Aircraft Design
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Lecture - 41
Numerical - Elevator Effectiveness

Hello friends. In a previous lecture, we saw small problems on EAV design and we discussed what will be the tail setting angle, what will be the relative position of wing and tail and today also, we will be continuing the same numerical, but today we will be looking at your elevator effectiveness.

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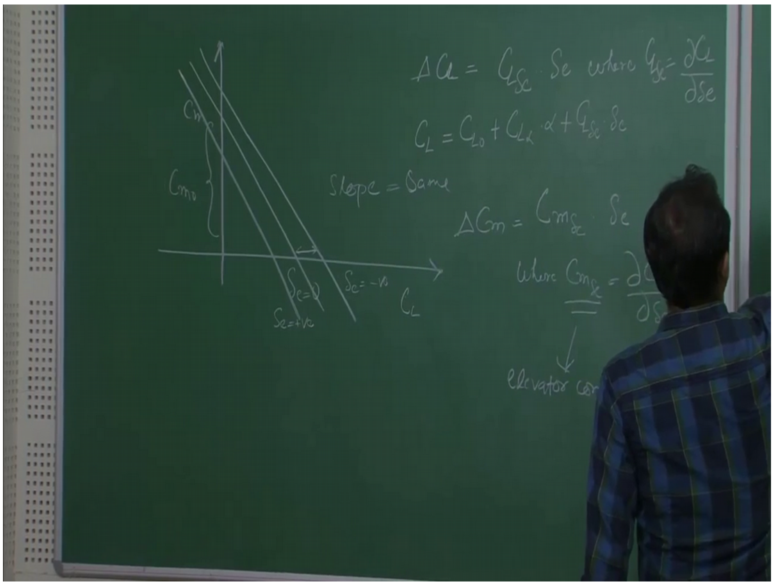

Now, what is the significance of a elevator? As you know we saw yesterday that for an aircraft to be statically stable, now Cm versus alpha or Cm versus Cl curve should have negative slope and it should have a positive Cm naught. We calculated yesterday in order to trim an aircraft at particular Cl what should be your tail setting angle if it is required.

Now, suppose we want to trim your aircraft at a higher value, you can change the elevator setting and it will shift your Cl trim to a higher value. So, this was for deflection of elevator, that is 0 and this is for negative values. Similarly, for positive value, your curve will shift in this direction. From this plot you can notice that the slope remains the same. Slope will be same, but you can see there is a difference between Cm naught or pitching moment as well as lift changes when you use your elevator control surface.

Now, the change in lift due to elevator deflection is given as Cl delta l equals to C l delta e into delta e, where Cl delta e equals to change in lift coefficient by control surface deflection. In this case, this is elevator. We also know that lift coefficient Cl equals to Cl naught plus Cl alpha into alpha and now, this new term will be introduced Cl delta e into delta e.

Similarly, due to elevator deflection, your pitching moment will also change. So, slope is same as you know it is same. So, change in pitching moment coefficient is C m delta e into delta e, where C m delta e equals to change in pitching moment coefficient by deflection of control surface that is elevator in this case. Now, this term Cm delta e, this is known as your elevator control. How effective is your control surface or elevator, it is determined by this aerodynamic coefficient.

Now, the larger the value of Cm delta e, you will have more control over your pitching moment and we have already seen that.

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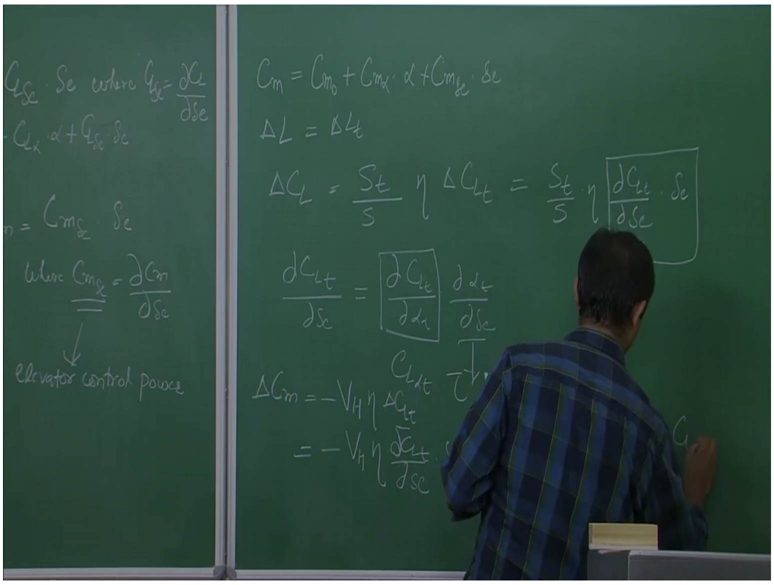

Total pitching moment of an aircraft is given by Cm naught plus Cm alpha into alpha plus Cm delta e into delta e. Now, this change in left is given as change in lift which is given as change in lift equals to ratio of the tail area to your wing area into eta by change in lift to elevator deflection which is equal to S t upon S into eta d Cl t by d delta e into delta e.

Now, this term d C l t by d delta e, this can be written as change in lift coefficient of tail by change in angle of attack at tail into C d alpha t by d delta e. This you already know. This is represented by Cl alpha t and this you can represent by tau. Similarly, your increment in pitching moment coefficient is minus tail volume ratio eta into delta Cl t. We have already seen Cl delta t. What is the value of this term? So, we will replace that term here which equals to minus Vh eta C eta d C l t by d delta e into delta e which will give me C m delta e equals to minus V h eta C l alpha t into tau.

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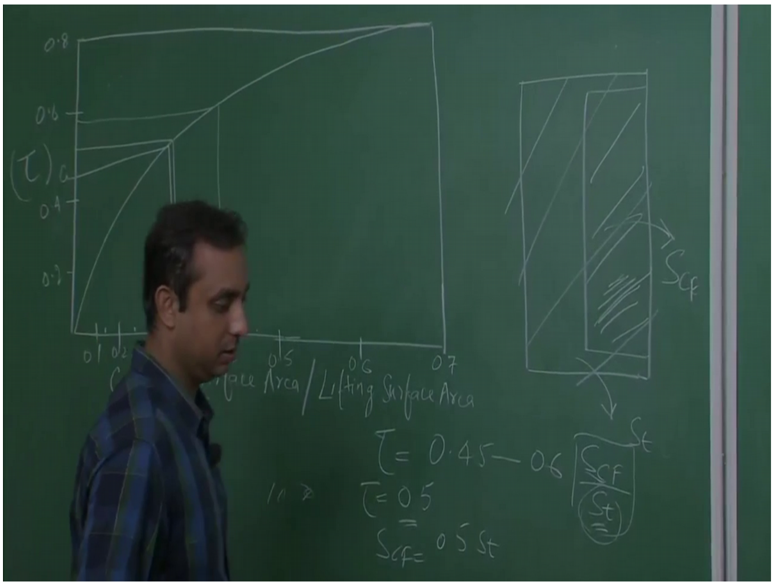

Now, we have introduced a new term tau. Now, this value of tau is taken from a graph which is usually given. It is a design parameter. I am giving you that graph. This is value of tau and this is your control surface area divided by lifting surface area, that is suppose this is your tail and this is your elevator or control surface. So, these value will be area of control surface participation by three area control surface and this is your whole area of tail. So, this will be represented by S t.

Now, this ratio is basically S Cf by S t. Now, the value of tau versus control surface area by lifting surface area where is like this 0.20.40.60.80.7654310.1234567. So, for any control surface whether it be your tail or not, you can basically calculate the value of tau using this formula. You have to calculate what is the surface area and usually it is good to take tau value ranging from 0.4520.6.

So, suppose if I take tau equals to 0.5 and I already know what will be your tail surface area, then I can calculate your control surface area which is 0.5 into S t. Now, based on these all calculations, now value formula which we have derived and the value of tau we will be calculating, what will be the delta e deflection in order to trim your aircraft at an higher value or desired value you can say. So, before that we have to find a relation between what will be the delta e trim required.

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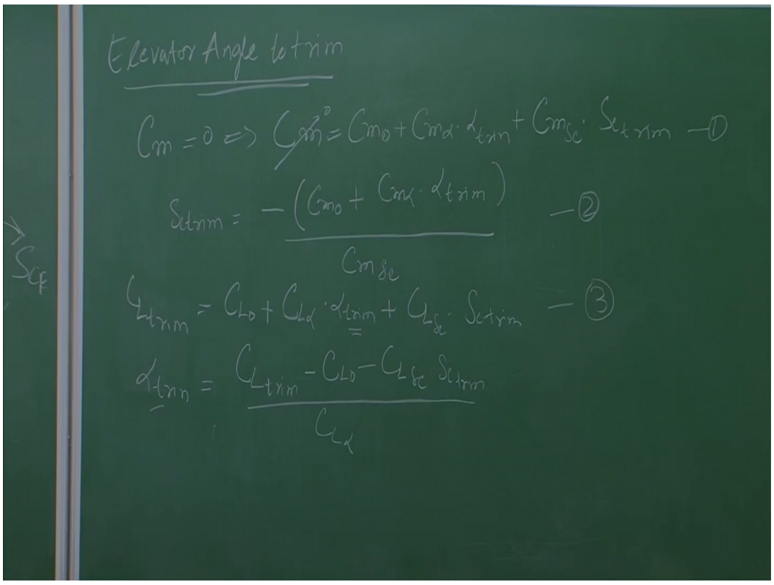

So, elevator angles required to trim as you know in order to trim an aircraft, the resulting switching movement should be 0 and we also know that pitching moment coefficient is given as Cm goes to Cm naught plus Cm alpha into alpha a. This is alpha trim plus Cm delta e into delta e trim. From this equation taking this term as 0, we will get delta e trim equals to minus of C m naught plus Cm alpha into alpha trim whole divided by Cm delta e.

We have another equation for lift coefficient that is Cl trim is given by Cl naught plus Cl alpha into alpha trim plus Cl delta e into delta e trim. This is equation 1 2 3. From equation 3, we can calculate what will be the value of alpha trim which will be alpha trim will be Cl trim minus Vl naught minus Cl delta e into delta a trim whole divided by Cl alpha.

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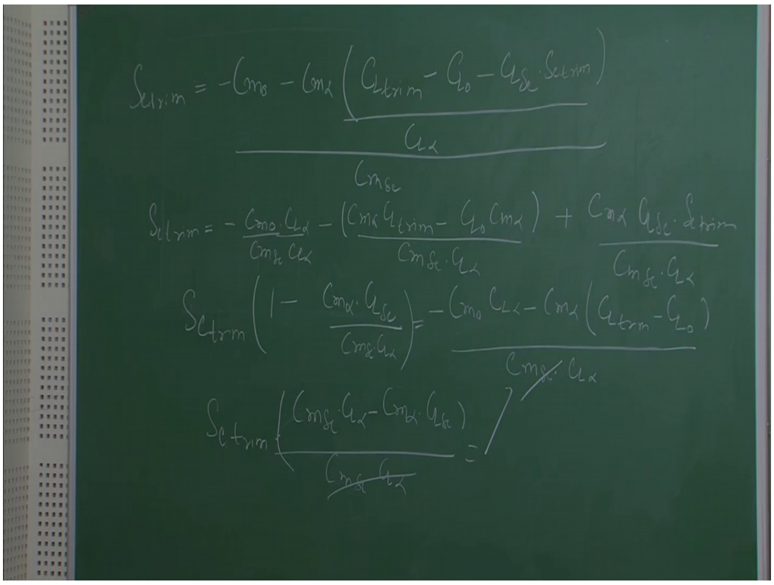

Now, substituting this equation or value of alpha trim in equation 2, we will get substituting the value of alpha trim in equation 2, delta equals to minus Cm naught minus Cm alpha into Cl trim minus Cl naught minus Cl delta e delta e trim this divided by Cl alpha whole divided by Cm delta e n. Now, rearranging the equation, we will get Cl alpha trim equals to minus Cm naught by Cm delta e into Cl alpha minus Cm alpha Cl trim minus Cl naught Cm alpha by Cm delta e into Cl alpha and plus plus Cm alpha Cl delta e into delta e trim by Cm delta e into Cm alpha.

Now, rearranging this equation Cl delta trim 1 minus Cm alpha into Cl delta e by Cm delta e into C alpha equals to minus Cm naught Cl alpha minus Cm alpha into Cl trim minus Cl naught whole divided by Cm delta e into Cl alpha or delta e trim equals to delta trim into Cm delta e into Cl alpha minus Cm alpha into Cl delta e whole divided by Cm delta e into Cl alpha equals to this value.

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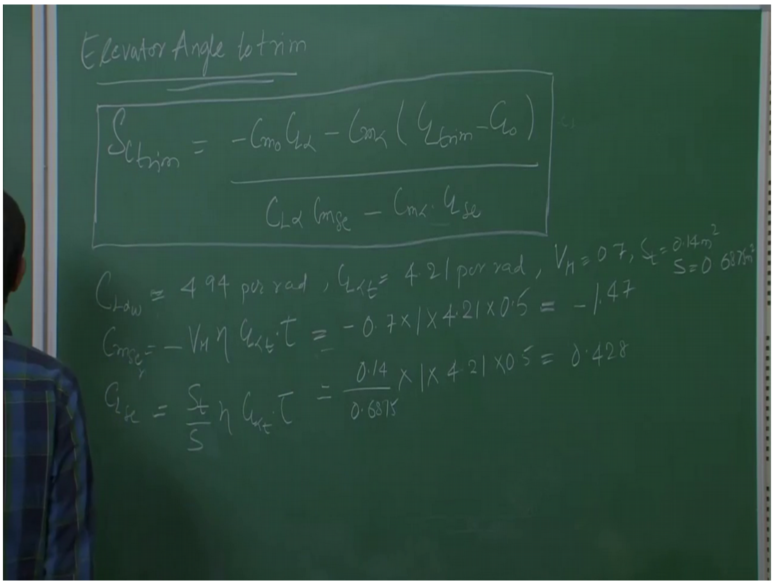

This and this will cancel and from this, we will get the value of delta e trim required which is equal to delta e trim which is equal to minus Cm naught Cl alpha minus Cm alpha Cl trim minus Cl naught whole divided by Cl alpha into Cm delta e minus Cm alpha into Cl delta e.

Now, this is the value of elevator angle deflection in order to trim your aircraft. Now, after derivation what is trim angle elevator angle required in order to trim? Now, we will be revisiting that numerical which we did yesterday. So, some of the values which we got from previous numerical were Cl alpha for wing was 4.94 per radian and the formula which we derived today that is Cm delta s minus VH eta Cl alpha t into tau Cl alpha t. Yesterday we derived or yesterday we calculated it was 4.21 per radian.

Similarly, Cl delta e we derived which was s t upon s eta Cl alpha t into tau. We took VH equals to 0.7. So, substituting this value in this equation, we will get 0.5 minus into eta, we will take as 1 Cl alpha t is 4.21 and tau let us take it as 0.5. We will get the value of minus 1.47. Similarly, for Cl delta e st, yesterday we derived st was 0.14 meter square and s was 0.6875 meter square.

Substituting these values 0.14 0.6875 into eta, we have taken 1 Cl alpha t. We have taken as 4.21 into tau 0.5 which will give me 0.428.

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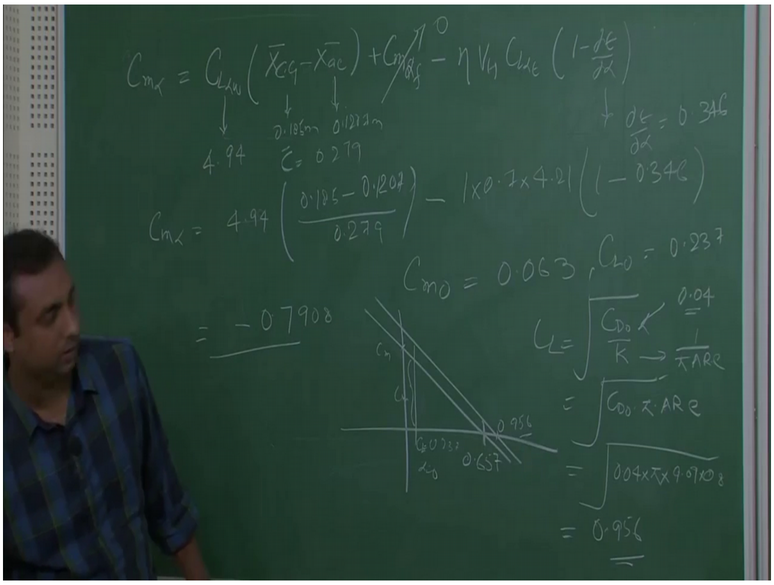

Now, Cm alpha of aircraft will be Cl alpha of wing into x cg of bar minus x n ac bar plus Cm alpha of fuse, Cm alpha of fuselage minus eta VH Cl alpha t into 1 minus d epsilon by d alpha.

We have already seen Cl alpha of wing was 4.94 xcg. We took at 0.185 meters, ac was 0.1207 meters and c, it was 0.279. For simplification we will take same alpha fuselage as 0 and the value of d epsilon by d alpha we calculated in previous lecture was 0.346. Now, substituting this value will get Cm alpha equals to 4.94 into 0.185 minus 0.1207 by 0.279 minus 1 into 0.7 into 4.21 into 1 minus 0.346 which will come around minus 0.7908.

Now, we have calculated all the terms we require in order to calculate Cl delta trim. We already know from yesterday that Cm naught was 0.063. We for simplicity we will calculate that instead of Cl alpha, this is lift coefficient for whole aircraft, but will be for simplification will take it as Cl alpha of wing. So, we have already seen, we already know value of Cm naught. Same alpha wing Cm alpha we have calculated Cl Cl trim. We will take that condition at what value of Cl I have to trim aircraft. I know value of Cl naught which was 0.237, Cm alpha Cm delta we have calculated and Cl delta here we have calculated.

So, we have derived all the parameters. So, yesterday we had seen that we were to trim the aircraft at 0.654 and that we calculated what was the value of Cm naught at Cl equals to 0.237 or alpha equals to 0. So, today I want to trim my aircraft somewhere here and what is that condition that is at minimum drag. For minimum drag, your Cl should be under root of Cd naught by k. I am assuming Cd naught as 0.04. Initial approximation is 0.04.

Now, substituting this value we will get v d naught into pi aspect ratio into e n. Since k is 1 by pi aspect ratio into e, we will get 0.04 into pi aspect ratio was 9.09 e. We took as 0.8 and that will be around 0.956. So, my new trim Cl value is 0.956. So, I have to trim my aircraft as point at 0.956 and in order to do that what will be the delta trim required. I have to calculate that and we have calculated all the parameters required to get that deflection.

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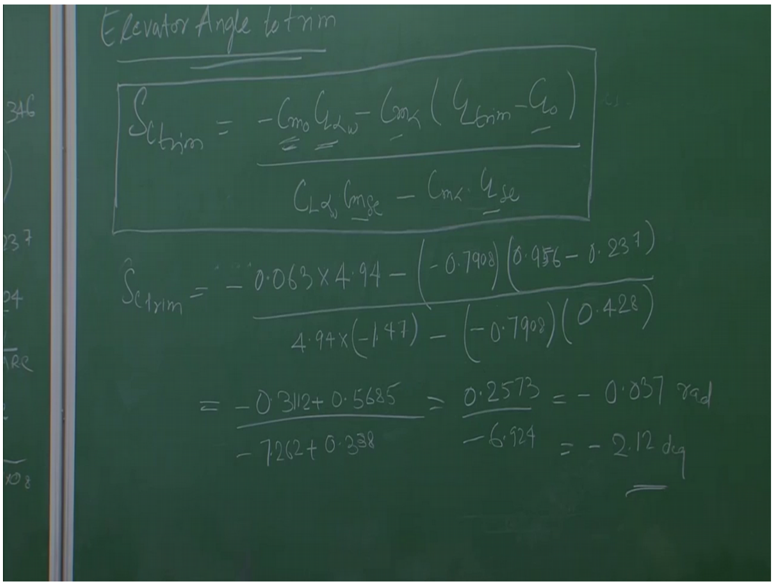

Now, substituting this value delta e trim equals to minus C naught was 0.063 into Cl alpha of wing was 4.94 minus cm alpha was minus 0.7908 into Cl trim. We have calculated 0.956 minus Cl naught is 0.237 whole divided by Cl alpha of wing as 4.94 into Cm delta e. We have calculated as minus 1.47 minus Cm alpha as 0.7908 into Cl delta e was 0.428 which will be equal to minus 0.3112 plus 0.5685 divided by minus 7.262 plus 0.338 equals to 0.2573 divided by minus 0.924 equals to minus 0.037 radian dispersed deflection or equal to minus 2.12 degree. So, that is a value of delta e trim required to trim my aircraft at Cl equals 29.56.

Thank you. That is all for this lecture.