically tau or the wall this is the shear stress at the wall, right. And which is when you integrate this you will get, this 1.3 to 8 root over r Re L and keep in mind that this L is the length here, if I consider this is my length L at which I am considering thus a drag coefficient for the laminar flow, right. And b is your width, whatever it is considered which is perpendicular to that.

So now what will be considering basically will be looking at the species equation, I am considering a one-dimensional flow, right. And in that, what will be considering is basically a one-dimensional flow.

(Refer Slide Time: 01:19)



Let say, in this is the one-dimensional flow is taking place, and with the species, species it can be y i, it can be fuel it can be oxidizer, right which is moving along the x directions, right to the species. And there is a we are considering one dimensional steady flow, right. For our consideration, and let us consider, right single step chemistry, right.

Let us consider single step chemistry. That is fuel is reacting with nu, you know, moles of oxidizer going to the product of 1 plus nu, I can say rather I would put a one kg of fuel is reacting with nu kg of oxidizer getting into product of 1 plus nu kg of product, right. And if you look at the continuity equation if you want to write, continuity what it would be for one dimension one dimensional steady flow, right. One dimensional, oh my god, one dimensional steady flow what it would be? It will be rho V x y dx is equal to 0, right. And species equation for, right will be writing for fuel let say species equations, right for fuel what it would be? V x by dx is 0, I can write down d rho V x y f d x is equal to dx rho y f.

And d is your diffusivity, right this is your diffusivity, mass diffusivity or molecular diffusivity plus, what it to be, right? Similarly, for oxidizer, I can write down rho V x y oxidizer, plus m dot triple dash oxidizer, and for product at the similar equations, right,

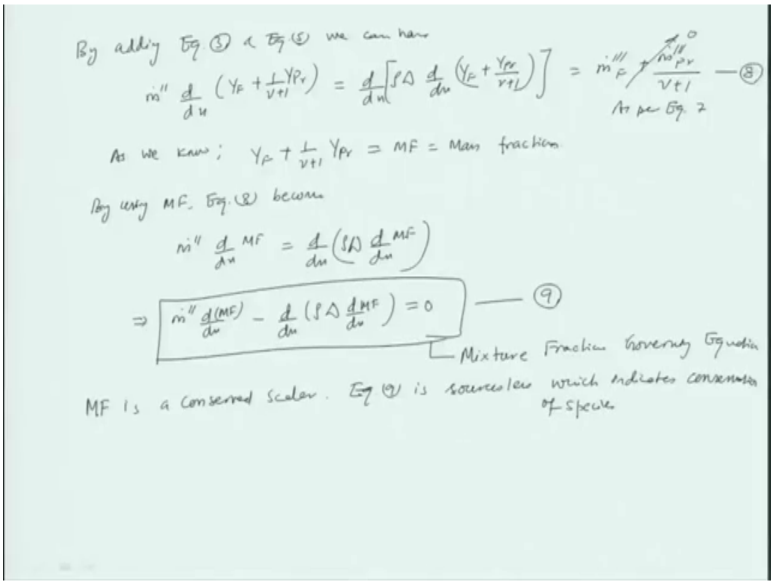
right. I can say this is my basically equation 1, I can say, and this is my equation 2, this is 3, this is 4, this is 5.

And he we will consider basically, we can apply the law of mass action for equation 1. We can apply law of mass action for equation for reaction one basically that is single step reaction. One we can write down that as f, this will be minus is equal to minus o oxidizer by nu, and is equal to product nu plus 1, right. Let say this is equation 6 and from these, I can write down as basically I can write down m dot triple dash f plus triple dash product by nu plus 1, right is equal to 0, yes or no? Can I write down, from this equation 6 from equation 6 because I am taking this that side, right. And that will be nothing but your 0 right.

Similarly, I can write down as what you call? I can write out m dot f minus these as you know I can write down also f minus nu is 0, right. That is that we would not be using I am just trying to make it, but what I will be using is this one this is I am saying 7 this is very important equation. And why we are doing this thing? Basically, we want to eliminate the source term from the equation, right. Are you getting because source term is causing the problem is Arrhenius form and then you know like it is e power to the e by r u t and then other things pre-exponential factors, right. What we will do? And we can use this equation 2 and 3, right? And I can write down in which form, I can write down as m dot triple dash I can say, right into d y f by d x. Can I write down, because m dot is nothing but your rho V x.

So, I will what I will do? I will take this and rho V x, this is this term, what is this term? I can write down as rho V x d y f by d x, yes or no? And rho V x is nothing but your mass flux, right. This is the term so, similarly I can write down here as y oxidizer, d x we did in the last lecture ok, in the similar way yes or no, right? Now I will add equation 3 and 5. I will add equation 3 and 5, I am going to the next slide now, right.

(Refer Slide Time: 19:14)



By adding equation 3 and equation 5 we can have; that is, I can take this out this is a mass flux, right? Dx y f plus 1 by nu plus 1 p r y pr is equal to by d x rho d keep in mind that diffusivity is same for all that I am assuming ok. It need not to be this is an assumption right; that means, diffusivity of fuel with respect to other mixtures you know other species need not to be same if you have the product with respect to.

Student: Fuels.

Other species or the other mixtures it not need not to same. I am taking considering same that you keep in mind, right. Otherwise I cannot this is an approximation, right.

By d x y f plus yp r divided by nu plus 1 is equal to f plus product divided by nu plus 1. And by equation 7, this is 0 as per equation 7. And what is this term? Can you recall yf plus 1 over nu plus 1 y pr, can you recall? This is nothing but your mass fraction; which I had discussed where in the when we had discussed thermodynamics, right. Na equivalence ratio and then mass fraction ok, as recov as we know that we know y f plus 1 by nu plus 1 yp r is nothing but your MF mass fraction, mass fraction right.

So, therefore, I can write down if I say this is equation 8, can I say this equation 8, right? Can I say this 8 equation? 8 just look at it. So, then I can write by using the definition of mass fraction equation 8 becomes MF, right equal to the rho d d by d x m f. So, this is the equation, right. Right I can also write down as maybe this is better I can write down this, minus I am just writing that is all taking that to left hand side, nothing else I am doing is equal to 0. And these equation is known as the mixture fraction equation, right. This is known as mixture fraction governing equation for one dimensional flow ok. For 2-dimensional flow you can also do that. I have just taken simplified form and keep in mind that this mixture fraction is basically a conserved scalar.

And important thing is that like the source term is not there, right. The source; that means, equation 9 is a source less; which is non-linear in nature, which indicates conservation of species, right. Because if you look at this is the convection term and this is a thing and this MF you know is the same is like a property, right. Now you are getting this is basically a variable, it can be used as a variable. And which is conserved, isn't it? Like the way species is conserved momentum is conserved right, but it is scalar it is not a vector, are you getting my point?

So, therefore, this is the things which we will stop over here, and we will be using this mixture fraction equation particularly when we are dealing with the z diffusion flame, right. I may not consider for droplet combustion, but droplet combustion also it can be utilized. Particularly when there is a diffusion flame, right? This is being used very much if you recall, when I had discussed about mixture fraction, I had told you this is meant for only for diffusion flame, not for pre-mixed, right. Because their meaning will be not having any sense if we use for the premix flame.

Thank you very much.