Aircraft Design
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Lecture - 40
Numerical – Pitching Moment

Hello friends, in the last week lecture we mainly focused on longitudinal static stability and how your wing and tail contributed to pitching moment. We saw what were the parameters in order to design your fuse large. What were the parameters in order to design your control surface? Today's lecture we will be focusing on the same topic we will be looking at an example, and that will make a better understanding of what we learned in the previous week.

But before that I want to focus on how an airfoil gives us information regarding your design process or what are the information we get or how to choose an airfoil. This will be we will be looking that first. So, first topic will be how to select an airfoil or what are the information we will get from a airfoil.

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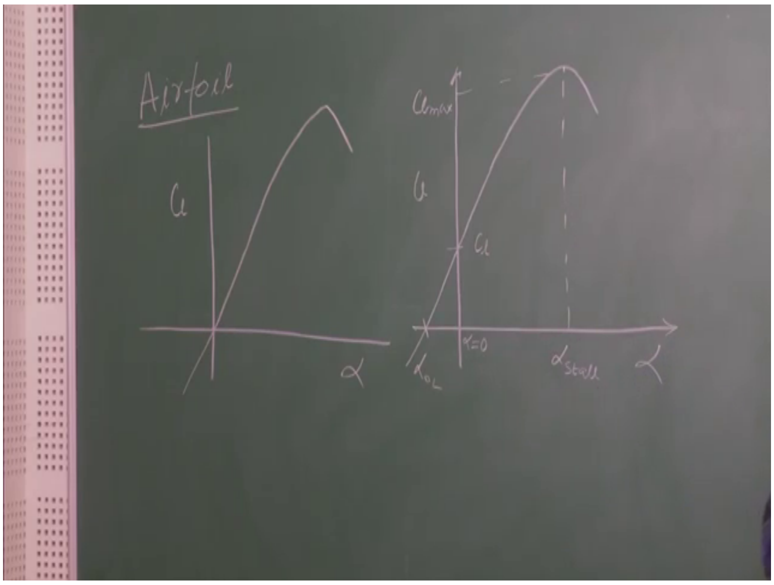

Now, big companies like Boeing or air base they basically go for their own design of airfoil, but for our initially phase we will be looking at our database of huge airfoil which is available, and what are the information we get from that.

The first graph which we will be looking is Cl versus alpha graph. Now for a symmetrical airfoil the graph versus graph for Cl versus alpha is like this, that is this passes through origin whereas, for a cambered airfoil your Cl versus alpha graph will be like this. And what are all the information we get from these graph is your maximum value of Cl. This is Cl max, yours stall angle alpha stall, your angle of attack at 0 lift and what is a value of Cl at 0 angle of attack. These are the information which we get from Cl versus alpha graph.

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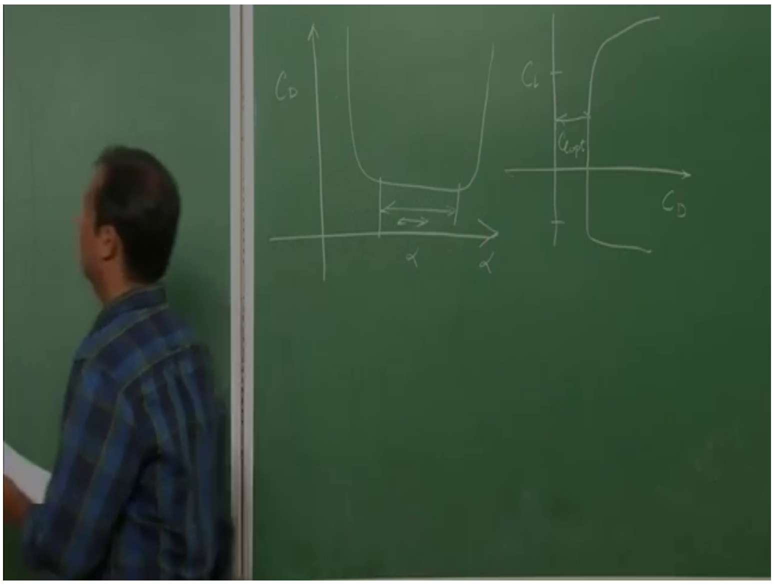

Now, the second graph which I will be focusing on is C D versus alpha, drag coefficient versus angle of attack or in some cases instead of Cl, C D versus alpha you will get to see Cl versus C D graph Cl C D. Now this is in the shape of a bucket and what is the significance this is, for a particular range of alpha or in this case in particular range of Cl you can see the value of C D does not now change.

So, in form a design prospect this is beneficially for me, because a file operating in this particular range, but drag would not be significant or that will remain constant now a part from these 2 graphs the third graph Which I will be looking is your Cm at c by 4 versus alpha graph.

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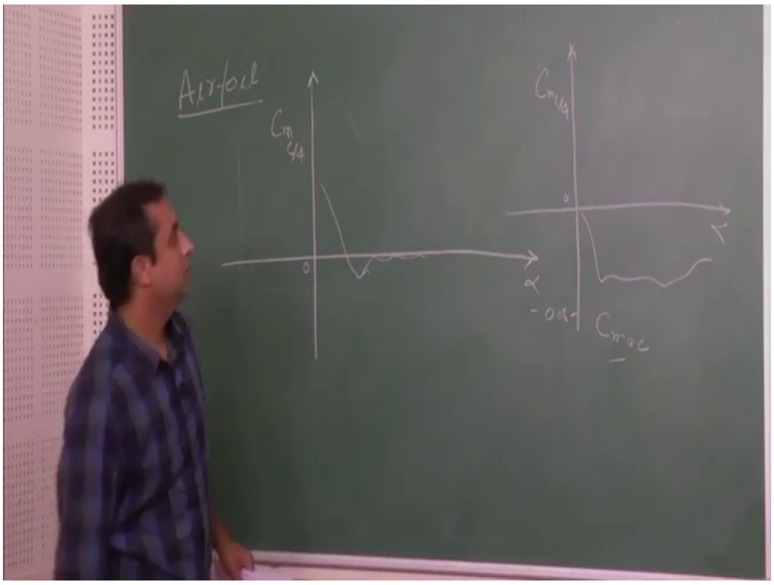

For an airfoil for an airfoil you will see a graph Cm at quarter chard versus angle of attack. And you will see it will vary a lot this is for if in case of a symmetrical airfoil we will see that Cm c by 4 is close to zero, but for cambered airfoils, this will be much lower than your 0, it will be in negatives negative values something like this.

So, from this I can get the values of C mac. So, these are the 3 main graphs which are helpful in order to go forward with my design process, because your wing as well tail will be using some sort of airfoil at the symmetrical or it would be cambered and these are the parameter required in order to go for a good design. Now to summarize what are the information we got from this are, first camber of an airfoil Cl optimum or we can see.

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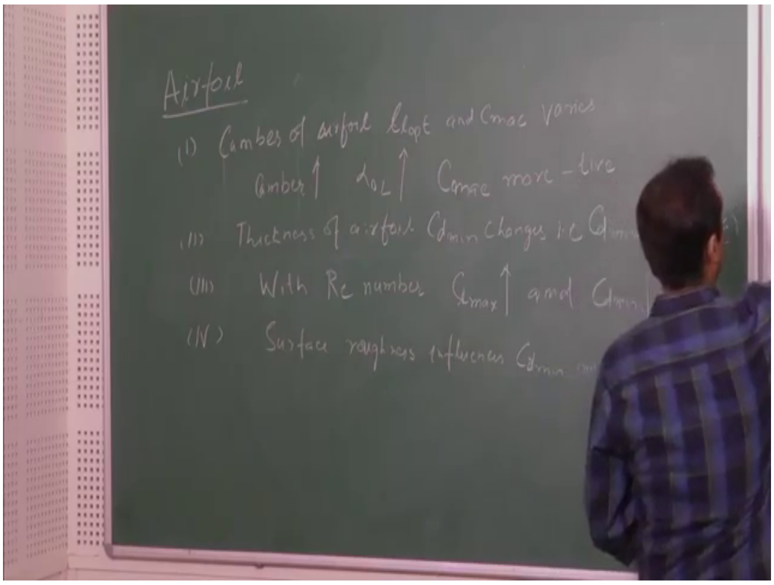

This is your Cl optimum this is your Cl optimum, because for this particular range your drag would not change or it will remain constant, for camber with camber of an air a camber of airfoil Cl optimum and C mac varies, that is with increase in camber increase of camber your angle of attack at is your lift increases and C mac will be more negative.

Second point, thickness of an airfoil thickness of airfoil your C D minimum changes, that is C D minimum increases with t by c value. Third point with reynold number Cl max increases and C D min decreases, and 4th point surface roughness influences your C D C dmin and Cl max. These are the points we have to take in to account when we look in an airfoil. Now there are different type of airfoils present. There are different series of airfoils.

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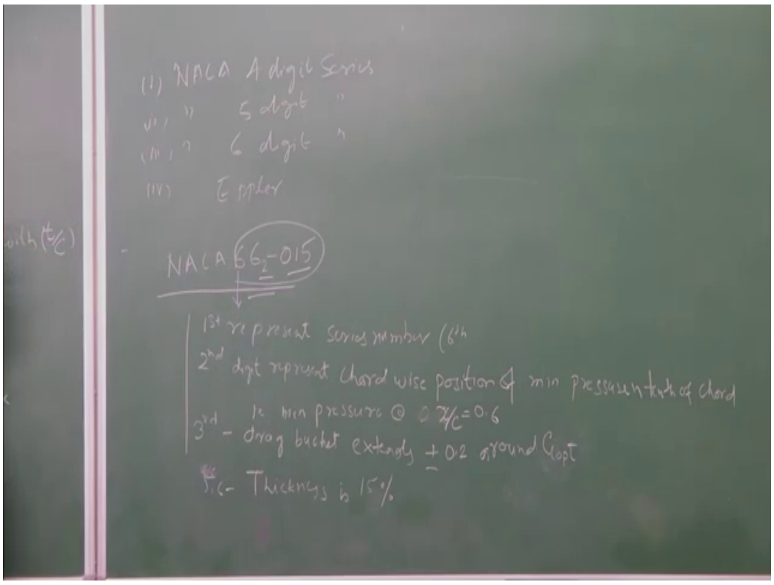

And some of them are your NACA 4 digit series. NACA 5 digit series, NACA 6 digit series, then there are some Eppler series. And each of this airfoils are followed by some numbers, for instance NACA 66 2-015 and these numbers haves significance.

For instance this first digit represents series number, that is 6th, your second digit represents chard wise position of minimum pressure in tenth of chard, that is minimum pressure is that at 0.6 at x by c equals to 0.6.

Now, this small 2 this third digit, this represents your drag bucket extends plus minus 0.2 around Cl optimum. And this last 2 digit 15, 15 this 4th and fifth, fifth and 6th sorry, fifth and 6th these represents your thickness is 15 percent. So, this is a information which we get from a particular series of airfoil. These are the what this series number tells us is these thing. And what are the plot or what are the information we get from plot we have already seeing few minutes before after airfoil design.

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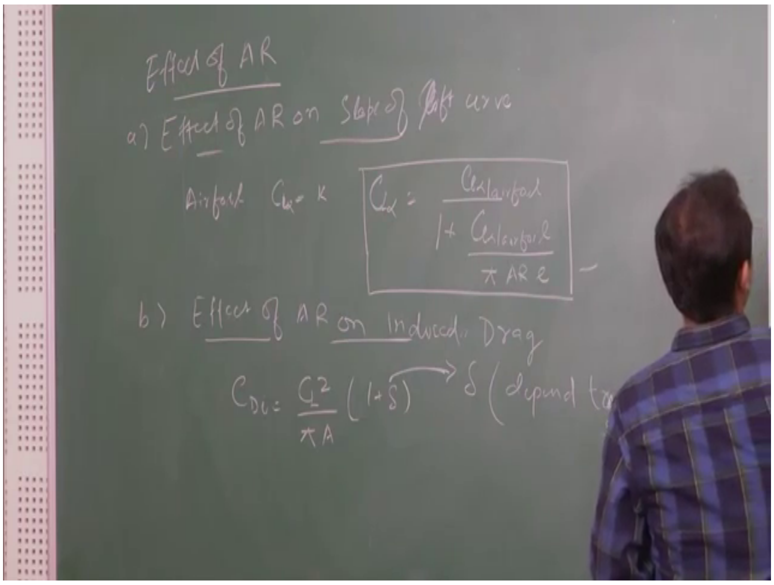

We will be looking at some wing parameters, for instance how do we choose your aspect ratio. So, effect of aspect ratio aspect ratio. First effect of aspect ratio on slope of lift curve, lift curve; we have already seen for a airfoil we know what is the value of Cl alpha some values say k. In order to transform this Cl alpha to 3 d we use formulas Cl alpha equals to, Cl alpha this is for airfoil divided by 1 plus Cl alpha airfoil by pi aspect ratio into e.

So, you can see as you change aspect ratio the value of lifts slope curve will also change, second effect of aspect ratio on induced drag. Now induced drag is given by C Di equals to Cl square by pi a 1 plus sigma. We have this delta this depends on your taper ratio sweep and other geometrical features of an aircraft. A third, third is your structural weight.

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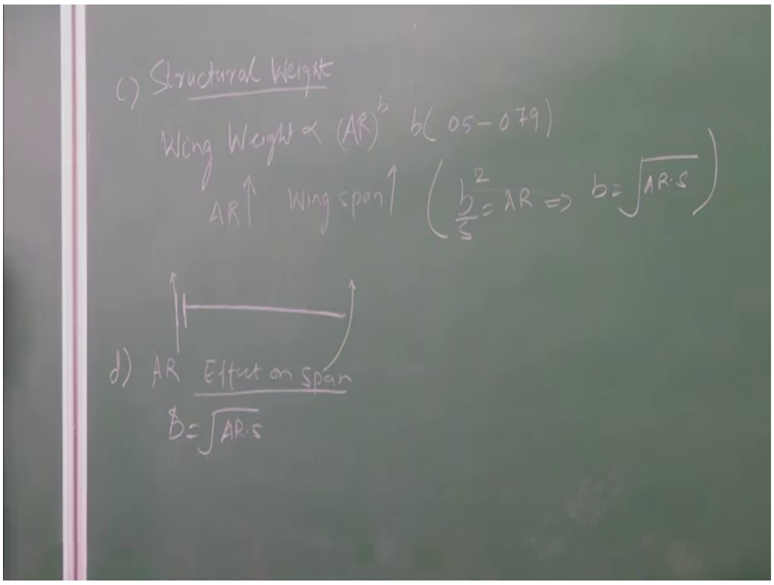

Now we know that wing weight is proportional to a to the or aspect ratio to the power of b where this b is in the range of 0 to 0.5 0.79. And we also know that as aspect ratio will increase your wing span will also increase, since we know that b square upon S is aspect ratio. So, b will be root under aspect ratio into area.

Now, wing span will increase. So, there will be more movement at this root, more wing span more will be the bending moment about this root, as a result your spars will have to encounter more bending force or more weight, that is why structure weight aspect ratio highly effects your structure weight. And point d is aspect ratio effect on span as we have already seeing that span is tatary or span is proportional to root under aspect ratio.

So, higher the aspect ratio more will be your span and rather than geometrical or aero dynamic features this will be the problem of storing. Higher aspect ratio your wing span will be too large and you will have a problem of storage under sort out thing. So, these are the effect of aspect ratio on wing design. Now we will be looking at the numerical based on the lecture we did on previous week, where we saw; what were the information we got from airfoil.

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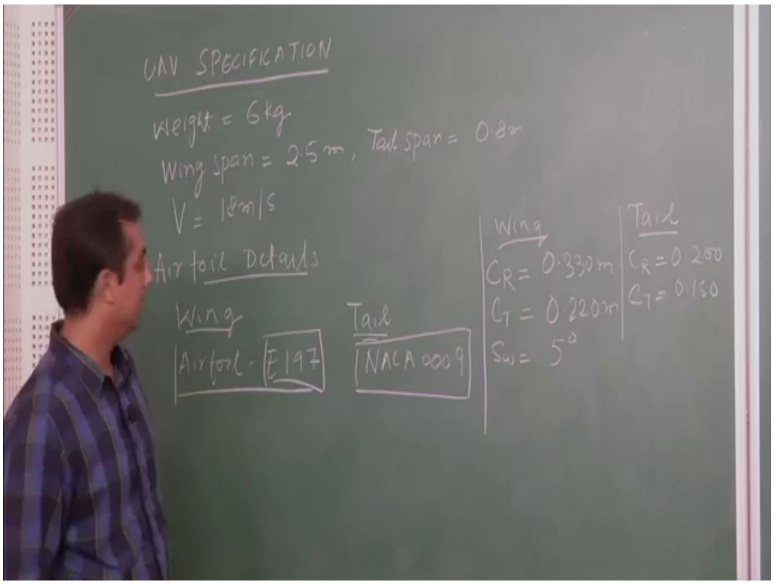

And what were the effect of aspect ratio on wing design. And now we will be doing a numerical based on the lecture we just saw and then what we did it in a previous week. So, we will be doing this for a small UAV.

So, these are the information which we have for a UAV or specification of UAV are, specification weight of UAV is 6 kg or wing span is 2.5 meters tail span is 0.8 meters (Refer Time: 17:20) speed or velocity is 18 meters per second. Now airfoil details for wing airfoil is E197 for tail it is NACA 0009. As you know for tail we use symmetrical airfoil and it is desirable to use a cambered airfoil for wing. Some other informations which are for give (Refer Time: 17:20) are wing root chard 0.330 meters tip chard 0.220 meters and sweep angle 5 degree for tail root chard is 0.200 tip chard is 0.150.

Now, using this information we have to design what will be the tail setting angle and what will be the arm what will be the position of wing and tail airfoil. First of all before starting we have to look at the plot of this airfoil E197 and NACA009 and we have to derive all the information required in order to proceed for design.

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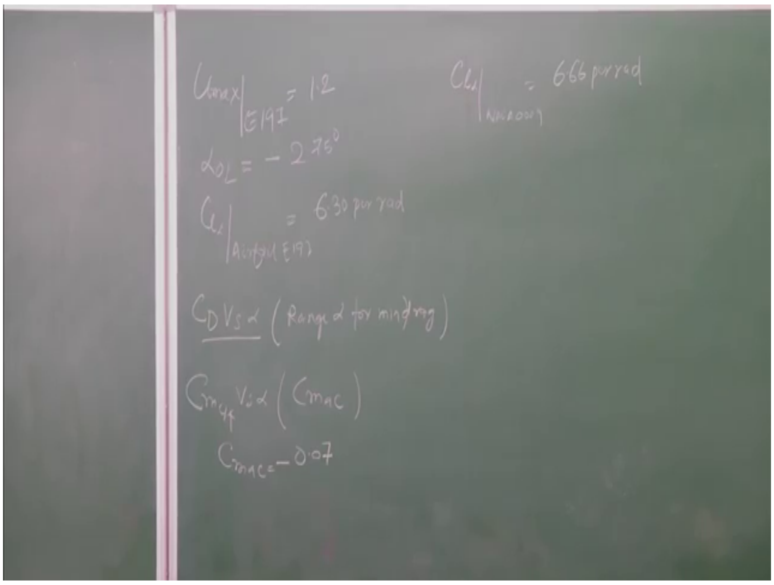

So, once you look at a data for a plot for E197 you will have following information from that plot, Cl max for E197 is 1.2 angle of attack at 0 lift is minus 2.75 degrees your slope Cl alpha of airfoil E197 is 6.30 per radian. Their you will be getting in degrees you have to convert into radian. Similarly for tail you will be getting Cl alpha for NACA 0009 equals to 6.66 per radian.

Second information we will get from C D versus alpha graph, that is in what particular range of alpha should I operate in order to have my minimum drag, range of alpha for minimum drag. And third Cm at quarter chard versus alpha in order to get C mac. And for Eppler 197 C mac is around minus 0.07.

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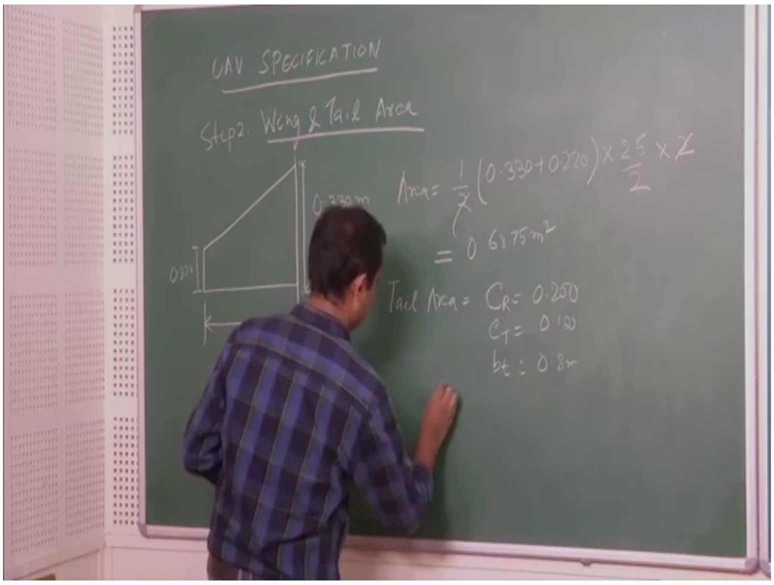

Now second step involves calculation of wing and tail area. Now for the wing it is mentioned that tip chard is 0.220 meters, the root chard is 0.330 meters. So, area will be using taking it as an trespasser. And span is mentioned as 2.5. So, this will be 2.5 by 2, area will be half into some of parallel sides into distance between them.

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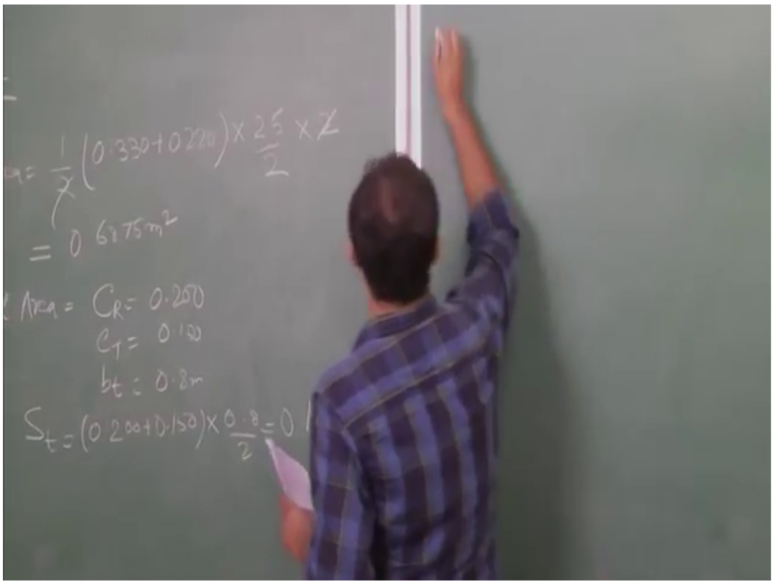

Since this is a area of half wing. So, multiply this by 2 we will get area of 4 wings. So, this comes around 0.6875 meter square. Similarly for tail area C R was 0.2 C T was

0.150, and span was 0.8 meters. So, area of tail will be 0.2 plus 0.150 into 0.8 by 2 equals to 0.14 meter square.

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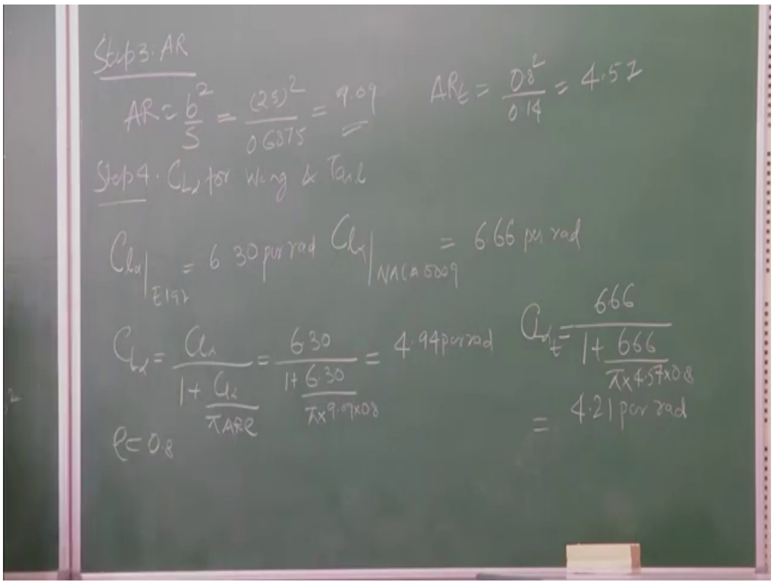

Third step is to calculate aspect ratio. We know that aspect ratio is given by b square by S, we already know for wing is 2.5 meters and area we got as 0.6875, which gives me aspect ratio of 9.09. Similarly aspect ratio for tail is 0.8 whole square by 0.14, which is around 4.57 and in case of tail aspect ratio usually between 4 and 5. Step 4 is to calculate Cl alpha for wing and tail and tail. The information which we got from airfoil is Cl alpha emmpler 197, slope was Cl alpha for NACA 0009 was 6.66, this hour larraine per radian.

Now using the formula for conversion from 2 d to 3 d, Cl alpha is given by Cl alpha by 1 plus Cl alpha pi aspect ratio into e. Here e we are taking as 0.8, which is equal to 6.30 divided by 1.1 plus 6.30 by pi into 9.09 into 0.8. 4.94 per radian and Cl alpha for tail equals to 6.66 divided by 1 plus 6.66 by pi into 4.57 into 0.8 which will come around 4.21 per radian.

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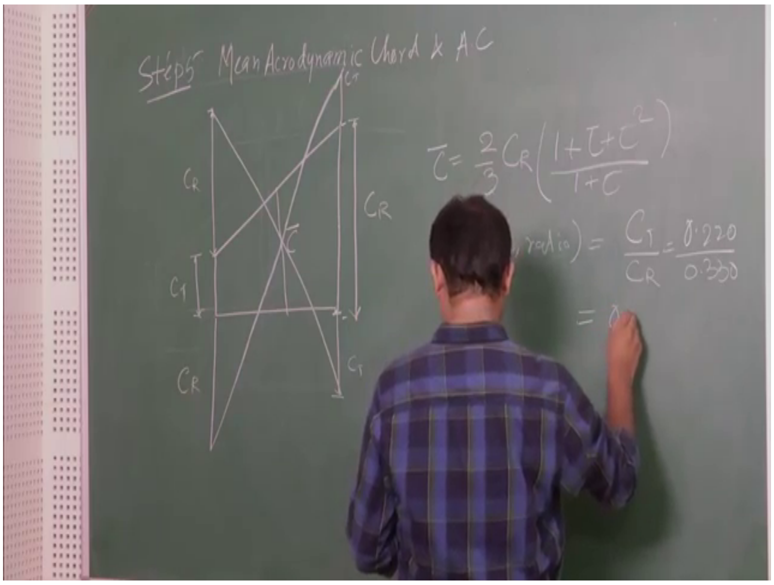

Step 5 mean aerodynamic chord and aero dynamic centre, mean chord and aerodynamic center. In order to calculate mean aerodynamic chord there are 2 ways, you can either use a geometrical method or you can use formula in geometrical method you know what are the tip and root chords this is your root chord, this is your tip chord. Measure this lengths and extend this joint this length on tip side. Similarly measure this length and extend this length on root chord C T. Joint these 2 points and either joint this center to this center or you can extend this line here also this will be a your C R, this will be your C T and joint these 2 lines. This will be your mean aero dynamic chord, this is geometrical method.

But in case you do not want to work on geometrical method because sometimes a wings are too large you cannot directly apply this. Go for mathematical formula, which is c mean equals or mean aerodynamic chord equals to 2 by 3 root chord into 1 plus tau plus tau square divided by 1 plus tau. Where tau is your taper ratio that is ratio of tip ratio of ratio chord root, in our case it is 0.220 by 0.330, which will be about 0.67.

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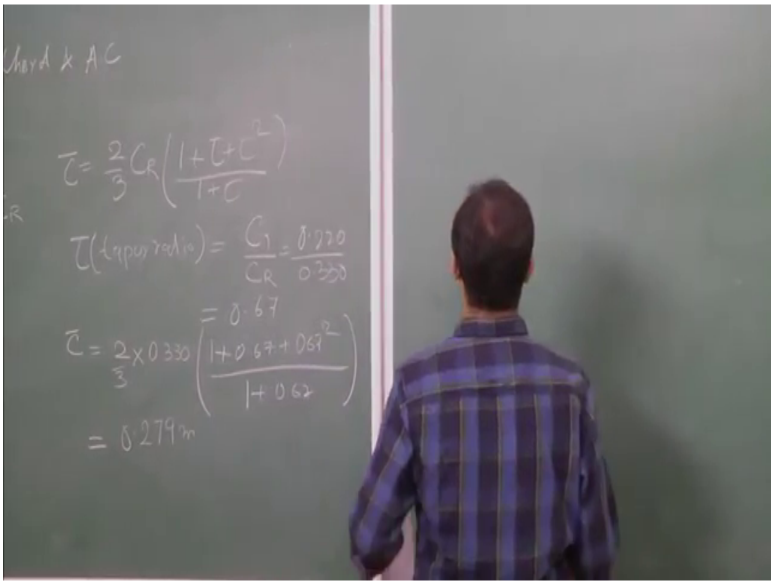

So, your mean aerodynamic chord will be 2 by 3 into root chord is 0.330 into 1 plus 0.67 plus 0.67 whole square by 1 plus 0.67, which will come about 0.279 meters.

Once you have calculated your mean aerodynamic chord, now you have to calculate aero dynamic centre.

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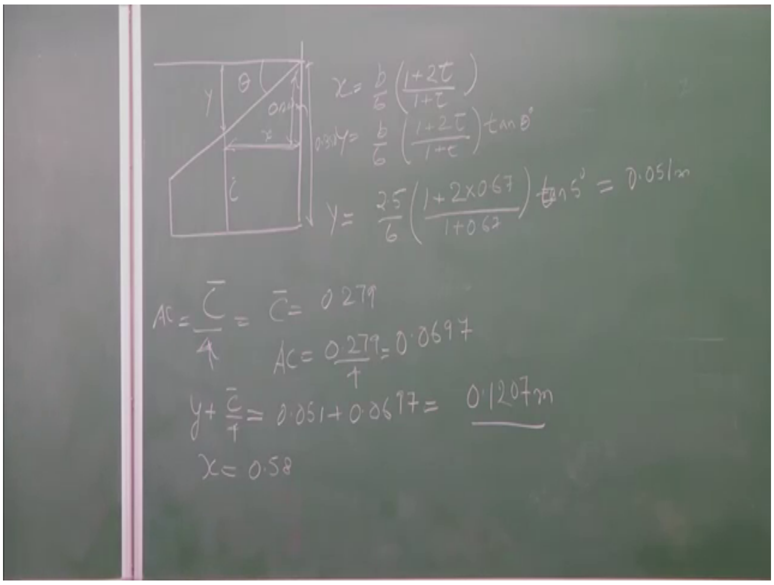

Now since in the question it is already given, that you have a sweep of 5 degree, this is suppose some theta then and this is your suppose mean aerodynamic chord. Then distance from the root x and y from horizontal line are given as x equals to b by 6 into 1 plus 2 tau by 1 plus tau. And y is given as b by 6 1 plus 2 tau by 1 plus tau into tan of theta in degrees. So, in our case y will be 2.5 by 6 1 plus 2 into 0.67 by 1 plus 0.67 which will be into tan of 5 degrees which will be equal to 0.051 as you know for aero dynamic centre we will usually take c by 4 as your aero dynamic centre. In our case c was 0.279. So, A C will be 0.279 by 4 equals to 0.0697.

So, your aerodynamic centre from this horizontal line will be at a distance of y plus c by 4, which is equal to 0.051 plus 0.0697, which is equal to 0.1207 meters. So, if this is your horizontal line, and this was your root chord which is 0.330, your aero dynamic centre from this point will be somewhere around 0.1207 meters. If you want to calculate x you can calculate it will come 0.58. After all this calculation you have determine what will be the location of A C from horizontal line or from starting of your wing position.

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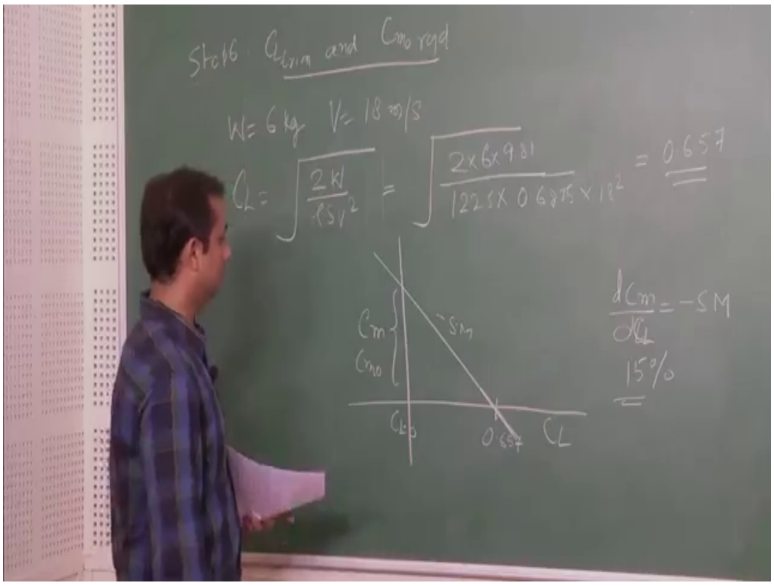

Now, after that step 6 will be to calculate Cl trim or particular value of Cl at which you want to operate your aircraft, and what will be the value of Cm naught required. We know that weight of aircraft is 6 kg and velocity is 18 meters per second (Refer Time: 33:41) velocity. So, using formula Cl equals to under root of 2 w upon though S v square. You can calculate the value of Cl trim, which is 2 into 6 into 9.81 divided by, this is at sea level we are talking about the sack of at sea level into area we got as 0.6875 into b is 18 square which will come about 0.657. So, I want to operate at 0.657.

Now, if you want to change these value you have to go for another set of this is a value I got first Cl trim. There can be different cases for which this Cl trim can be different for instance, you can operate at Cl equals to minimum drag that is Cl equals to C D naught upon k, where you can calculate C D naught you can assume at something 0.32 0.04, k you can calculate and you will get some Cl trim at minimum drag. Static margin is static margin is given by dCm by dCl, or slope of dCm by dCl equals to static margin.

We know that from previous lecture and various lecture before which we have done. So, Cm versus Cl graph if we if we plot this is minus static margin. And in order to operate at point 0.657 what will be the value of Cm naught, this is at Cl is equals to 0. This I have to calculate what will the value of Cm naught in order to trim a aircraft at 0.657. So, let us assume I want a static margin of 15 percent, you can take can you an static margin 10 15 20.

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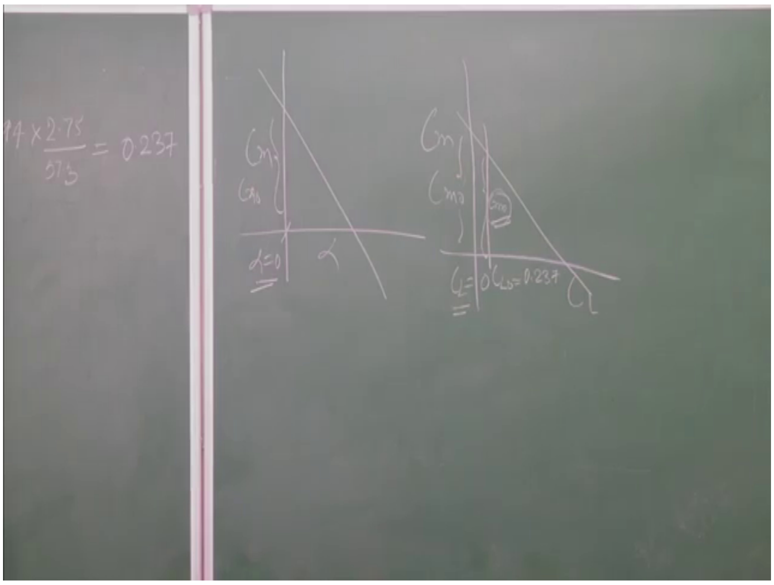

Whatever you like I have selected 15 percent as my static margin. So, you know that in order for an aircraft to be statically stable, your slope of Cm versus alpha or slope of Cm versus Cl should be negative, as well as Cm naught should be a positive number. But these 2 had different cases here this Cm naught is at alpha equals 0 here this Cm naught is at Cl equals to 0. So, I will be doing all my calculations at alpha equals 0. So, in this case your Cl will this the value of Cm naught will be not for Cl equals to zero, but at some value of Cl naught. So, this is the value of Cm naught I will be using for further calculations.

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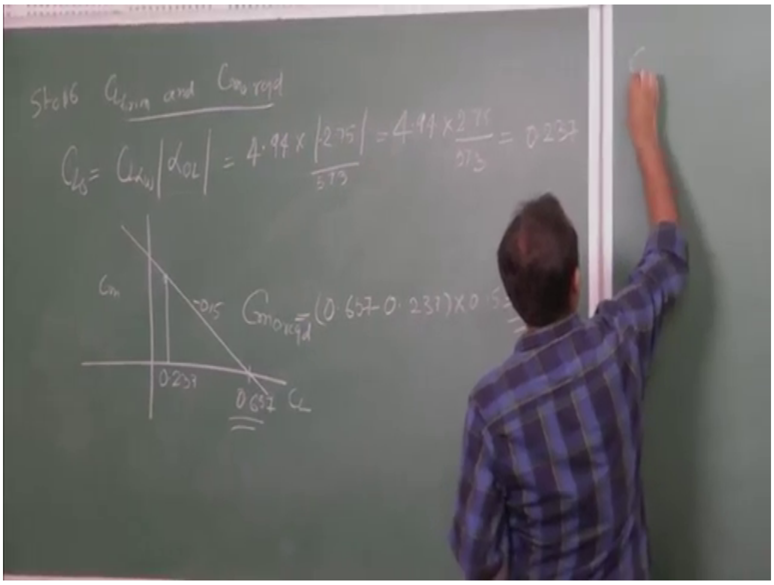

We are now in what will be the my Cm naught requirement as per Cm versus Cl graph. I have to calculate Cm naught at Cl naught value. So, first of all we have to calculate what will be the value of Cl naught, which is equal to Cl alpha of wing which I have already calculated and angle of attack at 0 lift, which we got from Eppler E17 airfoil which is for E197 Cl alpha wings was 4.94 into alpha at 0 lift was 2 point minus 2.75 we have taken modules which will be 4.94 into 2.75.

This is in degree we have to convert it into radian. So, I will divide it by 57.3. Which will give me Cl naught as 0.237. So, I have to calculate what will be my required Cm naught at Cl equal to 0.237. We have seen from the graph that my Cl trim position was 0.657 Cl versus Cm. I have to calculate my value at 0.237 and I also know thus this is minus 0.15 minus of static margin. So, I can easily get this position simply I have to do is 0.657 minus 0.237 into 0.15 which will give 0.063. So, in order to get a static margin of 15 percent at Cl trim of see 0.657, I have to get overall or Cm naught of overall aircraft as 0.063. So, now, we will see what will be the required condition what will be the tail setting angle whether we even require a tail setting angle or not, based on these calculations. So, this is the Cm naught required.

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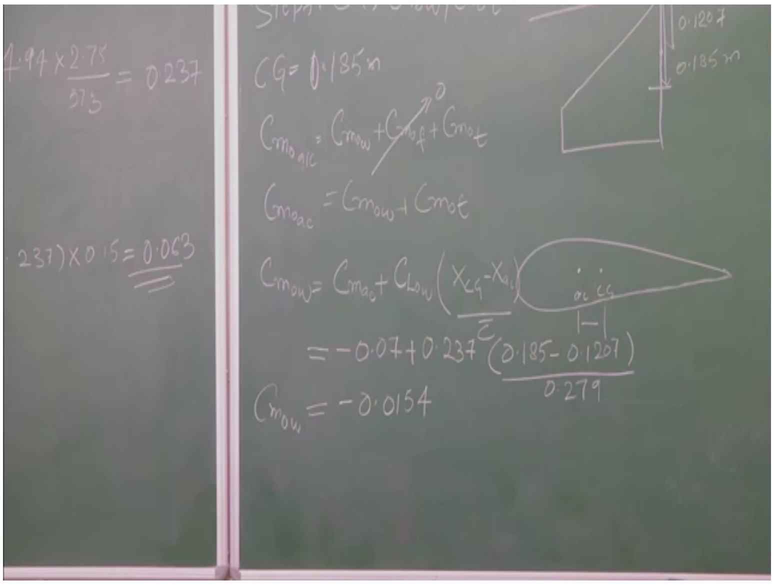

Now, step 7 C G the Cm naught of wing Cm naught of tail. Now earlier we derived what was if the position of aero dynamic center with respect to wing horizontal line. This was your horizontal line and we derived as this was 0.1207 as we have already seen in previous lecture, in order to have static stability your C G should be behind A C. This is your tail this is your tail your A C, C G should be behind of AC. So, for a fixed wing you have we use a lithium polymer battery. So, your mass does not vary with during flight. So, we can assume C G is fixed in case of UAVs, but in case of jet during flight your consumption of fuel your C G might change. So, you have to consider a range of C G for which you have to do this calculation, but in our UAV we do not have to worry about that since C G does not vary.

So, first of all let me take C G as 0.185 meters from wing horizontal position, 0.185 meters. You can change this C G cu what will be your result or what will be the tail setting angle. This is just for my calculation I have selected 0.185 meters. We derived that this was our requirement in order to have static margin of 15 percent at Cl trim equals to 0.657. So, Cm naught for an aircraft are required was 0.063 which is a combination of Cm naught of wing plus Cm naught of fuse large the Cm naught of tail. For simplicity of in our for in our case we neglect this value. So, this will be the combination of Cm naught of wing plus Cm naught of tail, equals to Cm naught of aircraft. Now Cm naught of wing is equal to C mac plus Cl naught of wing into x C G minus X ac by c bar.

We have already we already know what is a value C mac, from airfoil data which was minus 0.07. We calculated Cl naught wing as 0.2 0.237 x C G we have selected as 0.185 and next is c we calculated was 0.127. And c bar was 0.279 which will give Cm naught of wing as minus 0.0154. Once we get the value of Cm naught of wing, we can subtract that from total Cm naught required for the aircraft which is Cm naught of aircraft equals to Cm naught of wing plus Cm naught of tail.

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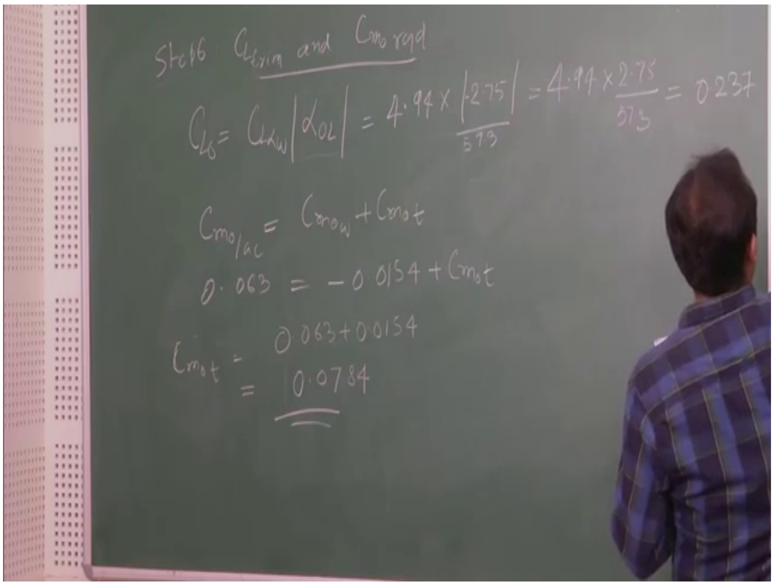

This was 0.063 equals to we got 0.0154 plus Cm naught of tail. So, Cm naught of tail will be 0.063 plus 0.0154, which will be equal to 0.074. So, this is a value I have to generate in order to get to total Cm naught of aircraft as 0.063. We have already seen the formula for Cm naught of t. Cm naught of t is given as eta V H Cl alpha t into cell naught plus incidence angle of wing minus incidence angle of tail.

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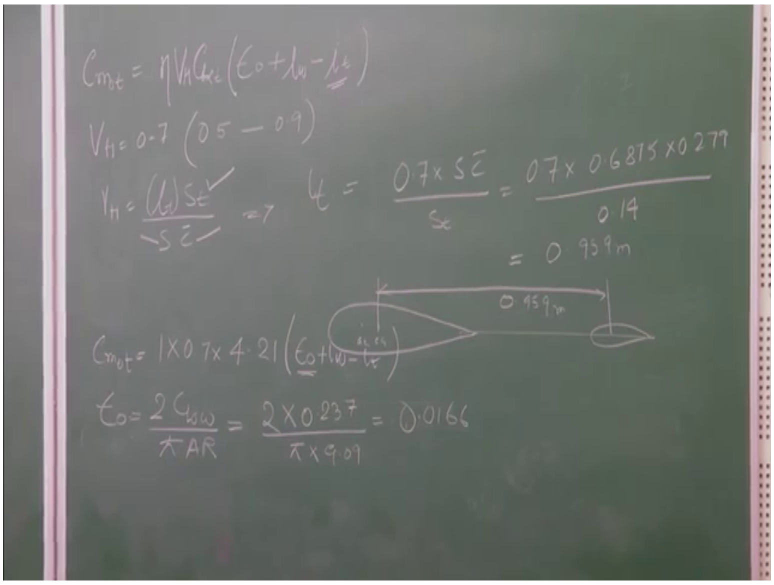

For our calculations we are taking V H equals to 0.7, typical range is 0.52 0.9. We are taking 0.7. So, we know V H equals to Lt tail arm into St area of tail by area of wing into c bar. We already know area of tail we already know area of wing we know mean academic chords. So, we can get tail arm, which is equal to Lt equals to 0.7 into S c bar by St equals to 0.7 into 0.6, 0.6875 into 0.279 divided by 0.14. Which will give me 0.959 meters. So, we got your tail arm as point 5.959. So, this is your A C this is your C G and this is your tail. So, distance between C G and aerodynamic centre of tail is 0.959 meters.

Now, to calculate Cm what will be the tail incidence angle in order to get Cm naught equals 0.0784. So, Cm naught of tail equals to, let us assume eta as 1 into 0.7 Cl alpha of tail we have already calculated, this is Cl alpha of tail. 4.21 and then epsilon naught plus i of wing minus i of tail. If epsilon naught is given as 2 Cl naught of wing by pi aspect ratio, which is equal to 2 into 0.237 by pi into 9.09. Which comes around 0.0166.

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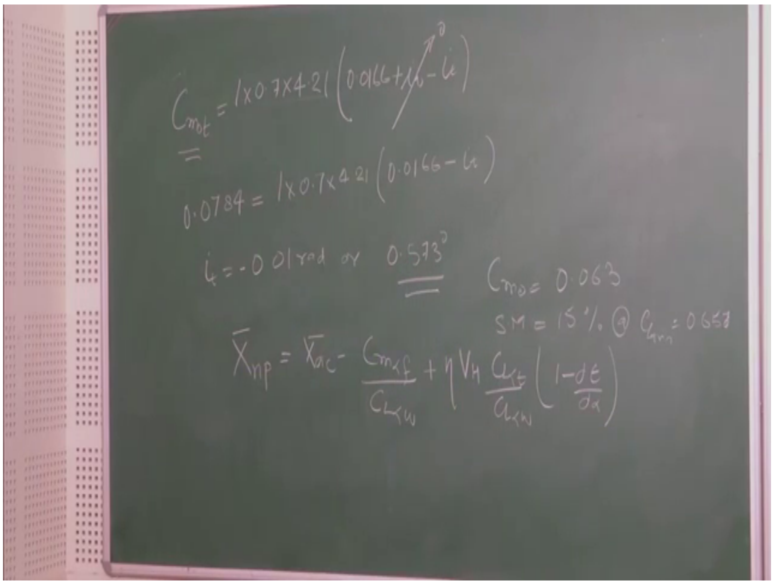

Now, substituting all these values in the equation, Cm naught of tail equals to 1 into 0.7 into 4.21 into 0.0166 plus iw minus it. I am not giving any incidence angle to wing. So, I will take this value as 0. And Cm naught of tail required was 0.0784 equals to 1 into 0.2 4.21 into 0.0166 minus i of t, which will give me i of t equals to 0.01 radian or something about 0.53 degrees.

So, this is the value of tail incidence angle we need to have in order to get total Cm naught of aircraft equals to 0.063 and static margin of 15 percent at Cl trim equals to 0.657. We have to calculate neutral point. So, neutral point is given as X np bar equals to X ac bar minus Cm alpha fuse large divided by Cm alpha of wing, plus eta V H Cl alpha of tail by Cl alpha of wing into 1 minus d epsilon by d alpha.

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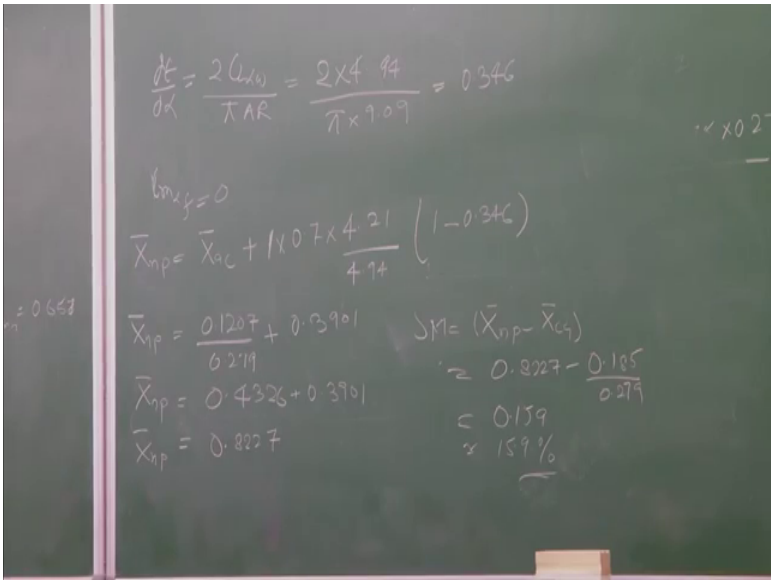

Now, d epsilon by d alpha is given by 2 Cl alpha of wing by pi aspect ratio equals to 2 into 4.94 divided by pi into 9.09 which will be 0.346 and Cm alpha of fuse large we are taking it as 0 for a time wing.

So, my final equation will be X np bar equals to X ac bar plus eta we are taking one V H we already taken as 0.7, Cl alpha tail we have taken 4.21. Cl alpha of wing is 4.94 into 1 minus 0.346. Which will be equal to 0.1207 this is a value of A C divided by your c bar which is 0.279 plus 0.3901 X np bar.

So, X np bar will be 0.4326 plus 0.3901, which will be 0.8227 your X np bar, if you want to calculate your static margin using formula, static margin equals to X np minus x C G bar. Once you get this neutral point you can verify whether my static margin which initially approximate is correct or not. So, you can substitute these values in this equation and you will be getting something 0.159, which is 15.9 percent, which is fair enough since we have taken lots of approximations.

So, this is determined what will be your tail arm length, what will be your incidence angle, where should be your wing and tail located. I want that you should practice first varying at different cgs and what at different Cl trim conditions.

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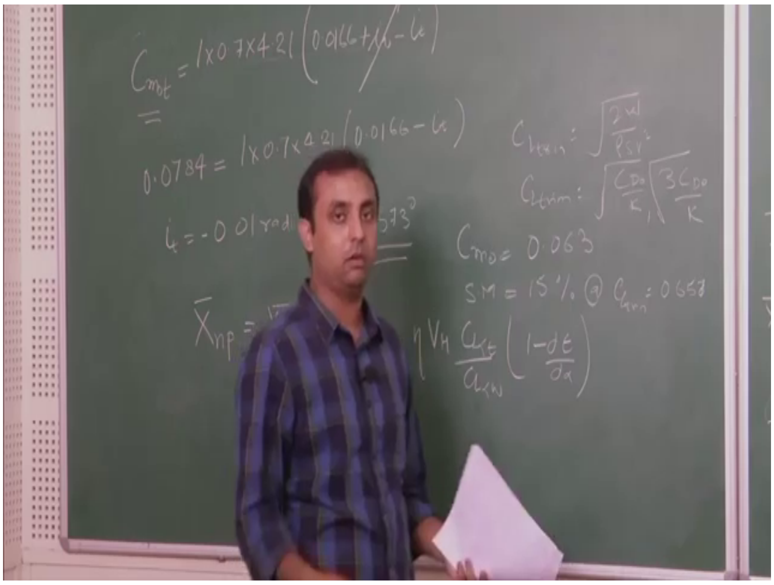

Since I took Cl trim at 2 w on row S v square. You can take Cl trim at minimum drag that is I already told C D naught by k or at minimum power 3 C D naught by k. Try this and you can formulate what will be the conditions or what we will be the tail setting angle. So, that is all for this numerical.

Thank you.