

Aircraft Design
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Lecture - 07
Fuel Consumption: Cruise Flight

Good morning friends. We were continuing our effort to understand how to compute how much fuel is consumed during cruise or maybe during loiter. And in that process we have seen depending upon type of aircraft.

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The image shows handwritten equations on a green background. The first two equations are for a jet engine:

$$\text{Jet: } R = \frac{V}{C} \left(\frac{L}{D} \right) \ln \frac{W_{i-1}}{W_i}$$

$$E = \left(\frac{L}{D} \right) / C \cdot \ln \frac{W_{i-1}}{W_i}$$

The next two equations are for a propeller engine:

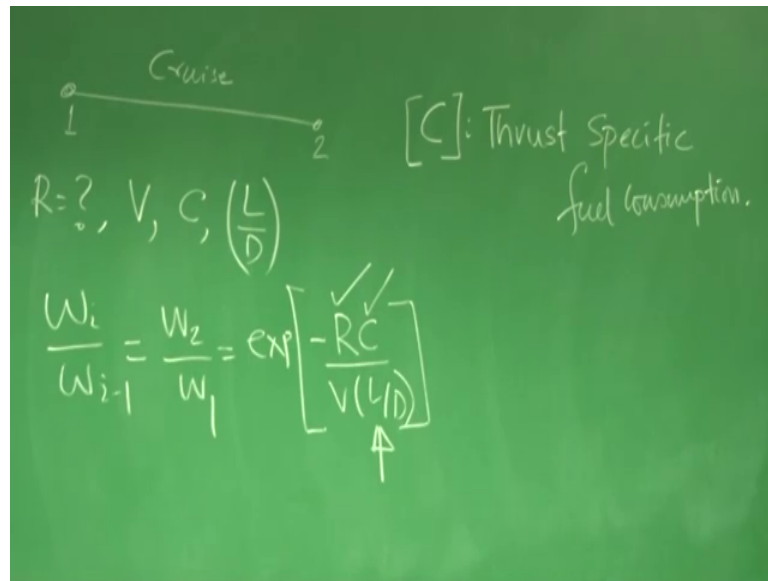
$$\text{Prop: } R = \frac{\eta}{C} \frac{L}{D} \ln \frac{W_{i-1}}{W_i}$$

$$E = \left(\frac{\eta}{C} \right) \frac{L}{D} \frac{1}{V_{\infty}} \ln \frac{W_{i-1}}{W_i}$$

We are using that is precisely what is the engine being used if it is jet driven engine then we have seen R can be expressed as V by C, L by D to Ln W i minus 1 by W i, and for endurance it is L by D divided by C into Ln W i minus 1 by W i.

Similarly, for propeller driven engine here we know that R is eta by C L by D Ln W i minus 1 by W i. Similarly endurance is eta by C L by D 1 by V infinity to Ln W i minus 1 by W i. This we have derived right and what we are looking for let us say we are talking about jet engine driven aircraft, and I am moving from point 1 to point 2.

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$R = ?, V, C, \left(\frac{L}{D}\right)$

$[C]: \text{Thrust Specific fuel consumption.}$

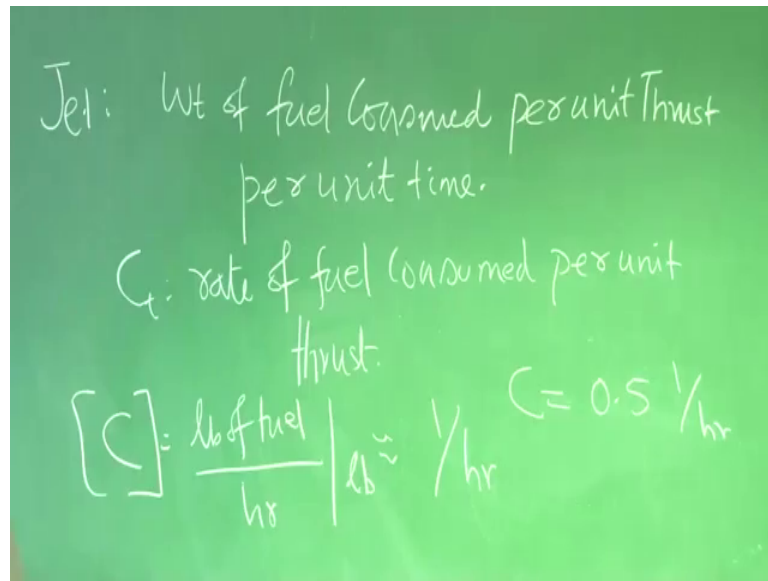
$$\frac{W_2}{W_1} = \frac{W_2}{W_1} = \exp\left[-\frac{RC}{V\left(\frac{L}{D}\right)}\right]$$

Which is let us say cruise at a certain speed, then for fuel consumption if I use this first expression, I need to know what is the R, what is the V with which I am cruising what is the value of C and what is the value of L by D. If I know these values they can I easily write W_i by W_{i-1} , which is nothing by W_2 by W_1 , if I refer to this diagram that will be equal to exponential minus R into C by V L by D, that will be coming from here right.

So, if I really want to know W_2 by W_1 means during this phase, how much fuel has been consumed I need to know the value of R that is how much range I want to fly and it is a jet driven engine what is thrust a specific fuel consumption for type of engine, and do not forget these are all initial sizing problem, we are handling some approximation will be there mostly they are based on historical data, V you know at what speed I want the cruise and the major question also comes at what L by D should I fly. As far as L by D is concerned we will be discussing little later we have already build up our background for selecting L by D.

Let us focus on C that is thrust specific fuel consumption. Let us revisit what is the definition of thrust specific fuel consumption and if you recall it is for jet.

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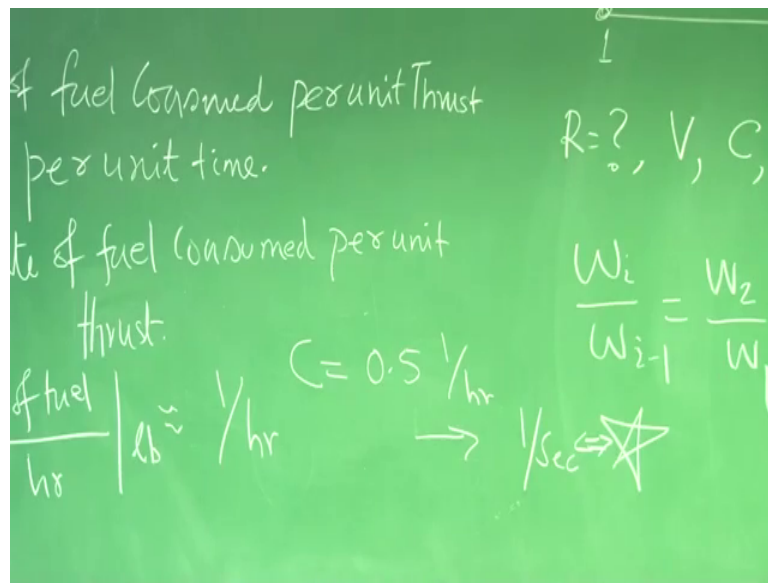


We define it as weight of fuel consumed per unit thrust per unit time. So, typically if the C_t is rate of fuel consumed per unit thrust. And if you see most of the handouts given or most of the design charts available, they are all in FPS unit or sometime inconsistent unit if you read, earlier books even recent books also the units will be in FPS the there are many attempts to convert into exile units.

But you can understand all those tools jigs and fixtures if you already made they have been made based on FPS system. So, there is a resistance in converting to exile units, but things are improving. Since most of the books we are talking about all those calibration all those correlations through FPS unit I will also use FPS unit with lot of word of caution. So, that you can correctly use it, you will find that most of the book C they have expressed in a unit pound of fuel per hour per pound of fuel right.

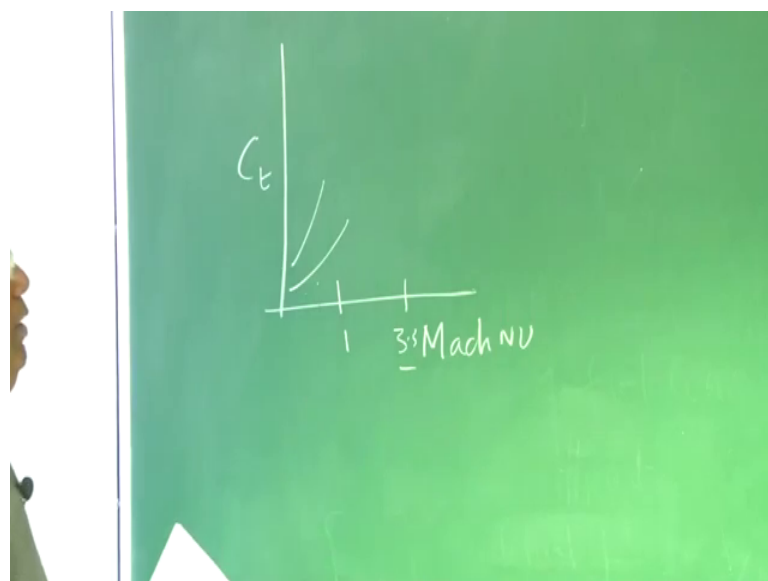
So, this is rate of fuel being consumed and per unit thrust which is in pounds. And if I expand it you will find this will be unit will be given in terms of one by hour. For example, you will find the values of C for such cases may be given as 0.5 per hour, but when you want to use this formula, here we have to be consistent in unit. So, my examples are with FPS unit. So, C I should convert it into per second, this is very important and you will see while I solve example I will be using that this is one observation.

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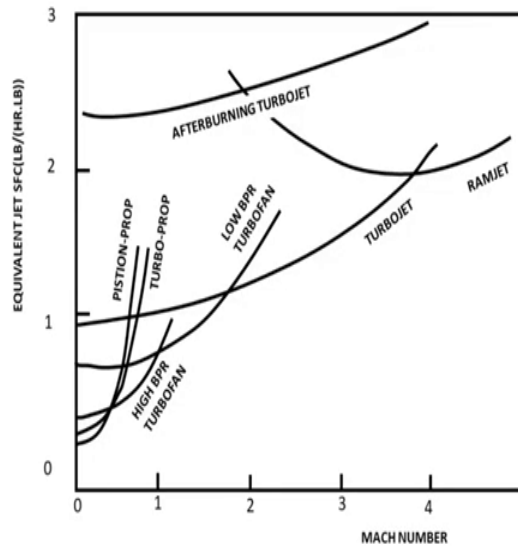
Second observation is that we although we will be giving example using an aircraft driven by jet engine, but we should also know how to handle if it is a propeller driven engine, in that case how do I handle this C the specific fuel consumption I thought I must mention at this point how to handle that. Before I come to that as I have been telling, what is the value of C typically I wrote around 0.5 or 0.6 for cruise some historical base number to ensure that we can pick those number depending upon type of engine you are using.

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You can refer one chart which I will be showing it to you typically that gives value of C_t versus Mach number for different type of engine, you can refer the figure.

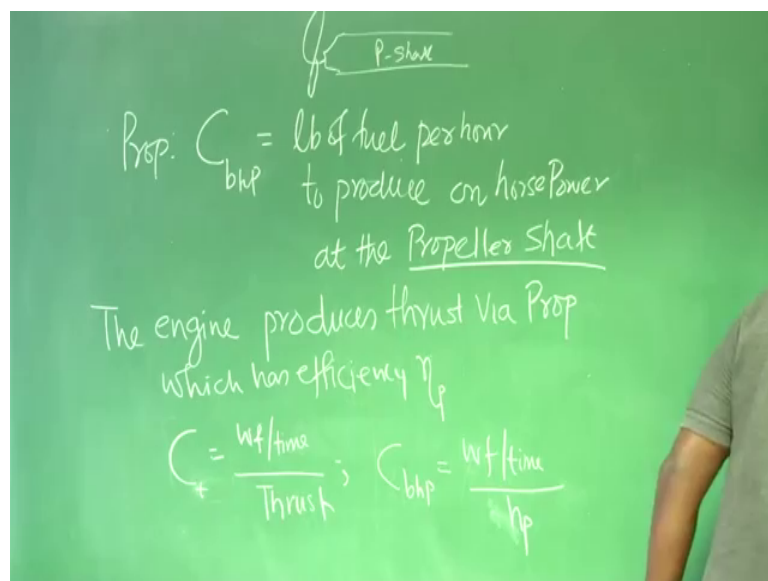
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There and for different combination of engine different type of airplane a type of Mach number you are flying you can pick the value of C_t based on historical data.

But when I say I repeat again these are all initial preliminary sizing. So, these are some guideline number. Finally, anyway we have to modify this improve this right.

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Now coming back to the propeller part most of the book will be using some notation and I will be also following those notations. And let us see for propeller we use something C_{bhp} . This is defined as pound of fuel per hour, to produce one horsepower at the propeller shaft. This is important at the propeller shaft. This here (Refer Time: 10:03) combustion what is the power available at the shaft and that is what we are talking about propeller shaft. And you know that the power is available at the propeller shaft how much you can extract that depends upon what sort of propeller you have coated and what is the efficiency right.

So, now the question comes how do I get a feel, from this power available and from the extraction of power, I want to know how much thrust it will be generating. That gives us a better understanding of design to fix initial numbers. So, there will be a similarity in interpretation between C_t and C_{bhp} , via how much thrust it will produce. So, we will be doing that now where the most of the book, books are doing that what is the physics behind it we write the engine produces, this is important thrust via propeller no problem which has efficiency, η_p , but exactly I was telling what is the power available at shaft, what is the propeller efficiency that will decide how much it has been extracted, and we are now telling the engine produces thrust via propeller which has efficiency η_p , this is the question we are going to address. So, that I have a similarity in my understanding between C_t and C_{bhp} right.

So, if I do that then I write C is weight of fuel per unit time by thrust that is what. In fact, C_t that is the thrust is specific fuel consumption and now also we know C_{bhp} is again rate of fuel, but per horsepower per unit horsepower. This is the 2 different ways of defining C_t , which is thrust a specific fuel consumption. And this is we call the specific fuel consumption for a propeller driven airplane. And we are trying to look for how to C_t and C_{bhp} are related. Or we want to know how much fast this gentlemen propeller driven you will generate for a given condition for a typical value of C_{bhp} .

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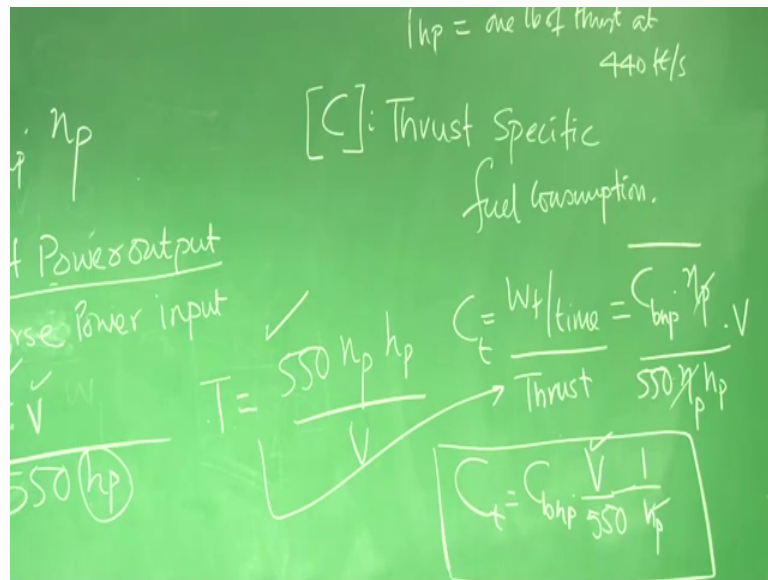
$\frac{W_f}{\text{time}} = C_{bhp} \eta_p$
 $\eta_p = \frac{\text{Thrust Power output}}{\text{Horse Power input}}$
 $\eta_p = \frac{T \cdot V}{550 \text{ (hp)}}$
 $T = \frac{550 \eta_p \text{ hp}}{V}$
 $C_t = \frac{W_f / \text{time}}{\text{Thrust}} = \frac{C_{bhp} \eta_p}{550 \eta_p} \cdot V$
 $C_t = C_{bhp} \frac{V}{550 \text{ hp}}$
 $\eta_p = 0.8$
 $1 \text{ hp} = \text{one lb of thrust at } 440 \text{ ft/s}$
 $[C_t]: \text{Thrust Specific fuel consumption.}$

Now, if I use this C_t , I write it as C_{bhp} let me write this. So, that there are no issues we are trying to find out how do I visualize what is the thrust, I expect for an aircraft which has particular value of C_{bhp} . In return I am looking for a relationship between C_t and C_{bhp} . So, we go back to definition C_{bhp} is pound of fuel per hour to produce one horsepower at propeller shaft, and we know that this is linked with the power available at the propeller shaft, depending upon what type of propeller efficiently, you are putting here is installing here you will get the power available which will be used for giving a thrust to the airplane. So, that is what is written here the engine produces thrust via propeller which has efficiency η_p .

Now, if we see that the definition of η_p we will come here. So, η_p is defined as the thrust power output divided by horsepower input. Because this is the horsepower being available at the brake and finally, what is the thrust power output you are getting which is making the airplane to move. So, if I right now a little more here I know thrust power output is nothing, but thrust into V the speed at which the airplane is moving, and what was horsepower input that was horsepower hp at the brake, but to be consistent in unit I have to multiply 550 because I know the relationship of converting horsepower into the hp power unit. So, once I know what is η_p by using this equation I can write thrust equal to thrust equal to 550 into η_p into hp divided by V no issue.

Now, come back what is our aim I want to build a relationship between C_t and C_{bhp} . So, I start from here C_t is W_f by time that is rate of fuel consumed divided by thrust and for W_f by time I use this equations. So, C_{bhp} into n_p this term of course, I write C_t here.

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Now, for thrust I use this expression for thrust. So, I get C_{bhp} , n_p into V by $550 n_p$ and V get cancelled. So, finally, you get a relationship C_t equal $C_{bhp} V$ by 550 , 1 by n_p this is propeller efficiency. So, this will tell you how much thrust is being extracted, was propeller R or engine is rated through power. And what is that meaning again designers input to us that if n_p is 0.8 , one horsepower you can visualize equivalent to one pound of thrust at 450 feet per second. This is extremely important when you are designing an airplane that is a field for number one horsepower means one pound of thrust at speed 450 feet per second.

But you could also see that this relationship demands the value of V η_p and C_t and C_{bhp} are also function of what sort of setting you have got in the engine, but as I am repeated telling. So, far the exercise is to get an approximate or number, so that I can start configuring the plane which is my preliminary configuration design. I thought this should be made clear to you before we go for further application. If you have any problem in this part, do write in the forum because as a designer, you need to understand these things very clearly and develop a feel.

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Typical Values:

Jet Engine

	Cruise	Loiter
Pure turbojet	0.9	0.8
Low bypass turbofan	0.8	0.7
High bypass ratio	0.5	0.4

Typical values I am using typical values for jet engine aircraft, pure turbojet for cruise and for loiter, if I show here the value of 0.9, it is around 0.8 and for low bypass turbofan mostly you will be using turbofan in our example maybe you may pick high bypass ratio this value is around 0.8 for cruise and 0.7, for loiter and for high bypass ratio, engine this value is around 0.5 and for loiter it is 0.4 these are all typical numbers right for jet engine. Now when you come to propeller driven engine, then the values are for propeller

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$$C_{prop} = C_{bhp} \frac{V}{550 n_p} \quad [C]$$

	Cruise	Loiter
Piston Prop (fixed pitch)	0.4/0.8	0.5/0.7
Piston Prop (variable pitch)	0.4/0.8	0.5/0.8
Turbo-Prop	0.5/0.8	0.6/0.8

Loiter
0.8
0.7
0.4

You know C is equal to $C_b h p$ and V by 550 eta p. Say if I give the values of $C_b h p$ and next value is eta p because you understand that when I am trying to get the meaning of C_t via $C_b h p$ this will change for different value of propeller efficiency. That is why these values are given in this fashion say for a piston prop like fixed pitch propeller, this value will be for cruise this is 0.4 and eta p is 0.8 and for loiter it is 0.5 and 0.7 if you take as n p eta p. So, this is top one numerator is $C_b h p$ value and denominator is eta p value it is required because you have seen that C_t and $C_b h p$ are related to eta p.

Similarly, for piston prop, but variable pitch variable pitch this value will be 0.4 by 0.8 and 0.5 by 0.8, that is the value of $C_b h p$ will be 0.5 and eta p is 4.8 for turboprop which is very popular turboprop this value is 0.5 by point and 0.8 and then it is 0.6 and eta p is 0.8. These are typical numbers right L means loiter now you are expert as (Refer Time: 21:26) cruise.

So, this is a guideline to select the values of $C_b h p$ or C_t depending upon type of engineer using for what to find out what is the fuel consumption during cruise or loiter right. So, let us solve one example, we have been talking about. So, many formulas etcetera and there is a every chance you lose the focus let us come back to one application. So, that it goes to your mind for example, this is station one to station 2 which I call it cruise.

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The image shows handwritten notes on a green background. At the top, a diagram depicts a horizontal line representing a cruise segment between two points, labeled '1' and '2'. Above the line, it says 'Cruise' and 'V = 0.6M'. Below the line, it says '9,114,000 ft' and '≈ 1500 nm'. To the right of the diagram, there is a question mark and the ratio $\frac{W_2}{W_1} = ?$. Below the diagram, the text reads 'Jet Engine. High b-P, ratio V = 0.6M R = 9,114,000 ft'. To the right of this text, there are two formulas: $R = \left(\frac{V}{c}\right) \frac{L}{D} \ln \frac{W_{i-1}}{W_i}$ and $\frac{W_i}{W_{i-1}} = e^{\left[\frac{-RC}{V(L/D)} \right]}$.

Let us say this is around 9 1 1 4 0 0 feet which is roughly equal to 1500 nautical mile. I am writing this unit because most of the documents we find engineering documents in the shop floor will be in nautical miles, if it is a digital regulations nautical miles and feet FPS system you can always find the exile unit balance.

So, this is the range means in this case we are focusing that I want to compute the fuel consumption, if the airplane is cruising from 0.1 to 0.2 for a range of this much of feet at a speed 0.6 Mac. So, what is the fuel fraction that is what is question is what is W_2 by W_1 . If I know this ratio I know how much fuel is consumed provided I know what is the weight at W_1 . So, how do I do that, we are assuming we are talking about jet engine and high bypass ratio this 2 I am speed is 0.6 Mac and range is 9 1 1 4 0 0 feet.

How do I use whatever you have learnt? So, talking about range, so range for jet I know it is V by C L by D to L n W_i minus 1 by W_i , that is W_1 by W_2 this is if I call it one I call it 2 that is reverse it I get W_i by W_i minus 1 is equal to exponential minus $R C V L$ by D . Once I want to apply this formula for a range I need to ask (Refer Time: 24:39) question what is that L by D should fly. So, that R is maximum and again revisit that question is what is that L by D should fly at a given speed v . So, that the range is maximum.

You know the answer already. We will come back when we actually solve this problem we have talked about this if you are forgotten.

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Handwritten notes on a green chalkboard:

- Top left: $\frac{W_2}{W_1} = ?$ with a question mark and a small diagram showing a horizontal line with a vertical arrow pointing up from its center.
- Top right: "Cruise L/D" and "Zet" with a note "0.866 $\left(\frac{L}{D}\right)_{max}$ $\left(\frac{L}{D}\right)_{max}$ [C]".
- Middle left: "1500 n.m." and the range formula $R = \left(\frac{V}{c}\right) \frac{L}{D} \ln \frac{W_{i-1}}{W_i}$.
- Middle right: Fuel consumption rate $C_t = 0.5/hr \approx 0.001389/sec$.
- Bottom left: Weight ratio formula $\frac{W_i}{W_{i-1}} = e^{\left[-\frac{RC}{V(L/D)}\right]}$.
- Bottom right: Cruise speed $V = 0.6M = 0.6 \times 340 m/s$.

So, let me remind you we have seen for jet for the part of cruise I should fly at $0.866 L$ by D_{max} . And if I am loitering then I should fly at L by D_{max} , this already I have addressed in earlier lecture. So, now, you know that if I want to know W_2 by W_1 , I need to know the value of C value of R is known V is known how much L by D_i should fly because it is range for jet. So, L by D should be $0.866 L$ by D_{max} . So, I need to know what L by D_{max} I will be flying. So, I will be talking that in the next lecture, but today I am focusing more on C , how do we use this C and you still remain you need by consistence to get a correct number because most of us come it mistake here because most of us fail in handling C because the units are given in a very funny manner that is why I am putting this time here.

So, now the value of C_t for high bypass ratio turbofan, we have seen this value to be roughly 0.5, 0.5 per hour. If you see any literature or any design handbook even old edition of Raymer the units are like this new edition yes there are changes, but I am following the old edition, but we know that C_t , I have to convert into second because I am working in FPS. So, this is equivalent to 0.0001389 per second just divided by 3600.

So, now I am consistent, although it is given per hour are converted into per second while we using FPS system of unit. So, C_t , I have corrected R , I have taken in feet. So, consistent V_v is 0.6 in Mac. So, I write 0.6 into 3 what is that should I write 3 forty meter per second here let me.

Student: We have to converted into (Refer Time: 27:46).

No this will be wrong because 0.6 Mac I have through working FPS unit. So, I have to write V equal to 0.6 into velocity of sound in.

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$\frac{W_2}{W_1} = ?$
 $C_t = 0.5/\text{hr} = 0.001389/\text{sec}$
 $V = 0.6M = 0.6 \times 340 \text{ m/s} = 0.6 \times 994.8$

Student: FPS unit.

F p s unit and that is 994.8 feet per second that is the speed of sound in FPS.

Now, everything I know. So, I have to just put this numbers here.

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Pipe
 $V = 0.6M$
 $9,114,000 \text{ ft} \approx 1500 \text{ m}$
 $\frac{W_2}{W_1} = ?$
 $\frac{W_2}{W_1} = e^{\left[\frac{-9,114,000 \times 0.001389}{0.6 \times 994.8 \times 0.866 \times 16} \right]}$
 $R = \left(\frac{V}{c} \right) \frac{L}{D} \ln \frac{W_{i-1}}{W_i}$
 $\frac{W_i}{W_{i-1}} = e^{\left[\frac{-RC}{V(L/D)} \right]}$
 $\frac{W_2}{W_1} = 0.852$

So, now, this W 2 by W 1 will be equal to exponential minus R minus R (Refer Time: 28:33) minus 911400 triple 0 into C. C we know it is 0.0001389 divided by V that is 0.6 into 994.8 into L by D L by D we know 0.866 into L by D max, let us take L by D max by 16

we have not talked about how to select L by D we will talk about selecting L by D_{max} in the next class for this computation you take L by D in max to the 16, and 15 16 7 teen these are typical values of L by D_{max} or most of the airplane.

What is important here because it is the range part you are computing and we have to ensure that for jet driven engine I need to fly at an L by D which is equal to 0.866 time L by D_{max} . And if I do that then I get W_2 by W_1 equal to 0.852. So, from this ratio if I know W_1 I know during this length how much fuel is consumed right. Similar exercise I can do it for loiter also only equivalently the numbers will change right in the final example we will be taking everything together, but it is better that piecewise we are very clear about the units, as I repeatedly I am telling the units are very inconsistent right especially C and C_t etcetera right.

Thank you very much.