Fundamentals of Combustion (Part 1)
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Lecture - 31
Mean Free Path Length

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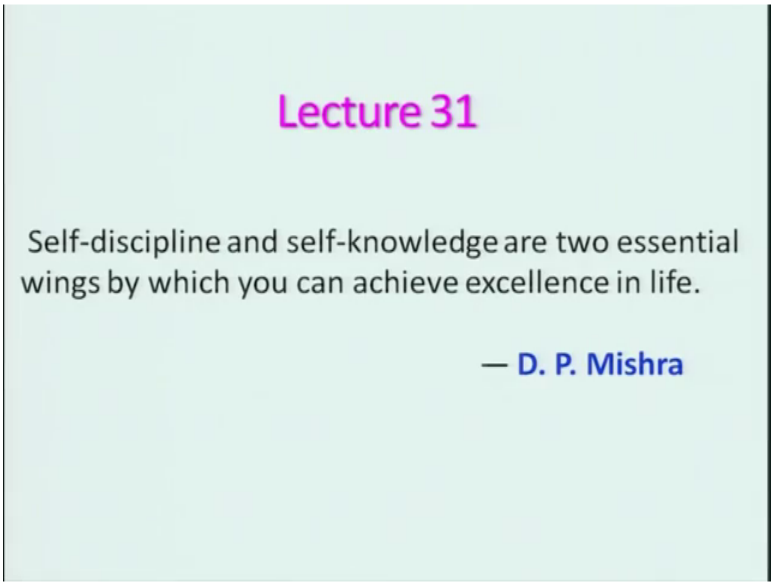

Let us start this lecture with a thought process self-discipline and self-knowledge at two essential wings by which you can achieve excellence in life. If you recall in the last lecture, basically we derive relationship for fix law; and later on we made it every what you call generic relationship for relating all three constitutive laws Newton's law of viscosity, and Fourier's law of heat conduction and Fick's law of mass diffusivity right.

And then we try to invoke the theory molecular theory of gases to relate you know to the mass fluxes right, so that we can find out you know express this, so that we can express this properties in terms of molecular properties right. And in the process I also emphasize that we need to look at the mean free path and already we have a derived relationship for the average velocity of a molecule or the molecular speed average molecular speed.

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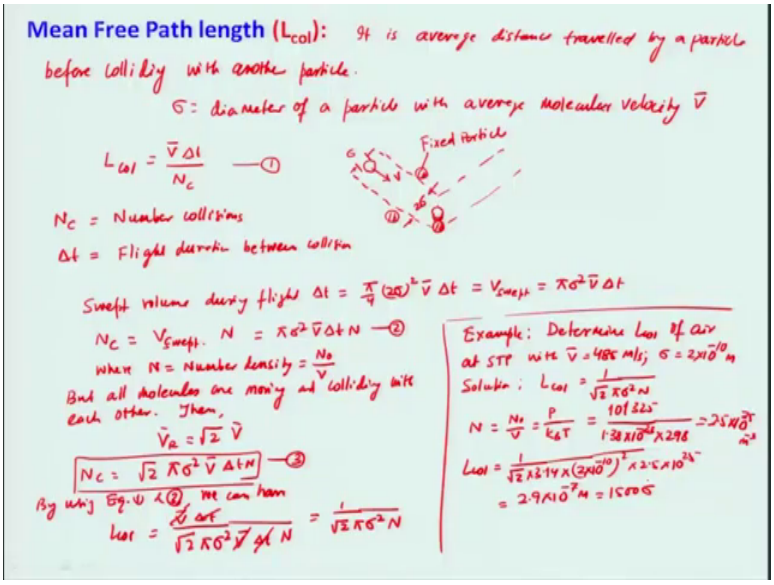

And let us now derive a relationship for a mean free path right, you can say length or mean free path only. So, considering that this for an ideal gas we are considering that is contains the molecule of same diameter d right, and we know that these molecules will be moving at an average molecular speed d dash. Mean molecular path is basically what, it is average distance right it is average distance travelled by the molecule right. It is average distance travelled by a particle; I will be saying particles right. And particle basically you know it can be atom, it can be molecule, it can be radical, before colliding with another particle.

And keep in mind that this particle is d sigma is the diameter of a particle with average velocity average molecular v dash and right. So, what is this L then L if I look at this L collision is nothing but your v average and delta t. What is the delta t that means, the time taken between during the flight between the collision two collision right divided by this is number of colliding particles right.

Let us consider that there is a particle right which is moving at a velocity v. And this diameter is basically sigma right. And keep in mind that we are considering this particle is moving and it will be colliding with others stationary particles right. Let us to make it simplified right. Now, I can say that this will be let us say moving in a in a volume there that is a another maybe fix particles which is away there might be fixed particles here, these are all fixed particle right. This is fixed particle this is of course moving that.

Now, this diameter if you look at this one right what will be this, this will be 2 d because this diameter distance if you look at it will be 2 sigma right because this and this, and this, there is another these thing right 2 sigma. If you look at the volume you know it is swept by this the swept volume, what it would be, it will be this cross sectional area 2 sigma right and into this velocity with which it will be coming so that that is the thing. Now, keep in mind that let us say this is the one and N c is number of collisions; and delta t is basically flight duration between collisions right. And in this volume, if you look at that might be several molecules which will be combined or coming and colliding right.

So, therefore, like we will be looking at the swept volume right during flight delta t would be basically pi by 4 2 sigma square into v average delta t right. And number if you look at number of collisions per unit time what it would be because I am really find out N c, what will be N c, N c will be this swept volume right you can I can say this is basically swept volume into what it would be into number density. N is the number density right where I can write down where that is number of particles divided by volume right that is your number density. So, that will be basically if you look at this is nothing but your pi sigma square v delta t this is pi sigma square v delta t.

And keep in mind that here we are assume that one molecule particle is moving and rest of the things are stationary, but in real situation all the particles will be moving with let us say same velocity. Therefore, the relative you know velocity what you call relative velocity, average velocity will be basically that is , but all molecules as I told are moving and colliding with each other. Then the velocity relative velocity I can say will be basically root over.

Then N c if I say this is N c, N c would be basically root over pi sigma square v dash delta t right. Now, what I will do, I will then by using equation 1 and 3, we can have L collision will be v delta t root over 2 pi sigma square v delta t into N, N should be there here into N, this will be N N. So, this will cancel it out. We will cancel it out it is nothing but 1 by root 2 pi sigma square N. What it indicates, it indicates that the mean free path is basically proportional to the diameter of the particle or the molecules.

If diameter is bigger, the mean free path will be smaller; and if number density is higher, the mean free path will be smaller is inversely proportional right. Because if it is the more number of particle for the same volume are there that means, it will going and colliding is very easily keep in mind that in actual situation the mean molecular path will be zigzag. But in this thing, we have considered as if it is going in a particular volume in a that may not be really the case in accuracy, but here for simplicity will consider.

Now, let us take an example right. Determine L collision of air molecule of air at STP - standard temperature and pressure and with which is moving with velocity V 485 metre per second right. And we will have to determine that as I told the pressure is standard pressure, so we will have to basically find out a the N. And the diameter if looked at sigma is given as 2 into 10 power 10 minus 10 metre; this is the diameter of air molecules right. Of course, it might be you know oxygen and nitrogen, but we are considering as a single one right ok.

So, if that is the case, then what will happen we know that L collision is equal to 1 over root 2 pi sigma square N. So, N we will have to find out N is basically equal to N naught by V which is nothing but your P by K B T right K B is here Boltzmann constant right. So, T we know 101325; and K B is 1.38 10 power to minus 23 into standard temperature. So, we are taking 298, 298, so this is coming to be 2.5 10 power to the 25 metre minus like 1 over metre cube right unit 1 over metre cube basically because this unit is there this is number of volume.

So, now, if I substitute this values L collision will be 1 over root 2 into pi 3.14 into 2 into 10 power to 10 whole square into 2.5 10 power 25. So, that is happening something 2.9 into 10 power to minus 7 metre which is happen to be 1500 diameter right. So, if you look at the mean free path, it is you know is equal to order of something generally something for gases around 1000 sigma here it is 15 order of 1000 not that exactly 1000. And the distance between these two molecule will be order of something 10 sigma's that is the way we use ideal gas law. Now, we will be looking at basically how we can relate this thing with the properties will be now using the mean free path with the properties.

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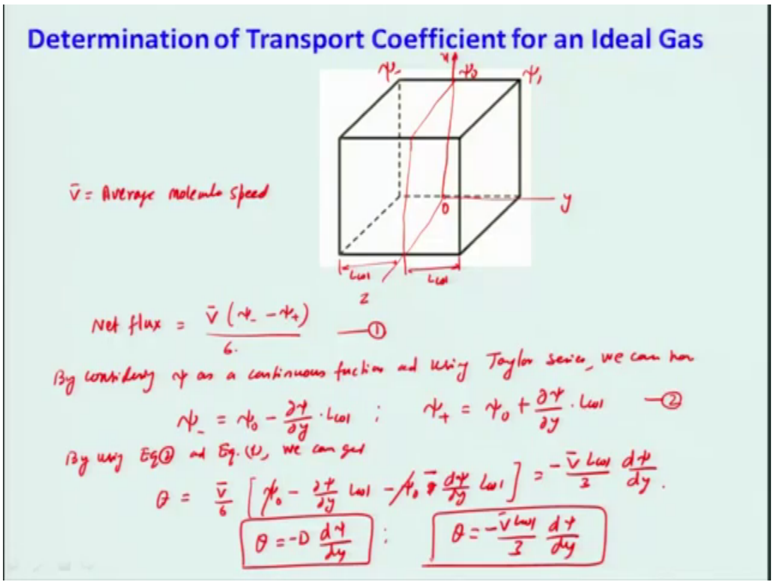

So, let us consider that these you know a chamber or a cube in which the small particles are moving with certain velocity v you know average molecular speed. And we will assume that it is having some flux which is continuous in nature some properties which is continuous nature right. If I consider this let us take a plane, and this plane you know let us take this as basically y-direction, and I can take this is x, and this is your z-direction. And these distance we are considering because the collision will be taking place with the mean free path you know average distance, we are considering this as a L collision right. And similarly this also we are considering as a L collision.

And in this plane, the lets say this properties you know will be psi naught. And here we are considering psi plus, and here considering psi minus. And this you keep in mind that this what are this properties, properties can be mass, it can be energy, it can be what you call momentum right. So, these properties are continuous in nature. Keep in mind, this very important, are you getting? In the when you are considering molecules, these are discrete ok. Here we are considering as a continuous right.

Now, what will be the net flux if I considering this plane where it is 0, this is basically 0 right, 0 plan and this is going L collision distance this side, and L collision distance that side right. It is very-very small, we have seen, it is 10 power to minus 7 per area you know. It is a very-very small distance we are considering between two plane just two. And what will be the net flux then net flux would be right, and keep in mind that there is a total we are considering along let us say y-direction, but there might be what you call x-direction and then z-direction, so therefore there will be totally basically six planes which will be coming.

So, number of particles you know if you look at average wise it is the you know total is six. So, therefore, it is moving with the velocity the particle is moving with a average molecular speed right; and that will be V, and this properties will be psi minus psi plus divided by 6 , because this is the velocity with which it is moving this is basically you can call as flux right. And if you look at like this psi like I can use because they for a continuous function by considering psi as a continuous function and using Taylor series, we can have psi minus will be psi minus d psi by dy into what? L collision, this distance, yes or no, this will be not, yes or no right?

Similarly, if I say this is equation 1, similarly I can write down psi plus will be psi naught minus this psi by dy into L collision mean free path right. Now, if I use let us say this is equation two both the thing together if I will use equation 2 in equation 1, 2 and equation 1, we can get net flux, if I say net flux is nothing but your theta is if I say this is V by 6 and psi naught is coming psi psi minus psi minus psi naught minus psi by dy L collision minus psi naught plus d psi by dy L collision.

This will be plus ok. So, therefore, this will be minus because this is the positive direction therefore, this will plus this will be negative right and what you will get basically that means, this will cancel it out you will get L collision by 3 with the negative sign d psi by dy.

Now, if you look at our constitutive you know or the compact form what we derived is basically theta is equal to diffusivity minus d psi by dy this is 1 right, this already you have derived right. And the this one is basically V minus L collision divided by 3 minus d psi by dy, is it not looking similar right only what we have done the diffusivity is nothing but your average velocity into collisions sorry mean free path divided by 3 that is all right. Are you getting?

This is simple because I have derived this relationship or I have look at this compact form right because of do this right. And now we look at how we can relate this thing to the properties various properties of course all right.

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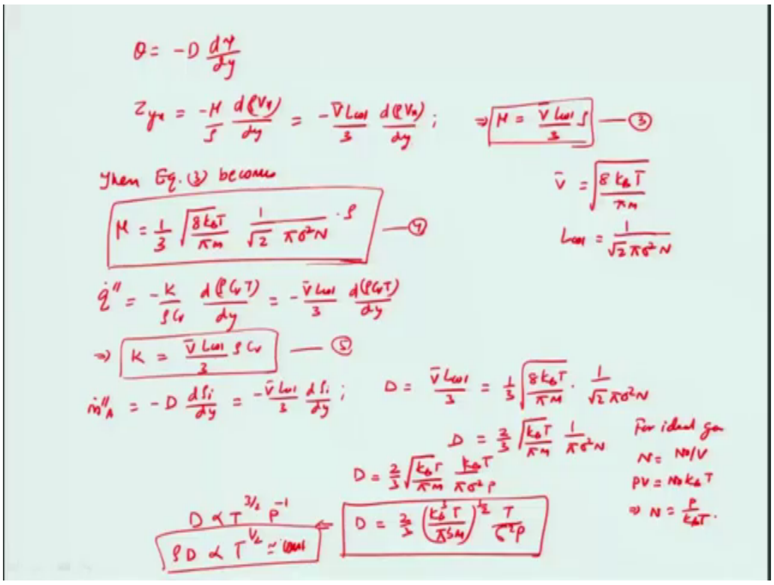

Now, if you look at we have this thing, derive this thing theta is basically diffusivity d psi by dy. And if you look at tau y x is nothing but your mu rho v x by dy right. And this is a thing we have derived and which is nothing but your what you call minus v rho v x. This is flux is basically nothing but you dy. So, therefore, I can write down mu is nothing but your v L collision divided by 3 into rho. Let us say I can say this as equation 3, can I say or equation 4, 3 right, 3?

And what will do we will now substitute the relations for the average velocity and the mean free path. So, the because average velocity we have seen that is 8 K B T by pi m root over N L collision we have seen what is that 1 over root over 2 pi sigma square N. This already you have derived. So, what I will do will use this thing right then equation 3 becomes mu is 1 by 3 8 K B by T pi m and 1 over root 2 pi sigma square N into density right. So, this is your expression you can look at it right.

And similarly, I can get the relationship for the q dash right. We know q dash is equal to minus k rho, I will say C v d rho Cv T dy is nothing but V collide V average right. So, therefore, I can write down the K conductivity will be what nothing but your V L collision 3 rho C v , let say this is 4, this is 5. Of course, you can substitute these values of V and L collision here; I am not doing that right.

And let us now look at mass diffusivity equation A D by dy which is nothing but V average L collision divided by 3 into. So, therefore, I can write down basically diffusivity as V L collision by 3 simply. And if I substitute here, V average here what is that 1 by 3 8 K B T by pi M root over into 1 over root 2 pi sigma N right. And keep in mind that I can simplify little bit here I can say this is basically 2 by 3, so I will get basically K B right T divided by pi M and 1 over pi sigma N.

So, what is this N? From for ideal gas right N is nothing but your N naught by V right and P V is nothing but your N naught K B T. So, I can write down N is nothing but your P by K B T ok. So, therefore, I can write down this as this is your D. So, I can write down D as 2 by 3 root over K B T pi M pi sigma square P by K B T. So, therefore, I can write down as 2 by 3 K B cube T by pi cube M.

This is half into t sigma square p that means, what is indicates it indicates that D is proportional to T, T what 3 by 2, and also it is proportional inversely proportional to (Refer Time: 30:52) right. But if you look at rho d is proportional to T half only right. And in some cases people is assumed to be constant because not really very much changing is not true, but they can assume is to be constant diffusivity right.

So, now you have seen considering basically the molecules right without rigid sphere and, but actually this sphere need not to be rigid right. And it is spherical, and the molecule need not to be spherical. And we have also assume the elastic collision, but collision is a need not to be elastic ok.

So, now we will be in the next lecture, we will be discussing how we can incorporate those things little indirectly, but not in I am not going to discuss at length about that, but little bit I will be touching upon how we can do that. So, with this, I will stop over.

And thank you very much.