Fundamentals Of Combustion (Part 1)
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Lecture – 38
Energy Conservation Equation

Let us start this lecture with a thought process, that a positive attitude creates positive thoughts.

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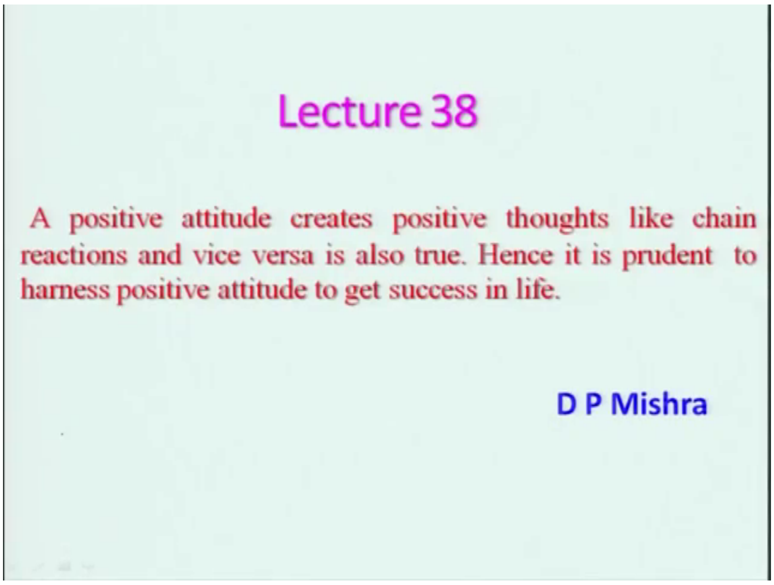

Like chain reaction, we have already seen chain reaction, how it leads to exposion, you know, and vice versa is also true. Hence, it is prudent to harness positive attitude to get success in life. If you look at in last few lectures, we are basically what you call derived the governing equation for mass conservation, and species conservation momentum conservations. And today what we will be doing we will be looking at energy equation or energy conservation equation.

So, in this energy conservation equation, what we will be doing we will be basically looking at a very simplified form by neglecting the radiation heat transfer heat you know transfer due to radiation and also, there will be dissipation due to the viscosity of the fluid, that we are not considering, right. These are the things we will be neglecting.

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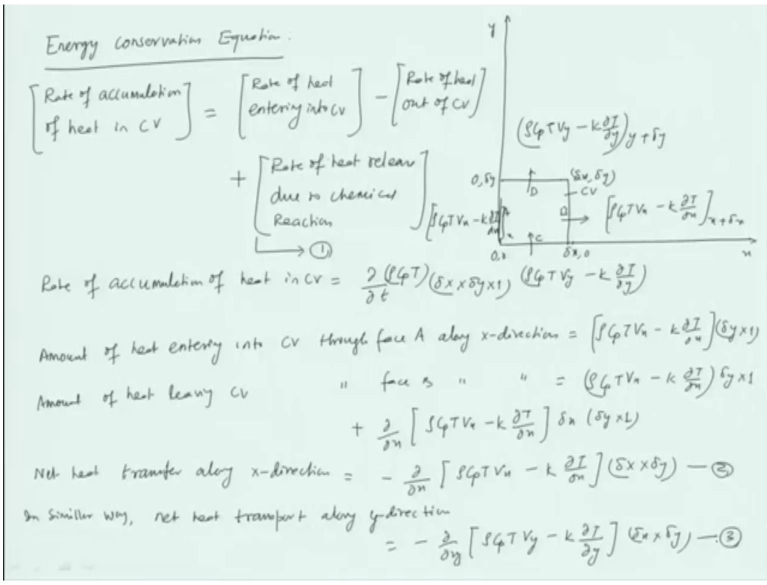

So, let us now look at energy transport equation or conservation equation, we will be deriving this, and of course, we will be considering basically also the heat generation due to chemical reaction, not due to nuclear reaction right.

So, let us consider basically a 2 dimensional infinitesimally fluid element. So, this is 0, 0, this is delta x 0, this is delta x delta y and of course, this is 0 delta y. And this is your face A face B, where C and face D, and that is you know this is nothing but you are what you call control volume. And it is infinitesimally small. That we have already done, you know several for other governing equations. So, if you look at that is we now to do a balance of energy right. So, I can write down rate of accumulation of heat in the control volume, right. And that is equal to the rate of heat entering into the control volume and rate of the heat entering out of the control volume plus what it would be? It will be the heat being released due to chemical reactions, right.

So, that is rate of heat entering into CV minus rate of heat out of CV, plus, the rate of heat release due to chemical reaction and as I told earlier that we are not considering what you call.

Student: Heat release.

Heat release due to the nuclear reaction; however, you can consider there is nothing as such you can derive the expression. But we are not considering because this is not concerned to us right, rather that is in rival for the combustion process, you know or for so for the power generation is concerned. So, coming back to that, what is this what you call the rate of accumulation of heat, lation of heat in this control volume, what it would be? That is basically dt rho Cp T, right into volume is delta x into delta y into 1, right. One is basically perpendicular to this plane in which I have drawn the fluid control volume.

And this if you look at certain rate of heat entering into control volume at the face A, right along the x direction would be what? Would be rho Cp T, then there will be some heat transfer due to what? Temperature due to temperature already I put, right. Right this is of course, this and then vx should be there this is a flow, right and then what else? There will be some heat conduction, right. Diffusion of the heat has to take place through conduction right. So, I can say k dt by D x, right. And what will be going out? Will be going out if you look at this is at this point, and then this will be what you call if I say this is at x, right. This will be then rho Cp T V x, right. Minus k dou T by dou x at what? X plus delta x, right I can do; that means, at this point if I say x is 0 here, right then it will be at delta x.

Now, what will be the heat? As a amount of heat, entering, entering into CV through face a of course, along x direction, what it would be? What will be rho Cp T V x, right? This is basically flux, right into k dt by dx, right into the area because this is a flux. So, this will be area will be what delta y into 1, this is at this face, right. And amount of heat leaving CV through face B along x direction would be it will be rho Cp T V x, right. Minus k dt by D x I can write down into delta y into one, right. Plus dou by dou x into rho Cp T V x minus k dou T by dou x into delta x, into I can write down basically, delta y into 1, yes or no that area through this area through the B.

So, therefore, the net heat along x direction, what it would be? It will be minus dou by dou x into rho Cp T V x minus k dt by D x delta x into delta y, right. One I am not writing, yes or no? Ok so, in the similar fashion, I can write down along the y direction; that is, rho Cp T V y minus k dt by D y, right. And similarly, I can write down rho Cp T V y minus k dt by D y y plus delta y. So, therefore, in the similar way, we can derive net heat transport, transported to transport along x direction would be dou by dou x, dou y rho Cp T V y minus k dt by D y into delta x into delta y right. So, keep in mind that I can say this as equation 1 and this is my equation 2, this is equation 3, right?

So, what will be the heat release, you know, rate of heat release due to the chemical reaction?

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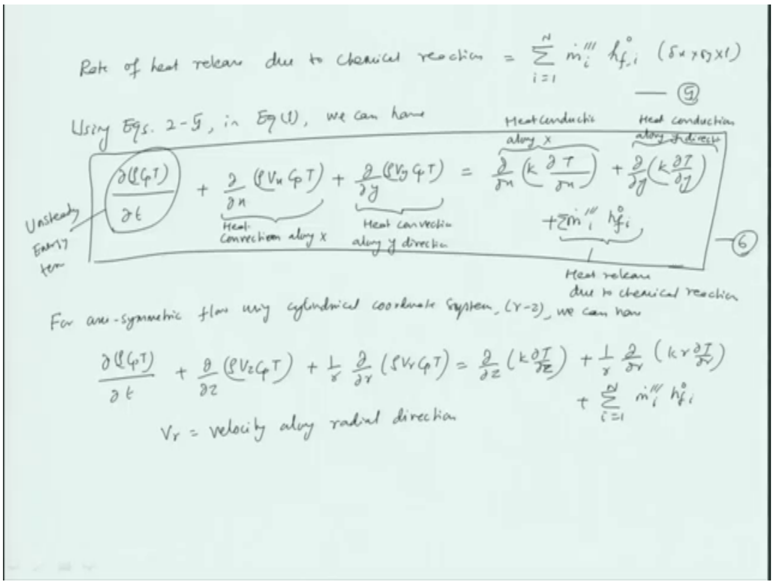

The rate of heat release due to chemical reaction would be basically what? Summation of m dot i triple dash i, right. H f i naught i is equal to 1 to n, right. It can be n number of species whatever it is participating e in the reactions, right into delta x into delta y, into 1, should be there is a volume ok, into one will be there.

So, this is your equation 4, right. Now using equations 2 to 4 in equation one, we can have rho Cp T, oh, I think I have forgotten another equation, ok. It should be actually this is 2, this is 3, this will be 4, this will be 5, right. So, this will be 5, 2 to 5 in equation 1, we can have, right. And you can divide this by the delta x into delta y across the equation, then you can get plus what will get? You will get the x row I can write out this better Cp T plus dou rho by dou y rho V y Cp T, right. And is equal to by dou x k dou T by dou x plus by dou y k D T by dy plus m dot triple dash I hf I naught.

So, keep in mind that, this is basically equation 6. And this equation is having this is your what you call unsteady terms, right term, and this is convective terms or heat convection I can say along x. This is your heat convection along y direction, and this is of course, heat conduction along x, heat conduction along y direction. And this is your source term, heat release due to chemical reaction.

And this term basically, which is also known as a source term that literally is due to chemical reaction is highly non-linear in nature ok. Why? Because you will be knowing, you know, these things provided you know the species conservation equation. And not only that, we know, if we will use ironies form of that that will be also dependent on temperature. So, this is a highly non-linear terms and which will be also these are these equations are basically coupled also has to be solved together.

So, keep in mind that, what we have here neglected is basically gravitational force. The energy due to gravitational force or electromagnetic or the anybody forces we have not considered, radiation we have not considered, and the viscous dissipation the energy lost due to the viscous dissipation we have not considered, you know, those are things if you look at you consider that then it will be quite complex, you want to see that you can look at the energy equation given in the principle of textbook, principle of combustion by Kk Kuan Kenneth Kuan, right K Kuan.

. So, that you can see is quite complex. So, but however, we do not need because those things are not relevant, and also particularly viscous dissipation is a very, very small as compared to the heat release. So, you do not need to consider that. And let us look at, you know, this let us rewrite this energy equation for cylindrical coordinate system or for axisymmetric, you know, for axi symmetric axisymmetric flow using cylindrical coordinate system. System I will be considering rz, we can have rho Cp T dt plus rho V z Cp T, plus 1 by r, r rho V r Cp T dz k dt by D z plus 1 by r dou r by k r D T by D r plus i is equal to 1 to n, m dot triple dash i h f i naught. Keep in mind that, this is basically V r is the radial velocity component V r is velocity along radial direction.

And we are assuming symmetric along the theta direction right. So, therefore, we are not considering that. And these are equations, like which are mean for a 2-dimensional situations; however, you can also write down 3 dimensional as such, but we do not need in this course. So, therefore, we are not considering it, and let us now look at for a steady flow, what will be this equation I will summarize, and also how we can simplify it little bit further.

So, we will do that. So, if you look at let us now summarize right.

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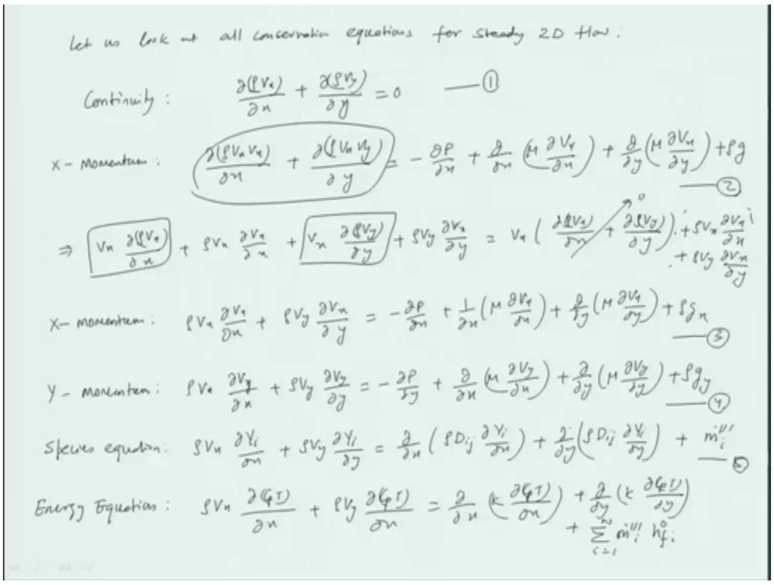

Let us look at all conservation equations, right equations for steady 2-dimensional flow, and we are using Cartesian and coordinate system only, we are considering now if you look at continuity, that is the mass conservation is nothing but rho V x by x plus rho V y D y, right is equal to 0. And let us consider x momentum equation or steady flow, that is rho V x V x x plus rho V x V y y minus dP by D x plus x mu dv x by D x, plus vx this will be, plus rho g I am saying for gravitational forces right.

Now, if you look at if I will take this portion, right. This portion and try to expand it, what I will get here? That I can write down as V x dou rho V x by D x plus rho V x into rho V x by dou x, right. I can write down similarly, V x V y by dou y plus rho V y by dv x. Now if you look at these 2 terms, this one and these if we will add, right what will happen, right? I can write down as let me write on V x, like common if I will take rho V x by D x plus y plus rho V x D V x plus rho V y dv x by dou y.

So, this will be 0, why? Because of continuity equation, if I say this is equation 1. So, right so, therefore, by using this, if I say the equation 2 becomes, right by I will use; that means, only this term will be remaining, right in this convection term, right. These are basically convection remain x mom. So, x momentum would be basically rho what I can write down? Rho V x dv x plus rho V y dv y a, right why is then rest of the things is similar, plus 1 by x mu dou V x by dou x plus dou y into mu dv x by D y plus rho g. Similarly, I can write down this is basically equation 3, I can write down as y momentum rho V x y by D x plus rho V y y y minus dou p by dou y plus y y dou x plus dou V y by dou y plus rho g y this is g x, right.

I mean gravity can be in one direction. So, any one of them has to be neglected 0, right. If I am considering in this case g x is the vertical direction, then actually g x will be there and g y it would not be there, right. It will be 0, so, but I am retaining it just for the you know completeness sake. And the species equation, I can write down similar way, y i by D x plus V y dy i by dy, and by D x rho D ij dy by D x, plus, by D y rho D ij y plus m dot i triple dash, right and this is my 5. And energy equation would be in the similar fashion I can write down.

Cp T by D x plus rho V y Cp T dx is equal to xk dcpt by D x plus D by dy k Cp T by D y plus, I will write down plus here, m dot i triple dash h f i naught i is equal to 1 to n. Keep in mind that this equation x momentum y momentum species equation all these equations are looks to be similar in nature, you know, if you look at these are your convection terms, along x direction this is y all. So, these are all convection term pressure drop of course, is there, but there is a lot of similarities between these diffusion terms like viscosity and diffusivity and conductivity. And of course, the source term are there in the species and energy equation, right.

So, we will be making lot of simplification when we will be dealing with that generally let say if you are considering y direction the diffusivity along the y direction is considered a whereas, the diffusivity in the x direction will be small. So, it is neglected we will be doing that thing later on.

We will stop over here. And we will see in the next lecture about how to deal with little bit you know boundary layer I will touch upon. Then we will see how we can eliminate the source term which is the culprit because that is a non-linear term that will do.

Thank you very much.