

**Aircraft Design**  
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**Lecture – 08**  
**L/D for Maximum Range and Endurance**

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The image shows handwritten equations on a chalkboard. On the right side, there is a simple diagram of a horizontal line representing a range  $R$  between two points labeled 1 and 2. Above the line, the velocity  $V_C = v$  is indicated with a checkmark.

For Jet aircraft:

$$R = \left(\frac{V}{C}\right) \left(\frac{L}{D}\right) \ln \frac{W_{i-1}}{W_i}$$

$$E = \left(\frac{L}{D}\right) \frac{1}{C_t} \ln \frac{W_{i-1}}{W_i} \rightarrow E_{max} \Rightarrow \left(\frac{L}{D}\right)_{max}; \quad C_D = C$$

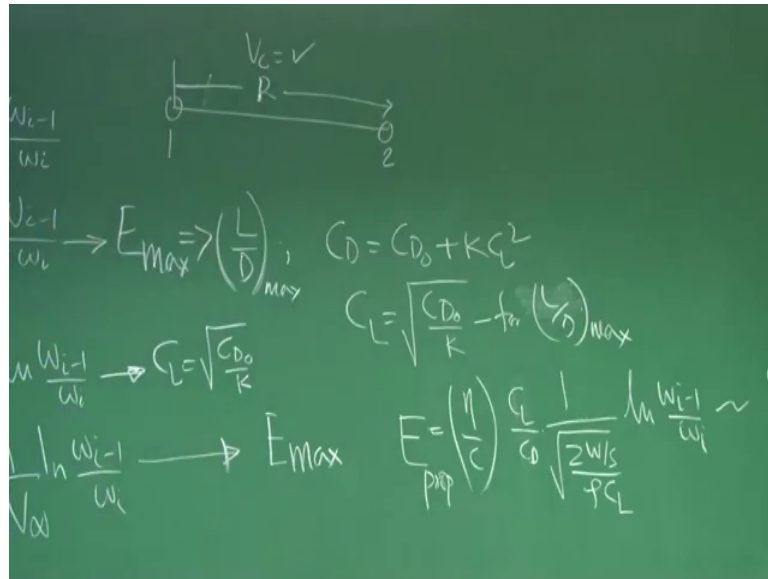
For Prop aircraft:

$$R = \left(\frac{V}{C}\right) \left(\frac{L}{D}\right) \ln \frac{W_{i-1}}{W_i} \rightarrow C_L = \sqrt{\frac{C_{D0}}{R}}$$

$$E = \left(\frac{V}{C}\right) \left(\frac{L}{D}\right) \frac{1}{V_{\omega}} \ln \frac{W_{i-1}}{W_i} \rightarrow E_{max}$$

Good morning friends, we are continuing our lecture on selecting initial values for C for L by D and in the last lecture. We are spend some time to see that how to handle the units of C because most of the handouts or literature will find C is given in a non consistent unit, and mostly the work is done in fps as far as books are concerned. So, we spend some time on C, so that you know how to handle this problem of units.

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But our main aim was to look for, if I am flying for a cruise from 1 to 2 if I know what is the value of R, I am aiming for, and if I know what is the cruise speed I am aiming for and type of engine roughly I know I have selected. We will discuss about how to select the engine. If we have all those things based on historical data, and how to compute this are handling the issues related to units of C. The next question comes how do I select the value of L by D what L by D I should fly to meet all these requirements. If you see, if you are using a jet driven aircraft as far as E is concerned, if I want to design for E max, I am very clear I should fly at L by D max rightly comes from here L by D max.

What is the meaning of L by D max for the pilot or for the designer L by D max means you know C D equal to C D naught plus L C L square. So, L by D max means C L have to fly such that C L equal to C D naught by K. Now, if you see here this expression for E if I see here which is for a propeller driven airplane. If I try to look for E max for a propeller driven airplane, which is here because see here L by D and then one V infinity. So, if I now rewrite E for a prop that will be eta by C L by D to 1 by V infinity to I will that V as 2 w by s rho C L for V and ln W i minus 1 by W i. And this L by D, I can always write as C L by C D. So, what you are saying if we want to ensure E max for a propeller driven airplane then this C L half goes on the top, it becomes proportional to C L 3 by 2 by C d.

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$$C_D = C_{D_0} + K C_L^2$$

$$C_L = \sqrt{\frac{C_{D_0}}{K}}$$

$$E_{prop} = \left(\frac{1}{C}\right) \frac{C_L}{C_D} \frac{1}{\sqrt{\frac{2W/S}{\rho C_L}}} \ln \frac{w_{i-1}}{w_i} \sim \frac{C_L^{3/2}}{C_D} \Rightarrow C_L = \sqrt{\frac{3C_{D_0}}{K}}$$

So, if I want to have e max for a propeller driven airplane the C L 3 by 2 by C D should be maximum which in turn means that I should fly such that C L equal to 3 C D naught by K. And now if I come for range for a propeller driven airplane - this one, it is straight forward I should fly such that C L equal to C D naught by K, because the range maximum for a propeller driven is when L by D is maximum. And for that you know C L equal to C D naught by K. This thing we know for minimum drag or we write L by D maximum. We have already done this thing.

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$$R = \frac{V}{C} \cdot \frac{L}{D} \cdot \ln \frac{w_{i-1}}{w_i}$$

$$V = \sqrt{\frac{2W/S}{\rho C_L}}$$

$$P_{jet} = \frac{P}{C} \cdot \frac{C_L}{C_D} \cdot \sqrt{\frac{2W/S}{\rho C_L}} \cdot \ln \frac{w_{i-1}}{w_i} \rightarrow \frac{C_L^{1/2}}{C_D} \Big|_{max} C_L = \sqrt{\frac{C_{D_0}}{3K}}$$

$$E_{prop} = \left(\frac{1}{C}\right) \frac{C_L}{C_D} \frac{1}{\sqrt{\frac{2W/S}{\rho C_L}}} \ln \frac{w_{i-1}}{w_i} \sim \frac{C_L^{3/2}}{C_D} \Rightarrow C_L = \sqrt{\frac{3C_{D_0}}{K}}$$

For a jet, this is the expression see. For a jet  $R$  equal to  $V$  by  $C_L$  by  $D$  to  $\ln W_i$  minus 1 by  $W_i$ . And now if I write  $V$  equal to  $2 W$  by  $S$  by  $\rho C_L$  under roots, what happens if I write this then for range for jet will be  $V$  by  $C_L$  by  $C_D$ . For  $V$ , now I will write under root  $2 W$  by  $S \rho C_L$  and of course into  $\ln W_i$  minus 1 by  $w_i$ . So, what is the message you get if you want to maximize range for a jet driven airplane then it means you need to fly such that  $C_L$  half by  $C_D$  is maximum; nothing new we have done this in performance course which in term means  $C_L$  equal to  $C_D$  naught by 3 K.

So, if I repeat, if I want to maximize the range for a jet driven engine, I need to fly such that  $C_L$  equal to 3  $C_D$  naught by K. If we want to fly a jet driven aircraft for maximizing endurance, I need to fly at  $L$  by  $D$  max or  $C_L$  by  $C_D$  max which means  $C_L$  equal to  $C_D$  naught by K. Or propeller, if you want to maximize range, I need to fly such that  $L$  by  $D$  is max or  $C_L$  equal to  $C_D$  naught by K. And for endurance for a propeller driven aircraft, if you have to maximize  $D$  that means, this  $L$  by  $D$  divided by 1 by  $V$  infinity. And we have seen that it means you have to fly such that  $C_L$  3 by 2 by  $C_D$  s maximum which means  $C_L$  equal to under root  $C_D$  naught by K.

Why we are doing all these things, because we are addressing a question, we want to translate all this information through  $L$  by  $D$  max. We will ask a question, if I am flying at  $C_L$  equal to 3  $C_D$  naught by K and still maintaining level flight how much  $L$  by  $D$  I am supposed to fly. If I flying such that  $C_L$  equal to  $C_D$  naught by 3 K then how much  $L$  by  $D$  I require. For example, here for range maximum, for  $E$  maximum, for  $R$  maxima for  $E$  maximum, we are looking what is the value of  $L$  by  $D$ . And can I relate this value of  $L$  by  $D$  with respect to  $L$  by  $D$  max, please understand for a propeller driven airplane to get maximum range I need to fly at  $L$  by  $D$  max. But for endurance or maximizing endurance, I am not flying at  $L$  by  $D$  max; I am flying at a  $C_L$  which is equal to 3  $C_D$  naught by K.

And corresponding to that what is that  $L$  by  $D$ , I need to calculate, and then we will not like to know that  $L$  by  $D$  is what percentage of  $L$  by  $D$  max. So, for designer point of view, if I know what is  $L$  by  $D$  max, then I know for different, different operation what percentage of  $L$  by  $D$  max, I will be flying for a particular cruise or a loiter mission that is what we are looking for. With that is important we have to plug those numbers here. I have tried to mention this every part of my lecture, because this is all the most important parameter, and you will see how selection of  $L$  by  $D$  will decide feature of the wings

feature of the wing aerofoil (Refer Time: 09:15) shape because after all you are designing we are trying to design aerodynamically efficient airplane. And  $L/D$  is known as aerodynamic efficiency of an machine or an airplane.

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The image shows a green chalkboard with handwritten mathematical derivations. At the top right, it states  $D = C_{D0} + K C_L^2$  and  $V_c = \frac{W}{R}$ . On the left, it shows  $\frac{C_L}{C_D}$  for an airplane and  $\frac{C_L}{C_D}$  for a propeller. The main derivation shows  $C_L = \sqrt{\frac{3C_{D0}}{K}}$  leading to  $C_D = C_{D0} + K \frac{3C_{D0}}{K} = 4C_{D0}$ . Another part shows  $C_L = \sqrt{\frac{C_{D0}}{K}}$  leading to  $C_D = C_{D0} + K \frac{C_{D0}}{K} = 2C_{D0}$ . At the bottom, it shows the lift equation  $L = W = \frac{1}{2} \rho V^2 S C_L$  and derives  $V^2 = \sqrt{\frac{2W/S}{\rho \sqrt{\frac{3C_{D0}}{K}}}}$  and  $V = \sqrt{\frac{2W/S}{\rho \sqrt{\frac{C_{D0}}{K}}}}$ .

So, let us try to visualize the  $C_L$  by  $C_D$  for different combination, we know that  $C_L$  for minimum power, which is nothing but  $3 C_D$  naught by  $K$  and that became one of the conditions for getting  $E$  max for a propeller case. Similarly,  $C_L$  or I write is to not to confuse you  $C_L$  when  $C_L$  3 by 2 by  $C_D$  is maximum at  $C_L$  is  $3 C_D$  naught by  $K$  and when  $C_L$  for  $C_L$  by  $C_D$  maximum it is under root  $C_D$  naught by  $K$ . Clear? If we are maintaining this sort of a flight condition then what will be the  $C_D$  in this case experienced by the airplane because we know  $C_D$  equal to  $C_D$  naught plus  $K C_L$  square,  $C_D$  will be in this case  $C_D$  naught plus  $K$  into  $3 C_D$  naught by  $k$ . So, this  $K$  cancels this becomes 4 times  $C_D$  naught. So, this is  $C_D$  for  $C_L$  3 by 2 by  $C_D$  maximum just for understanding.

Similarly,  $C_D$  for  $C_L$  by  $C_D$  maximum how much it is  $C_D$  naught plus  $K C_L$  square  $K$  into  $C_D$  naught by  $K$ , so this becomes  $2 C_D$  naught. Please understand we are trying to ask a question what  $C_L$  by  $C_D$  I should fly for a particular mission. So, I am just giving an example, if you are flying such that  $C_L$  is  $C_L$  3 by 2 by  $C_D$  maximum case or replying  $C_L$ ,  $C_L$  by  $C_D$  maximum case then we will try to compare  $C_L$  by  $C_D$  for this and  $C_L$  by  $C_D$  for this. What does it mean and mean the question is both the time I

am asking a question what is the  $C_L$  by  $C_D$  I am flying, flying which mission I am thinking of cruise only at the stage.

So, if it is a cruise an lift equal to weight and how much is the lift, lift will be half  $\rho V^2 S C_L$  of the aircraft. At the same weight for these two conditions then I am trying to find out what will be the  $C_L$  by  $C_D$  corresponding to this conditions. It is fair enough to assume weight is same. But remember when you are going for the range estimation when you going for a loitered estimation in practice there will be change in weight. So, will see how to incorporate that, but today a lecture is primarily focused on getting designers perspective on  $C_L$  by  $C_D$ . So, I am trying to build this subject.

So,  $V$  for and this is called  $W$ . So,  $V$  for  $C_L^{3/2}$  by  $C_D$  max will be equal to  $\sqrt{2} w$  by  $s$  by  $\rho$  into  $3 C_D$  naught by  $K$ . Similarly,  $V$  for  $C_L$  by  $C_D$  max equal to  $\sqrt{2} w$  by  $s$  by  $\rho$  under root  $C_D$  naught by  $K$ , which is clear because for this case  $C_L$  equal to  $3 C_D$  naught by  $K$ . And for this  $K$  is  $C_L$  equal to  $C_D$  naught by  $K$ . Once I know this, let us have a closer attention to this. If I ask you a question between case 1 and case 2, if the altitude is same then which case in which case the airplane will experience larger dynamic pressure that you should easily know that. For this case, you are flying at a higher  $C_L$ , so if speed required will be less and density being same the dynamic pressure for this case will be less; that means, when you are flying side the  $C_L^{3/2}$  by  $C_D$  is maximum then actually you are at a slower speed. And just underline this statement, I will try to correlate this when you are designing for indoor ends at all.

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The image shows a chalkboard with the following handwritten equations and annotations:

- Top left:  $\frac{D_{C_L^{3/2}}}{C_{D_{max}}} = \frac{1}{2} \rho \left( \frac{2W/S}{\rho \sqrt{3C_{D_0}}} \right) \cdot S \cdot 4C_{D_0} \left( \frac{C_L}{C_D} \right) = 0.866 \left( \frac{C_L}{C_D} \right)_{max}$
- Middle:  $\frac{D_{C_L}}{C_{D_{max}}} = \frac{1}{2} \rho \left( \frac{2W/S}{\rho \sqrt{C_{D_0}}} \right) \cdot S \cdot 2C_{D_0}$
- Bottom:  $\frac{D_{C_L^{3/2}}}{D_{C_L}} = \frac{2}{\sqrt{3}} = 1.1547$  (IMP)

There are also some scribbles and a circled 'IMP' at the bottom right of the board.

This is true then what is the drag for C L 3 by 2 by C D maximum case that will be half rho V square we will be this is 2 w by s by rho under root 3 C D naught by k. So, this is mistakenly half row V square into S into C D naught, C D naught is you have seen for this case C D naught is 4 C D naught or to correct myself, the C D is 4 C D naught, see this 4 C D naught. So, half rho V square s into C D and C D is 4 C D naught we have seen that.

Similarly, drag for C L by C D max will be equal to half rho into V square which is 2 w by s by rho C D naught by K into S into 2 C D naught which you could see somewhere here. This is the case. Let try if I try to see that ratio between D for C L 3 by 2 by C D maximum by drag reflying at C L by C D maximum, this will be equal to what and both the flying at same altitude. And then this gentleman goes, this goes, W i is goes is goes rho goes 2 w by s these 2 goes. So, what is remaining, remaining is this C D naught C D naught goes, this is 2. There is something like 3, everything goes right, this gentleman goes, this goes. So, it is like 2 by root 3, which is roughly equal to 1.1547.

So, what is the message here message is when you are flying suggest C L three by do C D is maximum you are flying at a lower speed. But see here the drag when you are flying at C L 3 by 2 by C D is 15 percent more than drag, when C L by C D is maximum. This is very, very important to understand. So, we see that drag for a configuration where C L 3 by 2 C D max and drag force configurations C L by C D max, their equal to ratio is

equal to 1.15. And we realize that drag when you are flying at C L 3 by 2 by C D is 15 percent more than what I am flying at C L by C D maximum.

I repeat again what it is showing is that drag experienced by the airplane at the lower speed flying side, their C L 3 by 2 by C D 1 maximum that drag is 15 percent more compared to the drag experienced by the airplane when it is flying at C L by C D maximum. So, this can be easily extended that C L by C D when you are flying at C L 3 by 2 by C D maximum will be 0.866 L by D max or I write it C L by C D max once empty where from this 0.866 is came this is nothing but 1 by 1.1547. What is done here for a given C L the C D for this case is 15 percent more compare to C D for C L by C D max. So, I can simply write C L by C D when a flying as C L 3 by 2 by C D max will be 1 by 1.1547 times C L by C D max which is this number is 0.866 that is why you are seeing that number 0.866.

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$$\frac{C_L}{C_D^{3/2}} \rightarrow \left(\frac{L}{D}\right) = 0.866 \left(\frac{L}{D}\right)_{max}$$

$$\left(\frac{C_L}{C_D}\right)_{max} \rightarrow \left(\frac{L}{D}\right) = \left(\frac{L}{D}\right)_{max}$$

$$\text{Jet. } R = \frac{V}{c} \left(\frac{L}{D}\right) \ln \frac{W_i - 1}{W_i}$$

$$E = \left(\frac{L}{D}\right) \frac{1}{c_t} \ln \frac{W_i - 1}{W_i}$$

$$\text{Prop } R = \left(\frac{\eta}{c}\right) \frac{L}{D} \ln \frac{W_i - 1}{W_i}$$

$$E = \frac{\eta \left(\frac{L}{D}\right)}{c V_w} \ln \frac{W_i - 1}{W_i}$$

Now, let us go back to the range equation and (Refer Time: 20:14) equation. What we have learnt, if C L 3 by 2 by C D becomes a primary condition then I should fly such that L by D is equal 0.866 L by D max as simple as that. Another thing what we learn if C L by C D is the governing factor that it should be max like here it should be max then L by D should flies as is equal to L by D max, these two things we have learnt from here. We have not talked about anything or range or endurance here. Fundamentally we have seen whatever may be the condition if you have flies as that C L 3 by 2 by C D is maximum



then you have to look for an L by D which should be 0.866 L by D max. And if you are flying for the C L by C D is maximum then naturally we are looking for L by D equal to L by D max, this is the L by D required.

Now, see how this can be used. Let me write for jet we have seen R is V by C L by D into ln W i minus 1 by W i and endurances L by D 1 by C t ln W i minus 1 by W i. For a propeller driven R is eta by C L by D ln W i minus 1 by W i. And for endurance, it is eta by C L by D 1 by V infinity to ln W i minus 1 by W i. So, let us apply whatever you have learnt here. If I first take endurance, the question is asked for a jet driven aircraft what should be your chosen l by d, so that we have got maximum loitering time or endurance is maximum. So, I say jet is this an E is L by D, this, this, this keeping other thing constant and looking for only L by D. So, for E max this tells me I should choose L by D such that the C L by C D is maximum.

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The image shows handwritten mathematical derivations on a green chalkboard. The top section is for a jet engine, showing the range equation  $R = \frac{V}{C} \left(\frac{L}{D}\right) \ln \frac{W_i - 1}{W_i}$  and the endurance equation  $E = \left(\frac{L}{D}\right) \frac{1}{C} \ln \frac{W_i - 1}{W_i}$ . It concludes that for maximum endurance,  $\frac{L}{D} = \frac{C}{C_D}$ . The bottom section is for a propeller-driven aircraft, showing the range equation  $R = \left(\frac{\eta}{C}\right) \frac{L}{D} \ln \frac{W_i - 1}{W_i}$  and the endurance equation  $E = \left(\frac{\eta}{C}\right) \frac{L}{D} \ln \frac{W_i - 1}{W_i}$ . It concludes that for maximum endurance,  $\frac{L}{D} = 0.866 \left(\frac{L}{D}\right)_{max}$ .

Then only use the maximum which in turn means I should fly C L equal to C D naught by K if I really want endurance to be maximum. If I come here for range propeller driven again I am seeing, R will be maximum if we other thing constant when L by D is maximum. So, it is also similar when I am designing a propeller driven airplane if I am to go for maximum range what L by D I should select, so that I fly at that L by D and that is nothing but I should fly at an L by D which corresponds to C L by C D max. For a

properly driven, if I come to endurance then I find that E is proportional to  $C_L^{3/2} / C_D$ . I hope you could see that this is  $L/D$ .

So,  $C_L / C_D$  and  $V$  infinity has, so this time if I had  $L/D$  and for  $V$  infinity it is  $2w$  by  $\rho C_L$  and  $L/D$ , I write as  $C_L / C_D$ . So, this becomes  $C_L^{3/2} / C_D$ . So, what is the message? Now it is something to do with  $C_L^{3/2} / C_D$ . So, for  $C_L^{3/2} / C_D$  I know I need to select  $L/D$  which is  $0.866 L/D_{max}$ . So, for this, I should select  $L/D$  this is equal to  $0.866 L/D_{max}$  or  $C_L / C_D_{max}$ , but you having answered what should be the  $L/D$  when I am flying a jet machine and designing an aircraft for a range maximum. Let us look into that.

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$$R = \frac{V}{c} \left( \frac{L}{D} \right) \ln \frac{W_{i-1}}{W_i}$$
 (arrow pointing to  $L/D$ )

$$R = \frac{1}{2} \frac{\rho W^3}{c P C_L} \cdot \frac{C_L}{C_D} \ln \frac{W_{i-1}}{W_i}$$

$$R \propto \left( \frac{C_L^3}{C_D} \right)^{1/2}$$

$$C_L = \sqrt{\frac{C_{D0}}{3K}}$$

$$\frac{L}{D} = \frac{C_L}{C_D} = A \left( \frac{L}{D} \right)_{max}$$

$$A = ?$$

Now, we are looking for that condition, so that I design a jet driven airplane for maximum range. And what is our aim we want to ask a question, what  $L/D$  I should fly  $L/D$  in terms of  $L/D_{max}$ , because I will design my aircraft for a particular  $L/D_{max}$ . Let us see carefully this expression  $R$  is  $V$ ,  $V$  means again  $2w$  by  $\rho C_L$  here under root here into  $C_L / C_D$  into  $\ln W_i$  minus 1 by  $W_i$ . So, this range proportional to  $C_L$  to the power half by  $C_D$  and this range will be maximum if  $C_L$  to the power half by  $C_D$  is maximum. This you also not new nothing new, we have already studied in your performance course.

And this essentially means a  $C_L$  will be  $C_D$  naught by  $3K$ . It is clearly got  $C_D$  naught by  $3K$ . Now I ask you a question, can you tell me approximately what should be the  $L$

by  $D$  or  $C_L$  by  $C_D$  I should fly let say times of  $L$  by  $D$  max, some factor let us say  $A$ , what is the value of  $A$ , how will you find out? So, I leave this lecture here I would expect that when I come tomorrow we have already made an attempt to answer this question, then only I will think that really we have been really communicating well.

Before I end I must tell you our aim is we are looking for what is that  $L$  by  $D$  I should fly that is a big question. Theoretically, we will have some number then we have to ensure that the airplane is indeed capable of generating that much of  $L$  by  $D$  as dictated by these things and that is what is a major task of the designer. So, what I am doing making you understand why this is required, so that I can comfortably go to a situation is how it can be achieved unless you know why it is required, answering how it can be achieved becomes less motivated.

So, next class may be another two, three class will be all focusing on  $L$  by  $D$ , so that we have indirect exposure to aerofoil shapes, its lifting characteristics, drag polar, all those things will be touched upon, so that you can pick some number right that is what a designer will do.

Thank you very much.