Fundamentals Of Combustion (Part 1)
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Lecture - 34
Mass Conservation Law

Let us start this lecture with a thought process that is for leading a materialistic modern life we burnt fossil fuels like oil gas coal etcetera.

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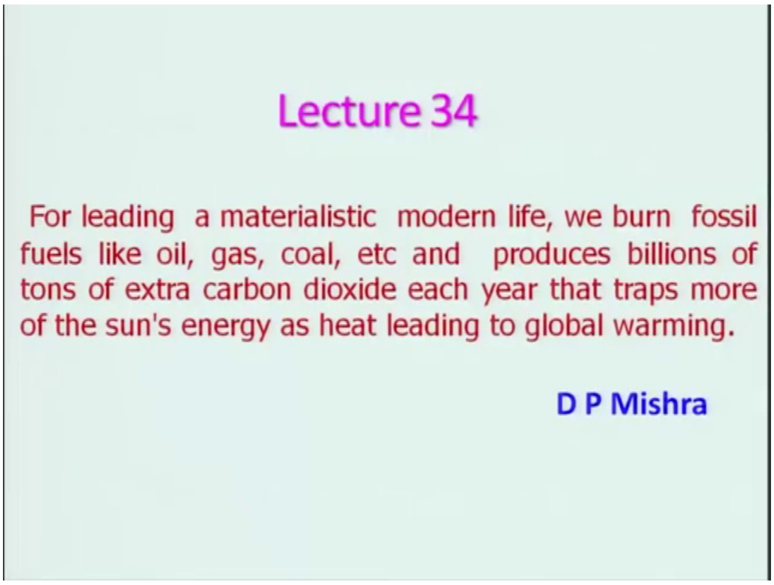

And produces billions tons of extra carbon dioxide each year, that traps more of the suns energy as heat leading to global warming. We are all now experiencing the global volume right and (Refer Time: 00:41) it.

So, we need to also you know learn combustion such a way that will minimize the you know carbon dioxide emissions of course, you cannot really, because you know you will have to use less fuel that is the thing; that means, if you mo makes the process more efficient so, you will have to use less fuel. And also you will have to be a little bit ka you know misery in using the modern gadgets so that you can really deduce this global volume.

So, in the last lecture, if you look at that we consider about the mass diffusivity, conductivity, and the viscosity, properties of the mixtures and also the individual spaces.

And today what we will be looking at we will be looking at how we will take care of transport of mass in a bulk flow right, there we are looking at individual microscopic effects looking at and then we connected those microscopic you know properties into the microscopic properties like; viscosity and conductivity and diffusivity, but here we will be looking at bulk; that means, those properties will be using here now.

And which of course, you will be knowing let us consider the mass conservation equation; that means, mass has to be conserved in a fluid flow, keep it in mind that we are here considering what we call the continuous continuum Hypothesis right.

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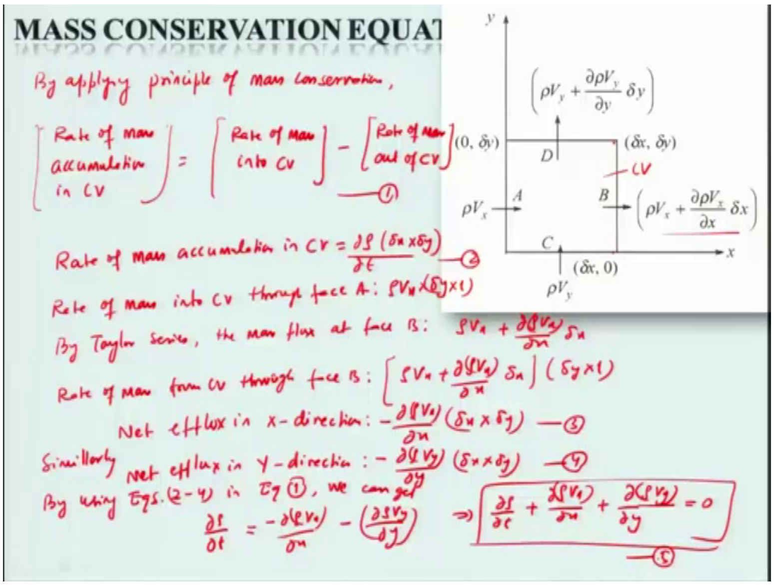

We are not considering that as of it is a discrete particles it is all are coming together right a right. So, therefore, because you know like therefore, we are not considering each individual particle now it is a continuous. So, the function will be also continuous in nature and we will be considering a control volume approach. And let us say that we are considering a very small infinite decimally control volume this is my control volume right.

And which is having a face a right and this distance is basically delta y and this distance in the face B you know and this is face C and this is basically delta x this portion this distance will be this distance and this is of course, the D is again like coordinate if you look at this point delta x delta y this is there. And we are considering 2 dimensional in nature, but the z direction will be perpendicular to this plane right x and y plane.

Now if you consider the let us say mass is entering to the face A and leaving along the x direction through the face B right. And similarly some mass flux rho V y is entering through the face c and it is leaving with the face D.

Now, I we want to look at basically the mass conservation; that means, we will have to apply the principle of right, by applying principle of mass conservation right. We can have what is happening here in this control volume we will have to look at rate of mass accumulation in control volume, that is rate of mass accumulation right in CV is equal to what is that, whatever the mass is being accumulated here which will be equal to mass in and mass out right; that means, rate of mass into cv minus rate of mass out right out of CV right.

Now what is the rate of mass of accumulation in CV; that means, mass right, what will be rate of mass accumulation in CV? What it would be rate of mass accumulation will be will be a rho by dou t into volume.

Right, that what will be the volume the volume will be the delta x into delta y of course, one we are taking perpendicular to this I mean by that is the 1. So, we did not take that is basically volume right you need depth we are taking. Therefore, it is volume fine. And rate of accumulation of this things you know now we will have to find out rate of mass into CV, what will be; that means, I will have to instead of these I can say rate of mass entering into CV right, through face a how much that will be rho V x into delta y yes or no rate of mass CV through the face is basically rho V x into delta y of course, into 1 you can say right.

And if you look at how much that at the face B what it will be this is the flux change how it is coming, how it is? Because by Taylor series the flux the mass flux right this basically mass flux at face B would be what that is rho V x plus d rho V x dx into delta x.

The rate of mass from CV through face B is what rho V x plus V x by D x delta x into delta y into 1 is not it right. So, what will be the net efflux in the x direction, net efflux will be rho d v x by d x delta x into delta y yes or no right.

Similarly, net efflux in y direction V y into delta x into delta y yes or no; so now, if I put this in equation one and this I can say equation 2 and this is equation 3 and equation 4 by using equation 1, 2 and 3, 4 or using equation 2, 3, 4. In 1 I can get 2 to 4 in equation 1, we can get as basically d rho by dt right is equal to V x by d x minus this will be here, actually this will be f x will be in direction will be basically minus this minus this right. This will be minus this minus rho V y by d y. So, similarly I can write down d rho by d t plus d rho V x by dx plus d by dou y rho v y is equal to 0.

So, now that will be rho V x dx by rho V y d y if I this is 5.

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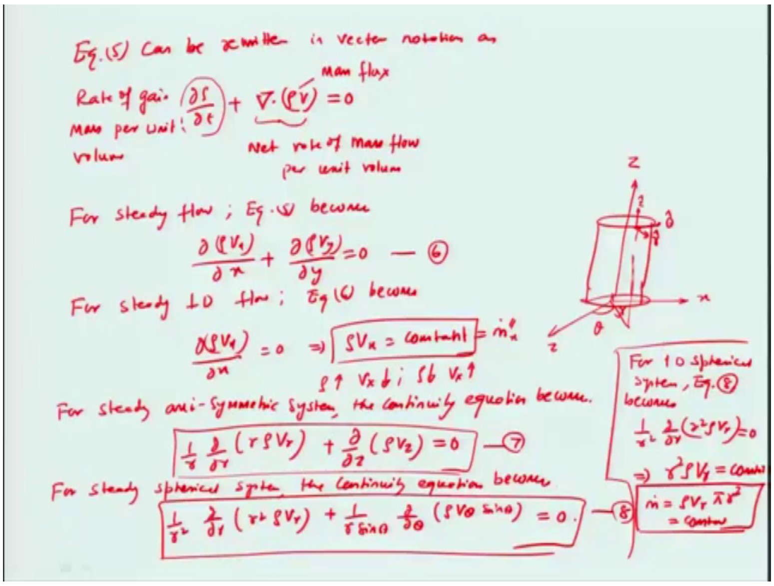

So, if you look at basically I can write down in a vector notation as equation 5 can be rewritten in vector notation as d rho by dt plus delta dot rho v is equal to 0.

So, if you look at del delta dot rho V is nothing, but your divergence of rho V and what does it indicates, if you look at this indicates the rate of gain of mass or accumulation of mass per unit volume and this is of course, the mass flux, which 1 rho V. And this term is basically net rate of mass flow rate per unit volume, I think net rate of mass flow per unit volume. And for a steady flow if you look at; we can write down right equation 5 equation 5 becomes rho V x dx plus rho V y dou y is equal to 0.

So, keep in mind that in our a course will be mostly dealing with one dimensional flow, but in some situations we may go for 2 dimensional right we will definitely not going to the 3 dimensional, because it is not tractable particularly for combustion situation. So, if I consider for steady one d flow right that equation 6 becomes rho V x by d x is equal to

0 implies that rho S V x is constant, what does it mean? It means that if the density is decreasing what will happen to V x V x has to V x a x has to.

Student: Increase.

Increase and in combustion problem, what will happen there will be increase in temperature particularly where the flame will be there. So, therefore, at that place what will happen to?

Student: (Refer Time: 14:53).

Density will be decreasing. So, the V x will be increasing. So, therefore, one as to carefully about it and you cannot really make that flow to be incompressible like you know are you getting.

So, that you should keep in mind because from this you know it can be say that rho if it is you know increasing the V x as to decrease and vice versa right. For example, I am going from let us say high temperature to the low temperature right. Then naturally what will happen density will increase and V x as to decrease right other around are going from low temperature to the high temperature region.

So, this has to be what we call a rho has to decrease V x has to increase. So, of course, this is for the Cartesian coordinate system are you getting. Now we will be also dealing with cylindrical like for example, there is a z flame right for that in that case Cartesian I can use, but that will be not that convenient. So, I will have to use cylindrical coordinate system right for the cylindrical coordinate system, if I look at here right.

Let us say this is the z and right. So, this is my I am saying this is x right and this I can say this is your z direction right, but I am having here a point right if I say that is coming over some where here; that means, this is my r right and this is my what we call theta that is azimuthal angle right and this is my z direction. So, right this will be z direction. So, if I will say this is my unit vector will be what you call that will be z right, this will be basically r direction right and this will be theta is it not visible here. So, this will be r unit vector.

So, I can write down this you know continuity equation, whatever we derived for Cartesian coordinate system right. For steady axis symmetric system the continuity equation becomes 1 by r d by dr into r rho V r plus z into rho V z is equal to 0 and keep it in mind that V theta is basically azimuthal direction, we can say this is basically 7.

And similarly for spherical coordinate right for steady spherical system right; the continuity equation right 1 by r square I am writing now basically for the may be what we call 2 dimensional right r and theta.

So, dou by dou r r square rho V r plus r sin theta into dou theta rho v theta sin theta is equal to 0, keep in mind that if you compare this equation with the cylindrical system what you can see this is in place of r, in case of axis symmetric system it is r square is coming right. Of course, this term is different like the second term is different keep in mind that like if I will consider for one dimensional flow right, because we will be using that for 1 d spherical system right what it would be it will be basically equation 8 becomes 1 by r square dou by dou r r square rho V r is equal to 0.

What is the meaning of that basically r square rho V r is constant right; that means, if you look at this is nothing, but your mass flow rate and r square is your area right. We can say that this is nothing, but your rho V r pi r square right I can write down this is nothing, but (Refer Time: 21:12) constant.

In this case of course, what is happening if you compare one dimensional flow in a coordinate system like what we call Cartesian coordinate system, that is the area is not coming into picture here the density will be decreasing; that means, we are also may increase may not increase depending on the area right are you getting. So, in r direction; that means, V r square also will be increasing.

So, density is decreasing and vr square is increasing now V r effect will be will be there, but that will be very less as compared to what you call cylind what you call co Cartesian coordinate system. Like if it is a straight line right going there is no effect of that sphere or the spherical and that comes into picture.

So, these things will be using particularly when you talk about what we call droplet combustion then we will be using spherical coordinate system. When will be talking about the ma z flame will be talking about cylindrical coordination? When you are talking about one dimensional flame will be talking about Cartesian coordinate system?

So, those things will be discussing and I think we will stop over now. And in the next lecture will be discussing about the momentum equations right conservation equation.

Thank you very much.