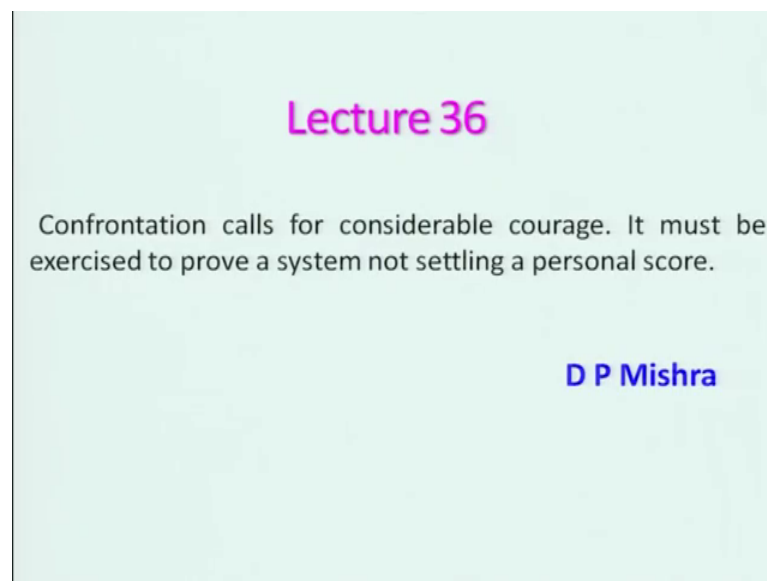


**Fundamentals Of Combustion (Part 1)**  
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**Lecture – 36**  
**Introduction to Mass Transfer**

Let us start this lecture with the thought process, confrontation calls for considerable courage.

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It must be exercised to prove a system, not settling a personal score. Let us recall that in the last lecture we derived the governing equation for momentum balance, right? We basically carried out the momentum balance, and eq and then governing equation pertaining to that. And today what we will do is basically looked at the mass or the species conservation, right? Because already we have talked about mass continuity or the mass conservation, right? And now we will have to look at species conservation. For that we will have to relook at basically mass transfer processes, right, mass transfer processes.

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Mass Transfer Process:

non-reacting mixture  
 $(CH_4 + H_2) \rightarrow$   
 $CH_4 \rightarrow V_{CH_4}$   
 $H_2 \rightarrow V_{H_2}$   
 $\rightarrow V$

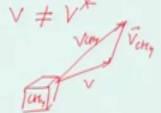
Let  $V_{CH_4}$  : velocity of  $CH_4$  w.r.t stationary coordinate system  
 $V_{H_2}$  : " " " " " "  
 $Y_{CH_4}$  : Mass fraction of  $CH_4$   
 $Y_{H_2}$  : " " of  $H_2$   
 $V$  : Mass average velocity of mixture  
 $V = Y_{CH_4} V_{CH_4} + Y_{H_2} V_{H_2}$   
 For a gas mixture of  $N$  components,  

$$V = \sum_{i=1}^N Y_i V_i$$
  
 Let  $V^*$  be molar average velocity, then for a gaseous mixture of  $N$  components  

$$V^* = \sum_{i=1}^N X_i V_i \quad \text{Note: } V \neq V^*$$
  
 Let  $\bar{V}_{CH_4}$  : Mass diffusion velocity  

$$\bar{V}_{CH_4} = V_{CH_4} - V$$
  

$$\bar{V}_{H_2} = V_{H_2} - V$$



So, let us consider a simple one-dimensional situation, right? And it is having a fixed one-dimensional flow situation we are considering. And we are having non-reacting mixture, right these are having non-reacting mixture of what let us consider methane, right? Gas and hydrogen this consists of that.

Now we are not talking about any reaction at this moment just for the simplicity sake ok, there is no production of the methane, or there is no destruction of methane or hydrogen in you know, this is a binary mixture what we are considering for simplicity case. And this if I say that is a molecule, right?  $CH_4$  which is you know this is let us say molecule is moving with a velocity of let say  $V_{CH_4}$ . And  $V_{CH_4}$  let is the velocity of  $CH_4$  molecule, right? With respect to what stationary axis or the in this case it is the x coordinate system, right; stationary or the fixed coordinate system.

Similarly, I can talk about  $V_{H_2}$  is velocity of  $H_2$  with respect to coordinate system, and this is what? Only for the x direction we are considering ok, we do not have considering 2 d 3 d kind of things just one. And to start with when you talk about mixture, there will be some mass fractions or mole fractions, right? Let say consider the  $Y_{CH_4}$ , right?  $Y_{CH_4}$  is the mass fraction of  $CH_4$  in the mixture and  $Y_{H_2}$  is the mass fraction of  $H_2$  in the mixture right; that means, I want to find out what will be the velocity mass average velocity, because each molecule as I told that there is a another hydrogen molecule which

is moving a velocity  $V_{H_2}$ , that let us say this is moving molecular velocity and with respect to x direction.

With respect to x direction, now that and also there, right? There will be also relative velocity, isn't it? And there will be some average bulk velocity which will be doing because both are moving both are moving average velocity will be there, what will be that mass average velocity? Let I say  $V$  is the mass average velocity of what? Of mixture, isn't it? Yes or no? Now what is that mixture? How can you find out a relationship for this  $V$ ? I will be expressing that in terms of.

Student: (Refer Time: 05:20).

Individual velocities, right? For example, methane and hydrogen here I can say  $Y_{CH_4}$  into  $V_{CH_4}$ , plus  $Y_{hydrogen}$  and  $V_{hydrogen}$ , right?

Now instead of 2, you know components in a mixture if there are n component of a mixture. What will be then average velocity; that means, summation of that. So, if I consider that for a gas mixture of n component, the mass average velocity would be we would be summation of  $Y_i V_i$ , i is equal to 1 to n ok, are you getting? Now in the similar way, I can talk about molar average velocity, right? Let us say, let  $V^*$  is basically be the molar average velocity, right? Then for a gaseous mixture, of n components, what will be the velocity? Molar average velocity would be summation of  $x_i V_i$ , i is equal to 1 to n, right?

See there are and is it  $V$  actually note that  $V$  is not equal to need not to be equal to  $v^*$ . It may be sometimes you know like that is a just a coincidence ok. Are you getting? Now the question arises if I look at a simple a molecule which is moving, right? And I can say let us say this is a molecule which is moving with; let us say this is a methane molecule let us say. And which is having a velocity bulk velocity is something a vector which is having  $V$ , and there will be some which will be having a velocity which you will call as a diffuse velocity, because it will be diffusing let say there is not no flow. If there is no flow it will be also moving or not. For example, if I put a what you call a perfume here. That mass is moving or not; that means, that is a diffusion velocity.

So, there will be some let say diffusion velocity I call it as a  $CH_4$ . And the total velocity what? It will be it will be this  $V_{CH_4}$  right. So, let I am coming again to the binary

mixture keep in mind ok, that is let  $V$  star sorry  $V_{CH_4}$  this thing is the mass diffusion velocity, right? That is means what? This is the meaning is that this is a relative velocity with respect to what with respect to bulk you can say, isn't it? And which will be equal to  $V_{CH_4}$ , we  $V_{CH_4}$  minus  $V$ , yes or no? You can say  $V$  this is a vector quantity  $V$  plus  $V_{CH_4}$  bar is equal to  $V_{CH_4}$  so that is a thing.

And similarly, I can write down also  $V_s$  minus  $V$  right, but you know we will be this thing, but what will be more interested in the flux, right. So now, let us look at the mass flux what will be more interested in, right? If I say that when we talk about this mass flux with respect to stationary coordinate system, right? What will be mass flux mass flux of methane, I am considering 2 cases, right?

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Mass flux of  $CH_4$  w.r.t. fixed coordinate system  
 $\dot{m}_{CH_4}'' = \rho_{CH_4} V_{CH_4}$

Molar flux of  $CH_4$  w.r.t. fixed coordinate system  
 $\dot{n}_{CH_4}'' = C_{CH_4} V_{CH_4}$

Relative mass and molar fluxes of  $CH_4$  will be  
 $\bar{m}_{CH_4}'' = \rho_{CH_4} (V_{CH_4} - V)$  — (1)  
 $\bar{n}_{CH_4}'' = C_{CH_4} (V_{CH_4} - V)$   
 Concentration of  $CH_4$

For any species:  
 $\bar{m}_i'' = \rho_i (V_i - V)$  — (2)  
 $\bar{n}_i'' = C_i (V_i - V)$

By using Eq. (1) we can have:  
 $\dot{m}_{CH_4}'' = \rho_{CH_4} V_{CH_4} - \rho_{CH_4} V$   
 $\dot{m}_{CH_4}'' = \rho_{CH_4} V_{CH_4} + \bar{m}_{CH_4}''$  — (3)  
 $\Rightarrow \dot{m}_{CH_4}'' = \frac{\rho_{CH_4}}{\rho} \dot{m}'' + \bar{m}_{CH_4}'' = Y_{CH_4} \dot{m}'' + \bar{m}_{CH_4}'' = Y_{CH_4} (\dot{m}_{CH_4}'' + \bar{m}_{CH_4}'')$   
 By Fick's law  $\int D_{CH_4} \frac{\partial Y_{CH_4}}{\partial x}$

2 as I am components of a binary mixture, right? With respect to fixed coordinate system, right? in this case is the x one dimensional we are considering what it would be into  $\rho_{CH_4}$  into  $V_{CH_4}$ . And if I consider you know molar flux, right of  $CH_4$  with respect to same thing, right fixed coordinate system.

We will be basically  $CH_4$  will be  $C_{CH_4}$  for  $C$  is the concentration and  $V_{CH_4}$ . And of course, I can write down also in a similar way for the hydrogen ok, right? For example, I can write down for hydrogen I can write down for even molar mass flux per hydrogen. Now if you look at what will be the relative mass flux mass and molar fluxes of methane will be rho, isn't it? And what is this  $V_{CH_4}$ ; which is basically relative velocity, that

we have looked at and this is  $\text{CH}_4$  should be density  $\text{CH}_4$ . That will be  $\rho_{\text{CH}_4}$  into  $V_{\text{CH}_4} - V$ . And similarly, I can write down of  $\text{CH}_4$  is equal to  $C_{\text{CH}_4} V_{\text{CH}_4}$  - star is always corresponding to what molar ok. And equal to  $C_{\text{CH}_4}$  into  $V_{\text{CH}_4} - V$ . Sir, what is  $\text{CH}_4$ ?  $C_{\text{CH}_4}$  is the concentration, concentration of  $\text{CH}_4$ , when you talk about molar we talked about concentration not mass, isn't it?

So now, if you look at  $i$  you know  $i$ -th species I can write down for  $i$ -th species, let us say there are  $n$  number of species that; that means, I will have to look at  $i$ -th species I means any species for  $i$ -th species I can write down as mass  $I$  is equal to  $\rho_i V_i$  is equal to  $\rho_i V_i - V$ . Similarly,  $i$  is equal to  $C_i V_i$  is equal to  $C_i V_i - V$  this is keep in mind that average, right? And this  $v_i$   $v_i$  star are different because one is molar other is mass.

Now, if you look at these, right what I can, right? Then I will take this let us say I am talking about now this is equation 1, I can rewrite the equation 1 is a little different way for the methane by using, or I can say this is equation 2, and this is equation one, right? And 2 by using equation 1 we can have, right? Mass flux of  $\text{CH}_4$  is equal to  $\rho_i V_i$  plus what? Keep in mind that ok, I am taking this also I am taking as average, this is average this will be average fine  $\rho_i$  I am taking this what I am doing? I am basically looking at mass flux of  $\text{CH}_4$ , eh are you getting this point or not.

If you are not getting let me do that again, this is average ok, because this is relative means due to the diffusion ok. Are you getting? And this mass flux let me write down here, I am in equation 1, what you did we basically looked at  $\text{CH}_4$  average is equal to  $\rho_{\text{CH}_4} V_{\text{CH}_4} - \rho_{\text{CH}_4} V$ . And this is what? This is your nothing but your  $\text{CH}_4$ , yes or no? Yes or no a, right? Are you getting? That is that or not.

Student: Yes.

So, therefore, I am taking this portion to the other side, because I want to write down with respect and this mass flux with respect to what with respect to stationary system. This is mass flux, because with respect to fixed coordinate system, right? And this portion is getting into  $\rho_i$  into  $V_i$ , I am taking this  $I$ . So, sorry this is  $\text{CH}_4$  I have already done here that is let us solve like I am taking to this is the equation. Let say this is equation I can say 3. In the similar way I can write down for what for  $i$ -th species. But let me little bit dabble with that before getting into that is  $\dot{m}_i \text{CH}_4$  is equal to  $\rho_{\text{CH}_4}$

CH 4 and instead of V what I will write down? I can write down mass flux of this into divided by rho, yes or no? I am talking about this term, this term I am taking here.

Can I not write down mass flux of this total mass flux divided by rho, and this is nothing but here what is that why i why CH 4, yes or no? Plus, I can write down this CH 4 average is equal to I can write down Y CH 4 mass flux of mixture plus m dot CH 4. What is this mass flux of this? This is nothing but your mass flux of CH 4 plus mass flux of hydrogen in this example, what we are considering binary mixture, yes or no?

Now, that is, right? Keep in mind that this will be nothing but that. So, I can write down Y CH 4 m dot CH 4 plus m dot hydrogen, I can write down minus, and what is this term by the Fick's law? By Fick's law that is nothing but your minus rho d CH 4 hydrogen into what? D CH 4 into d x right.

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The image shows a handwritten derivation of the mass flux equation for the i-th species in a mixture. At the top, the equation is written as:

$$\dot{m}_{CH_4}'' = Y_{CH_4} \dot{m}'' - \rho D_{CH_4-H_2} \frac{\partial Y_{CH_4}}{\partial x}$$

Below this, it says "For i-th species, we can have" and then the equation is written inside a box:

$$\dot{m}_i'' = Y_i \sum_{j=1}^N \dot{m}_j'' - \rho D_{ij} \frac{\partial Y_i}{\partial x}$$

Underneath the boxed equation, three terms are identified with labels:

- $\dot{m}_i''$  is labeled "Mass flux of i-th species".
- $Y_i \sum_{j=1}^N \dot{m}_j''$  is labeled "Bulk mass flux of i-th species".
- $-\rho D_{ij} \frac{\partial Y_i}{\partial x}$  is labeled "Mass flux of i-th species due to molecular diffusion (Fick's Law)".

A small circled number "4" is at the end of the boxed equation.

So, therefore, I can write down this as what? M dot CH 4, I can write down Y CH 4 m dot i, right? Minus rho CH 4 hydrogen dou Y CH 4 d x. And for i-th species we can have i is equal to Y i.

Student: There is a (Refer Time: 21:11).

Summation of so, and what you will do? You can write down write i, i is equal to 1 to n minus rho d i j Y i d x. Are you getting this? Is for i-th species and what it says it says basically this is what you call mass flux of i-th species, right? And this is this term? What

it called it calls basically, bulk mass flux of  $i$ -th species. And this is mass flux of  $i$  th species due to what? Due to molecular diffusion. Keep in mind this equation is very important, I can say this equation maybe 4, right? Can I say this as a 4 just check?

Student: (Refer Time: 22:39).

Right so, this equation is very important for deriving governing equation for species, right? By  $dx$  and these term is basically due to Fick's, law already we have derived that isn't it, right? Make sense.

So, this is a very important relationship what we will be using ok. I will discuss little bit more about that in the next lecture, then we will derive the mass species conservation equation.

Thank you very much.