Aircraft Design
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Lecture - 06
Range and Endurance: Propeller-driven Aircraft

Good morning. Let us continue the discussion on fuel estimation.

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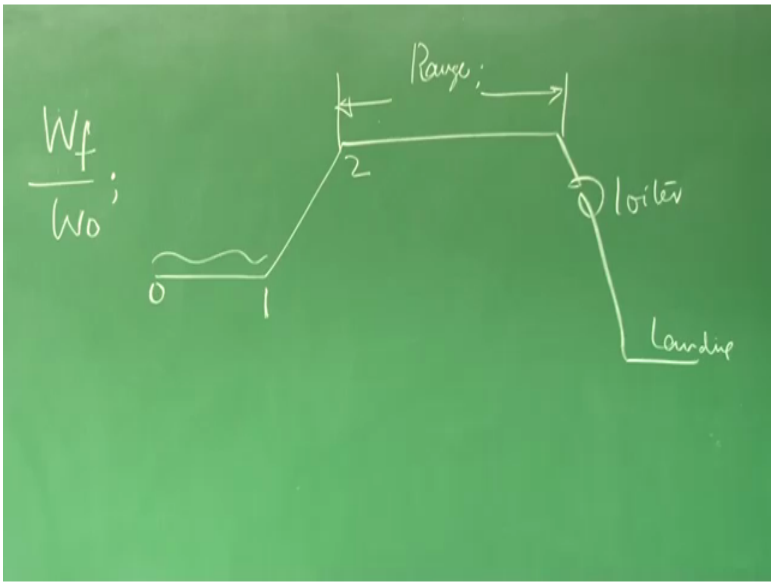

That is we are trying to find out or to be more precise; we are trying to estimate roughly, how much will be W f by W naught this fuel fraction and we have agreed and understood this is primarily decided by the mission the aircraft is supposed to complete, for example, if I take a simple mission where this part is warm up and take off climb then cruise loiter where landing.

Today we will just go back to some basic understanding for the cruise and loiter whatever you have studied in air airplane performance we will revise that. So, that that will help us in getting an idea how to estimate W f by W naught for mission as a range or the mission as a loiter mission as a range means how many kilometres; mission as a loiter how many seconds or hour going to be in air

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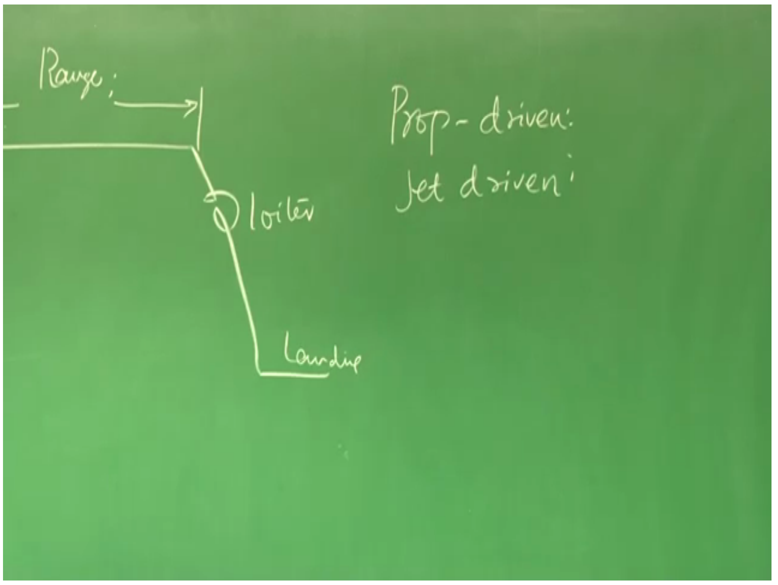

So, let us revisit range and endurance for 2 type of aircraft one is propeller driven let us say I C engine based another is jet driven.

So, basic question would be if I am using a propeller driven aircraft and then to complete a particular range or to be in air for particular time how much fuel will be consumed similar question will ask if I am flying the machine using a jet driven engine how much fuel it will consume for meeting a particular range requirement and particular endurance or loiter requirement; we want to go back to this airplane performance as a revision. So, that we can really use it for designing our aircraft as I have been always saying whatever we have understood in airplane performance when you are they applied in design you will find we do a certain different interpretation in terms of implementation.

So, that I can we can synthesize those understanding easily and keep your mind open on that and you see how beautifully people have helped us in creating such understanding. So, if I recall what was the definition of range or what is the definition of range next talk about range it means the ground distance the airplane will cover for a given tank full of fuel.

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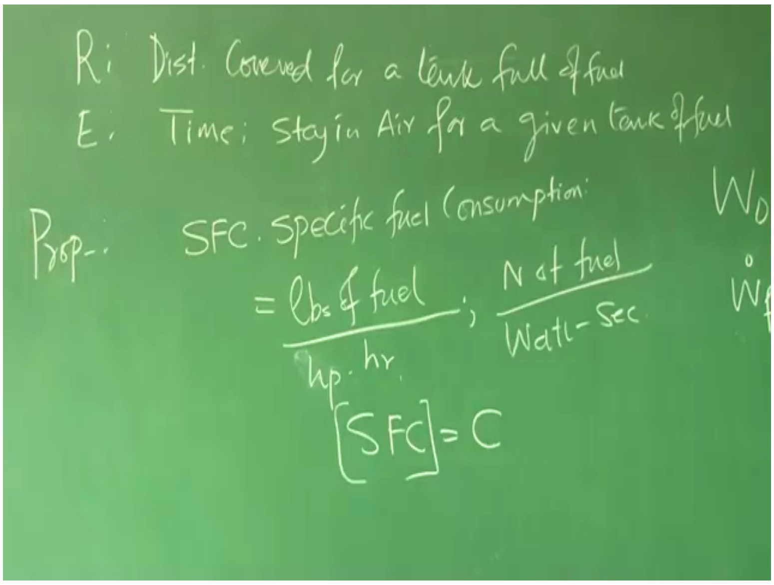

So, range is the distance covered for a tank full of fuel and endurance it is a time stay in air again for a given tank of fuel this is the basic definition we all understand.

Now, we will first talk about quickly you go by the propeller driven aircraft and you know that when you have analyzed propeller driven aircraft we talked about SFC that is a specific fuel consumption. This is specific fuel consumption and how do you define that this was the (Refer Time: 04:54) pounds of fuel per unit horsepower or per unit power and in FPS you may use horsepower and per unit time maybe hour this is a inconsistent unit, but understanding is how much fuel what is the weight of the fuel how much means weight of the fuel here consumed per unit power per unit time. So, if you want to be consistent it is Newton of fuel consumed per watt second that is SFC and relevant for I C engine driven propeller airplane.

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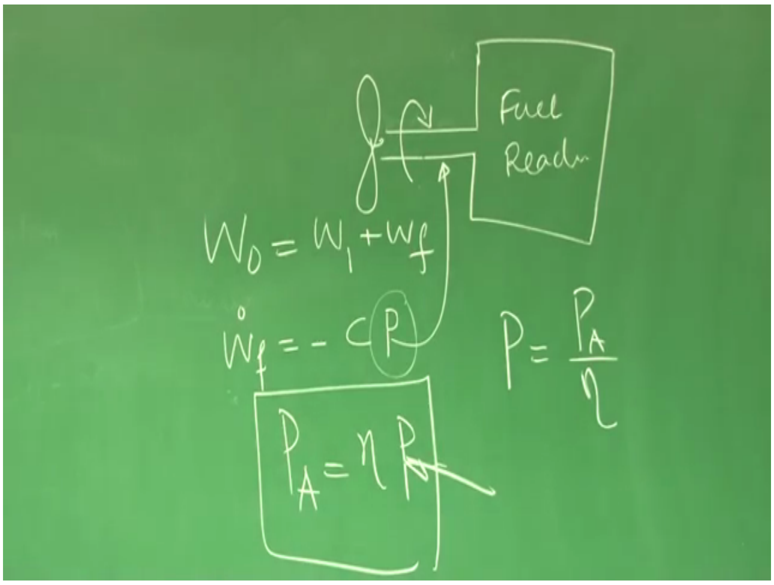

So, now if I write W naught is equal to W 1 plus W f; it is very clear what is W naught W naught is the gross weight what is W 1? All the weight accepts the fuel weight right then I can write W f dot is minus C into P; what is C here? This SFC my notation will use letter c which you have used there also since it is W f dot rate of change of fuel consumed. So, as per the definition W f dot can be expressed as minus c into P what is this P p is the power wire that should be very clear if this is here the fuel reaction going on and you are getting some power at the brake shaft is rotating.

So, this P is this P because the fuel has consumed to give A P at the brake, but how much power you are going to utilize to decide it by the moment you put a propeller and then you say power available is eta times P that is this P was available at the brake, because of fuel reaction combustion etcetera and power available for the machine is decided by how efficient is the propeller and that is propeller efficiency eta into power available at the brake.

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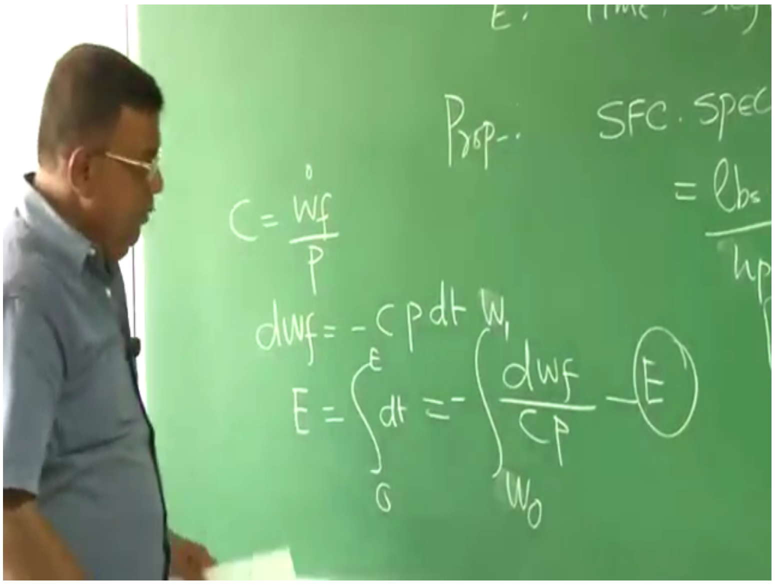

So, this is the power available this is nothing new I am telling. So, what I can do now I can write c is equal to W f dot by P and d w f will be equal to c p d t because d w by d t is minus c p the from here I am writing like this if I am looking for endurance the endurance will be 0 to E d t and from this I can write this is 0. So, E d w f minus sign by c into P and when I write here 0 to E since this integral is over w. So, what I have to do if started with W naught and the fuel was consumed it become W 1. So, this part is the definition of E and we should be very very clear that this P is the power available at the brake this P is not equal to power available. So, this P is actually is equal to power available by eta this should not forget it is extremely important.

So, we know this is the definition of E. So, let me erase this let me write E is equal to node W 1 to W naught.

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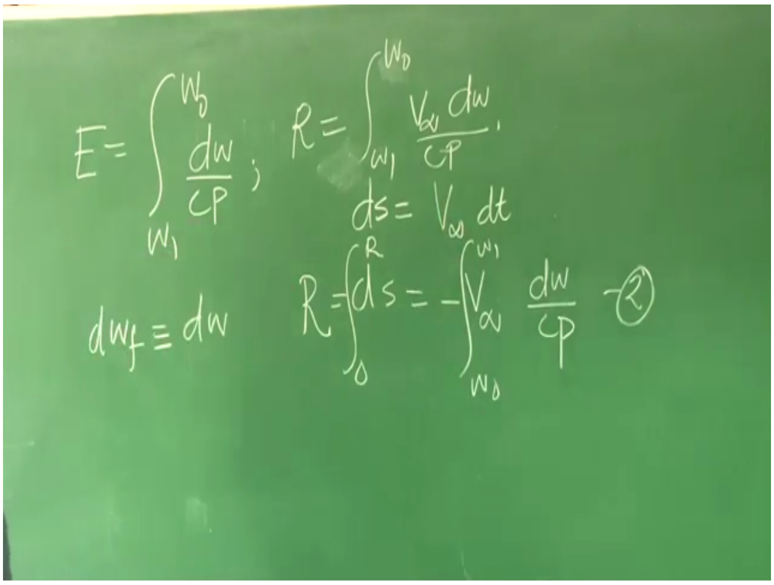

I have absorbed minus sign d w by c p, similarly for range we also do you have an expression like this how do I visualize range this range means I am talking about the range when it is completing cruise flight level unaccelerated flight. That means, the airplane is moving with constant speed. So, I can write d s is equal to V infinity into d t that is the distance travelled in time d t moving with a constant is the V infinity. So, I can write d s equal to minus V infinity for d t earlier we have derived expression I will write W naught to W 1 d w by c p.

Let us have a look here I am writing E is equal to W 1 to W naught d w by c p and you know that d w f is basically d w because the change in the weight of the airplane is primarily and in fact for all the purpose in this as a derivation we have assumed is because of change in the fuel the change in the weight is because of change in the fuel weight only. So, that is why d w is being substituted for d w f.

So, here d s we go V infinity d t for d t we have put this. So, this becomes your expression for the range, because I am now done the integration. So, from 0 to r this becomes your expression for range which I will write it somewhere here. So, that we can refer it equal to 0 or W naught to W 1. So, again I will change the order because minus sign is there so, W 1 to W naught V infinity V W by c p; that is all these two expressions.

So, these are the basic expression for endurance or range fundamentally and do not forget you are talking about level unaccelerated cruise flight from there.

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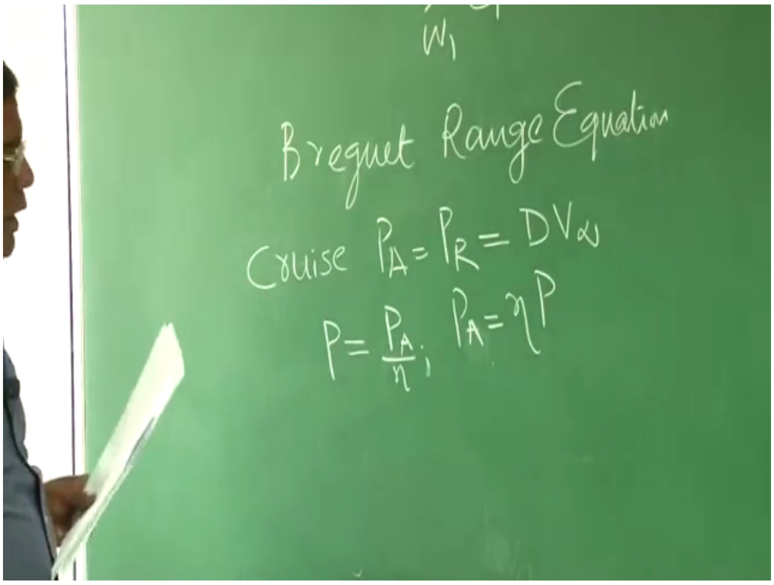

You if you recall we have Breguet Range and Endurance Equation. So, if I talk about range equation. So, if it is cruise in the P available is equal to P required that is power available equal to power required and this will be equal to D into V infinity and you know power at the brake is can be written as power available by eta because you know power available is eta times power at the brake. So, I am just writing it like this.

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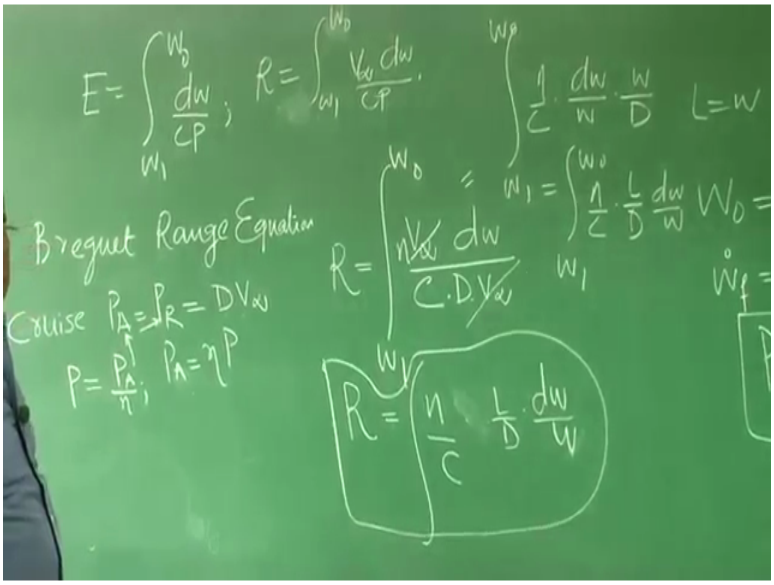

If I substitute this suitably what I will get in the range equation I will be little faster in this I assume that you have done all these things so V infinity d w by c for P for P to be P available by eta. So, P available is power; P available is power required which d into V infinity. So, I will write here d into V infinity and eta will go in the top right power P is P available by eta and P available is power required equal d V infinity that exactly have substituted there.

Now, further I can modify this as eta by c V infinity then L by D d w by W V infinity will go you could see from here and here (Refer Time: 13:23) now you will ask me how this was happened I am sure you know what was the step followed here this is since this gentleman got cancelled V infinity we have eta by c and what we did with a d w by w. So, multiplied and divided by W and what I have done divided by W multiplied by w. So, the expression remains the same and since if the cruise flight lift equal to w. So, I can easily write at W 1 to W naught eta by C L by D w by W that exact expression this is for the range do not forget this we have done it for propeller driven machine aircraft.

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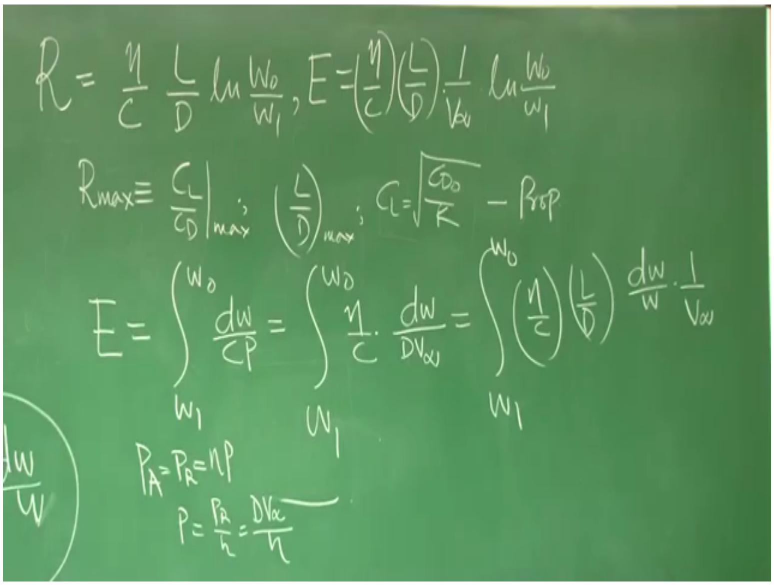

If we go further what sort of expression you will get you will get range is equal to eta by C L by D l n W naught by W 1 what are the assumption here assumption is that eta is not changing c is not changing L by D remaining constant. So, if I take this thing out of the integral I get an approximate expression as range as eta by C L by D L and W naught by W 1 please understand we have assumed L by D to be constant, but you understand also that as I am cruising the weight will go on reducing as weight goes on reducing the lift requirement will change. So, L by D will not remain constant, but what is the designers perspective we are saying we are aware of this we are assuming the change in weight is not large which is not a bad assumption.

So, this is a range expression. So, what is the message if you want to get maximum range you need to fly at such that C L by C D is maximum or L by D is maximum or typically C L should be c d naught by k these are very clear to you (Refer Time: 15:52) and performance we have been repeatedly, but this is for propeller driven airplane propeller now let us see what happens to the endurance; endurance if I start endurance by definition to W 1 to W naught d w by C into P. So, let me do this that then I will explain W 1 to W naught eta by C d w by P is D V infinity, because I hope you understand this is the P is what P available equal to power required equal to eta times p. So, this P is power required by eta and power required is d into V infinity so d into V infinity by eta that I have substituted here and eta and the numerator this is the expression for endurance, right.

Again I can do that little bit of manipulation or rearrangement and I can write it as eta by C L by D V W by W and one by V infinity and again if I assume eta constant c constant L by D constant. So, I can get to the expression for E as eta by C into L by D 1 by V infinity l n W naught by W 1 this is where I want your attention. So, let us have a relook in the last step.

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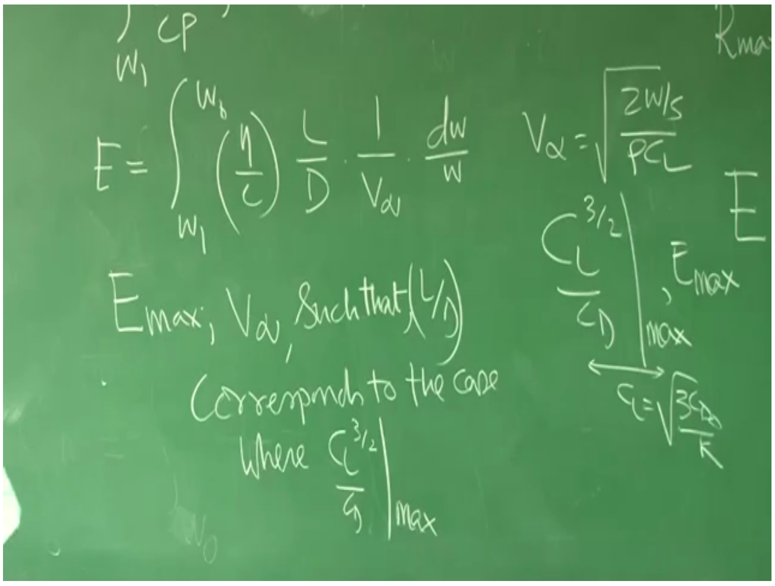

If we see the expression for E if I follow the procedure it comes to W 1 to W naught eta by C L by D and one by V infinity into d w by W if you go to the other performance notes you see what was the next step done was we wrote V infinity equal to 2 W by s rho C L and that is substituted here. So, then we got a condition that C L 3 by 2 by C D should be maximum for E max or a propeller driven airplane, it is clear.

In our performance source we have done is V infinity we replace V infinity by 2 W by s rho C L if I put it here. So, under root C L will be here at this is also C L by C D. So, you get C L 3 by 2 by c d and to maximize the understanding was C L 3 by 2 by C D should be maximum, but from designer perspective. Now see how beautifully it will be handled we will say we understand that for getting E maximum which is for the loiter case C L 3 by 2 by C D should be maximum or the airplane to be flown such a way C L 3 by 2 by c d should be maximum.

So, now we make a different way of interpreting this result we know C L 3 by 2 by C D maximum means particular C L that is C L is 3 C D naught by k. So, how do I interpret this result in terms of L by D I said that if I want E max I should fly at the speed V such that L by D corresponds to the case where C L 3 by 2 by C D is maximum here L by D is not maximum what we are saying for a propeller driven airplane. If I want the endurance at a given speed V I should fly at an L by D such that C L 3 by 2 by C D is maximum right for getting maximum range I should fly at a speed such that L by D is maximum, but here we are not flying at L by D is maximum we are flying at an L by D which corresponds to C L 3 by 2 by C D is maximum that should be the understanding.

Thank you very much.