Fundamentals Of Combustion (Part 1)
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Lecture – 35
Momentum Conservation Equation

Let us start this lecture with a thought process. Man can never be more intelligent than the Mother Nature. And in the last lecture, if you look at we had discuss about mass conservation equation, right.

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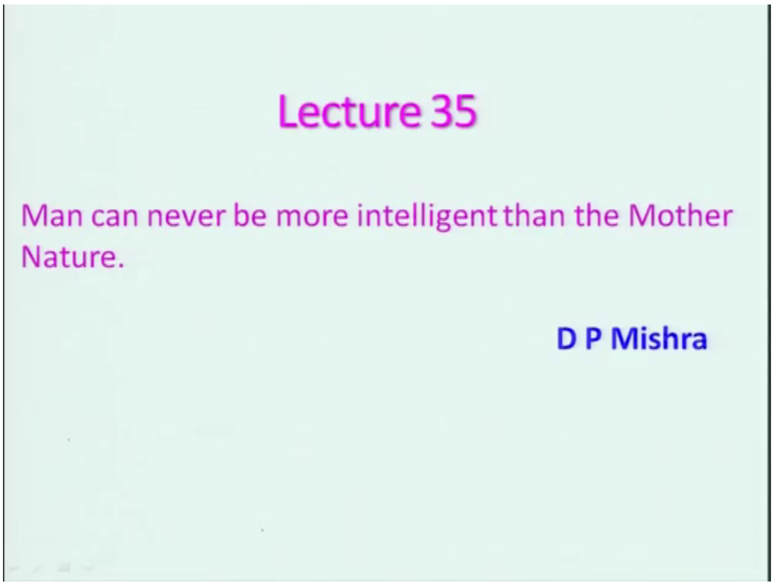

And what we call continuity, and continuity is very important, isn't it? Not only in the fluid mechanics, but also in our culture the continuity is being broken means, you will lose the cultural heritage. So, also the fluid mechanics and all are continuum, right, we are working on the continuum mechanics.

So, that is very important and what we will be doing, and when the continuum is there or it is continuity is there and also that we need to look at momentum. How much momentum is there that makes the fluid to move. And so, also the nation isn't it? Momentum plays a very important role. And if you look at momentum; momentum comes from where? What are the causes of this momentum, any idea? What are the causes? Is the forces, right? Which will be acting that makes it to have momentum, isn't it? Without force can you have any momentum? Yes or no?

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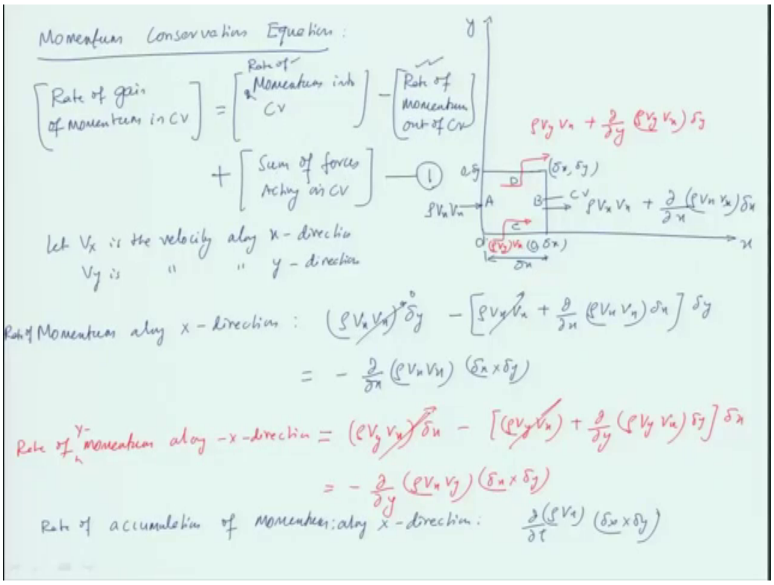

So, today we will be looking at momentum conservation equation that will be deriving. And for that what we will do? We will take as usual that is a infinitesimally control volume.

And as we are done in the last time, we are taking a 2 dimensional situation; however, it can be extended for 3 dimensional very easily. And we are taking Cartesian coordinate system. Let us consider a infinitesimally control volume, this is your delta x, delta x delta y and this is 0 delta y. And keep in mind that what do you mean by this infinitesimally small the delta x and delta y the distance if you look at if I say this is my distance, right? He is too small, right? That is the meaning. Now we want to look at this control volume, right. This is my control volume it is having less a face A and face B and this is your face C and B, right. Just to have a feel I am just putting this it is not required as such.

And now we will have to look at the momentum conservation. And what is that? That is basically rate of accumulation of momentum in the control volume, right. Rate of accumulation or gain, I can say gain of momentum in CV is equal to rate of control volume minus, I can say rate of momentum out of control volume, right. Is plus, what are those things? Like some of the forces acting on control volume. And this I can say is the equation one, right. And when you talk about these basically we will have to look at now the momentum conservation along the x direction, and momentum conservation in the y direction, we will be looking at it, but let us now consider the momentum you know conservation along the x direction, right along.

So, if you look at what is this momentum in the x direction, that is entering into this is rho Vx and Vx, what is V x? Vx is the; let Vx is the velocity along x direction, and V y is the velocity along y direction, right. And what is getting out th from this face B? That is rho Vx V x, right. And we will have to expand in the Taylor series and assuming that delta x is too small and higher order terms are not there. So, what will write down? Will write down the x rho V x V x into delta x. So, when you look at this momentum flux which is entering into these, and you will have to appla into multiplied by the area rate of momentum along x direction. What it would be rho V x V x into delta y minus rho V x Vx plus rho V x V x delta x into delta y type.

And this will cancel it out, right. This will be cancel it if I just you know take this delta y into this that will be cancel it out. What I will be getting is basically rho V x V x into delta x into delta y, right. This much I will be getting. Now what we will have to look at now? We will have to look at along with the x direction, right. Due to the fluid velocity V y how, much momentum flux is going in and out. So, that way if you look at it is going this way, what is that? That is basically rho V y into V x.

Similarly, here what is that going on? Rho V y V x plus by dou y rho V y V x into delta y. So now, it will be if you look at it is going through this delta x right. So, therefore, the rate of momentum this is basically y momentum, right; along x direction. In this control volume will be basically I will not write down is equal to rho V y V x into delta x minus rho V y V x y rho V y V x into delta y into delta x right. So, this will cancel it out, right this will be cancelling out. So, therefore, I will get minus V x V y into delta x delta y, right?

And if you look at rate of accumulation of momentum, what it would be? Along the x direction ok, along x direction what we are doing now only along the x direction, right; that will be what dou by dou t rho V x into delta x into delta y of course, the volume one is a in a perpendicular to this plane. So, therefore, that itself is a volume keep in mind that. So, a now if you look at we know in this term the left-hand side and, right. Hand side both the terms, we know only we will have to look at the forces some of the forces acting on the control volume along the x direction right. So, if we will do that. Now we need to look at what are the forces acting on the control volume.

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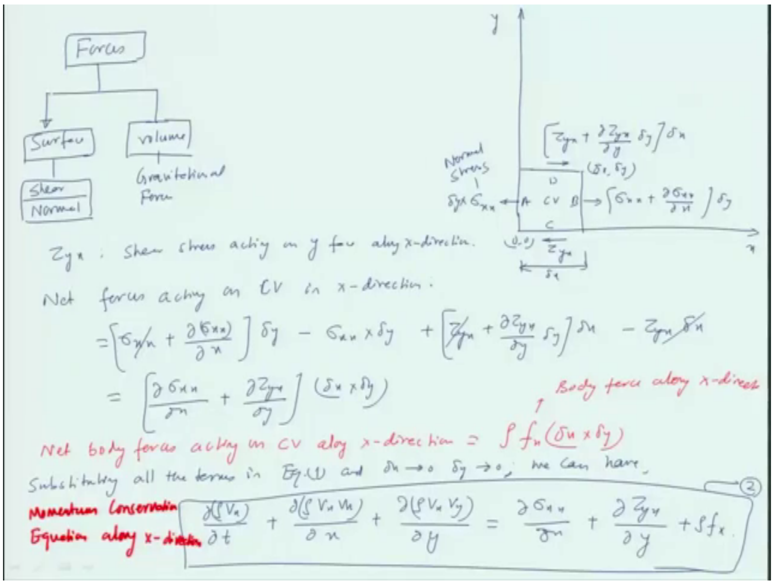

If you look at we need to now evaluate it sum of the forces, right. What are the forces acting on this control volume? What are the forces? Pressure? Pressure I can of course, pressure will be acting without that flow can take place, right? But I would like to look at in a little different way; that means, there will be some forces which will be acting on the surface. There will be farce forces which will be acting on the volume itself right. So, therefore, I can divide these forces into 2 categories. One is which will be acting on the surface, right. And other will be acting on the volume.

So, what are the forces which will be acting on the volume?

Student: Gravity.

Gravity, electromagnetic forces, right and other things which we are not interested in except gravity. Why will be interested in gravity? Ok, gravity is not really very important, right. When the convective flow is taking place gravity does not play a very important role, isn’t it? Yes or no? But however, we will be dealing with something diffusion flame jet diffusion flame. We will be talking about droplet combustion, where gravity will be playing a role, there we are not looking at convection as a predominant right. So, it is a natural convection which will be taking over shape therefore, we are keeping the gravity in our governing equation will be considering gravity, right gravitational force right.

And the surface forces can be due to the shear stress. Because it will be acting on the surface making the shear, right. And you know stress force per unit area that is the. So, therefore, it will be we can say that due to these forces there will be created shear stress, right. You can say shear force also and other is the normal, the 2 things which will be normal forces, right.

Now, let us look at what are the things which will be considering along the x directions, right. We will take again the control volume, right. now there will be force you know which will we call it as a normal force, right. Which will be acting and the that is we call normal stress in this control volume, this is a control volume, this of course, A B C D, right. And this is delta x delta y, I am just writing, I am not writing other any other point now. And if you look at the force in the shear stress which is acting on here is sigma x x, right. And which will be acting here sigma xx plus sigma x x x into of course, delta y, I can say this is into delta y right.

So now, the shear force which will be acting here and similarly, it will be acting in this direction just opposite because this force is opposite in direction, that will be y x plus y x by y delta y into delta x, because acting on this surface, you know, this area this is you know this thing is basically delta x. Now what is this tau? If you look at this tau y x means, this is acting on the surface y along the x direction ok. What is the tai yx is the shear stress, right. acting on why face direction face, right along x direction. Now the net force acting on this fluid element what it would be net forces acting on the inf what you call control volume in x direction would be equal to sigma x x plus x I can put this way, right into delta y minus sigma xx into delta y, right; plus, tau y x y delta y into delta x minus tau y x into delta x. So, this will cancel it out, this will be cancelling out.

So, I will get basically x x x plus y x y into delta x into delta y, right. And net body forces which will be acting on this body force means, gravitational force what we are considering. Acting on control volume along x direction would be nothing but rho f x. F x is the body force along x direction. Basically, it will be body force can be anything you know, you need not to be gravity if it is gravity it will be g ok, right and rho into g that is into delta x into delta y, right. And if I will substitute all these thing in equation 1, right.

What it would be? It would be by substituting all the terms in equation 1 and what you call delta x tending towards 0, and delta y tending towards 0, conditions, right we can have that basically what I will get I will get rho V x by dt plus rho V x V x x plus rho V x V y dy is equal to x by d x plus y x divided by y plus rho f x.

So, this is the expression what you will get, right. Let us say this is equation 2 ok. Now if you look at we do not know really; what is the normal stress, right. This I call it as a normal stress and this of course, I only told you shear stress along what we call x directions. And we can also you know do all these processes, all these by the similar way by the analogous way, we can derived the momentum equation along the y direction. Keep in mind that this of course, we call the momentum conservation equation along x direction right.

So, momentum equation along the y direction by using similar method we can derive momentum equation along y direction, right.

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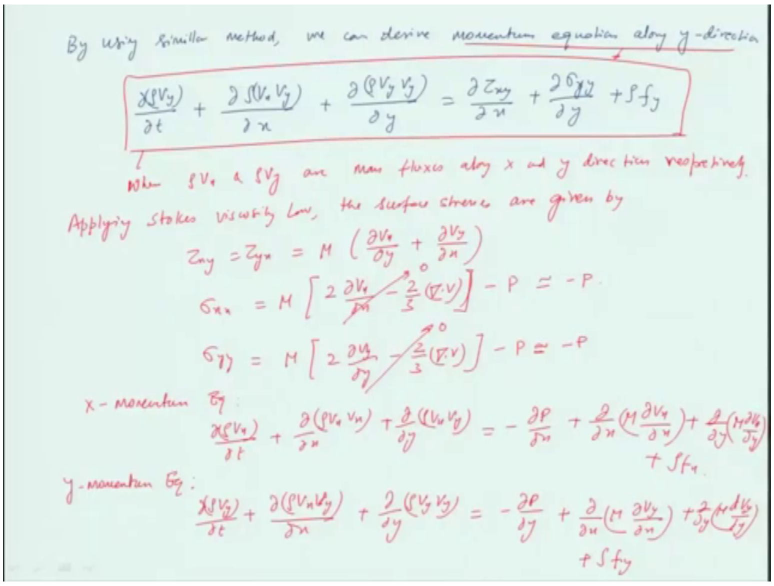

That is, Vy this is basically momentum equation in the y direction, right. This is your momentum equation y direction. Now keep in mind that these are what you call? If you look at rho V x rho V y is what are those things those are mass fluxes where rho V x and rho V y are basically what you call the mass fluxes isn't it? This is the mass flux which is basically a vector quantity, right. Because of the velocity is there, right. And mass fluxes along x and y direction respectively.

And of course, the sigma that is shear stress and the sigma that is basically the normal stress and the tau is the shear stress which is acting on the, you know, surface of the control volume. And how we will take care? What we will do? Because we will have to apply the stokes viscosity law, right. And of course, this one can derive which I am not deriving, but you can look at some standard books like you know any fluid mechanics book you will get that by applying stokes viscosity law, the surface stresses are given by tau xy is equal to tau yx mu y plus tau Vy x and sigma x x is equal to mu 2 x by V x minus 2 by 3 V minus P. And keep in mind that this generally will be very, very small or 0 tending towards 0.

Therefore, you can say approximately equal to minus P. Similarly, sigma y y will be mu 2 dv y by dy minus 2 by 3 delta V minus P is equal to basically this will be 0. For most of the cases, and then approximately equal to the P. So, then when we substitute this equation, what we will get is basically final equation we can get x momentum equation doe rho V x y doe t plus rho V x V x plus rho V x B y minus doe P by doe x plus mu V x doe x plus mu plus rho f x. Similarly, y momentum equation, you can get as rho V y dt plus rho V x and V y d x plus y rho V y y minus dou P by dou y plus rho d x mu y yd x plus y mu dou V y by dou y and plus rho f y. So, this is the equation for cartesian coordinate system.

And however, we may use for the cylindrical systems axisymmetric systems. What I am thinking that you please look at that and expressions. Because, or I can write it down for you that is for an axis symmetric flow.

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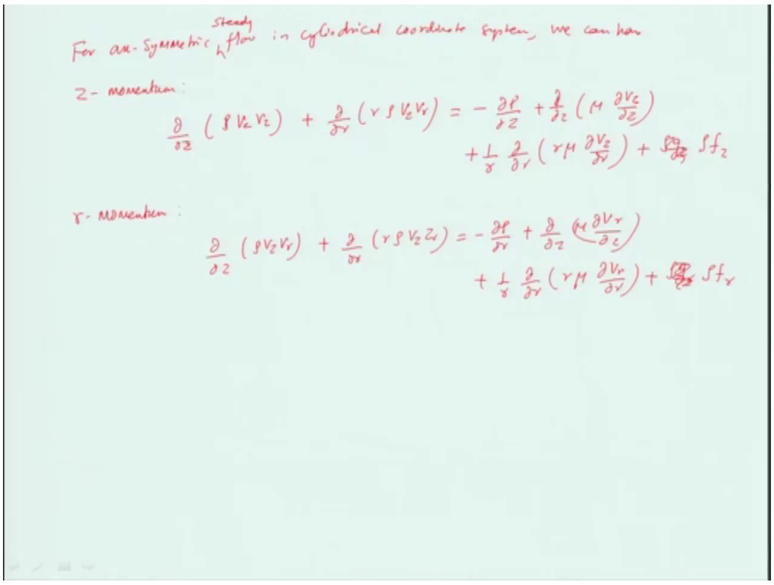

We are only considering in r and z directions, right in cylindrical coordinate system. We can have this is z momentum by d z rho Vz Vz or this will be V z V r is equal to minus dP by dz plus 1 by this is dou by dou z mu d V z by d z plus 1 by r r r mu d z by d r, and then rho g z.

So, and the r momentum equation we can write down as and keep in mind that this I have considered as for the steady process, right. This is for the steady flow unsteady term, I have not added in this. So, that is basically V z V r plus by r into r rho Vz Vr is equal to minus dP by d r plus rho d z mu Vr by z plus 1 by r d by dr r mu dv r by d r plus rho g r, right direction just to sake, but you know gravitational force you are considering, right. That will be rather I would like to write it down here rho f z this is general body force ok. Rho f r ok, these are the body forces what we are considering. And if it is gravitational it will be g ok.

So, with this we will stop over. Keep in mind that we will be revisiting these things later on, whenever we are looking at the flame kind of thing for analysis.

Thank you very much.