

**Aircraft Design**  
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**Lecture – 04**  
**Design Basics: Range, Endurance**

Good morning friends. In the last lecture and also in coming few lectures, I will be just sharing if you philosophy few dos and do not before we start rigorously, following a procedure to design an aircraft. In the last lecture I showed you that for the preliminary stage when our student the design aero models remote driven by thumb rule, and few things are heuristic as well. And that is exactly why I said that please do not bring those thing with you when you attend my lecture, but truly speaking if you find somebody is really a passionate aero modeler.

He is feeling for numbers are huge you have to respect an aero modeler truly aero modeler. I work with few of them captain amoolya captain Mukherjee and huge number of things I am learned from them, only you as it participant of this course should realize that the feel of an aero modeler truly aero modeler he may not be able to explain the law of aerodynamics the way you want, but if we have learn this course you should be able to translate his feel into number a radio procedure based on scientific assumptions. So, that is where the aero modeler and this course will merge, but not at this stage after we build up our self as appropriate time I will bring a aero modeler what is respectful aero modeler and share some thoughts with him to prove my a point.

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$\frac{W}{S}, \frac{T}{W};$   
 $T = D$   
 $L = W$  at  $\frac{C_L}{C_D} = C_{ruise}$   
 $\left(\frac{T}{W}\right)_{cruise} = \left(\frac{1}{C_L/C_D}\right)$

We will still continue on few things which will be useful, and it is also a part of revision we have seen that we are been talking about  $W$  by  $S$  we have been talking about  $T$  by  $W$ . And when it comes to  $T$  by  $W$ , how much  $T$  by  $W$  is required, but in normal say cruise dominating airplane, if I want to really see that then I know that  $T$  equal to  $D$  and  $L$  equal to  $W$ ; that means,  $T$  by  $W$  is  $1$  by  $C_L$  by  $C_D$ . If I write  $T$  by  $W$  cruise equal to  $1$  by  $C_L$  by  $C_D$ . So, what is the  $T$  by  $W$  required for cruise how do I answer? That means, I need to know at what  $C_L$  by  $C_D$ , I am cruising. For example, if I am cruising such that drag is minimum, it is an cursing such that drag is minimum.

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$T = D$   
 $L = W$  at  $\frac{C_L}{C_D} = C_{ruise}$   
 $\left(\frac{T}{W}\right)_{cruise} = \left(\frac{1}{C_L/C_D}\right)$   
 $\frac{C_L}{C_D} = 15$   
 $\left(\frac{T}{W}\right)_{cruise} = \frac{1}{15}$   
 $\neq \left(\frac{T}{W}\right)_{T_0}$   
 $D_{min} = \frac{C_L}{C_D} \Big|_{max}$   
 $\left(\frac{T}{W}\right)_{cruise} = \left(\frac{1}{C_L/C_D}\right)_{max}$   
 $C_L = \sqrt{\frac{C_{D_0}}{K}}$

Then I know that for this condition,  $C_L$  by  $C_D$  has to be maximum and; that means,  $C_L$  has to be  $C_D$  naught by  $K$ , this number. So, immediately I know  $T$  by  $W$  will be 1 by  $C_L$  by  $C_D$  and let us say  $C_L$  by  $C_D$  max this value. When I write  $T$  by  $W$  cruise I say cruise for minimum  $T$  by  $W$ ; that means,  $C_L$  by  $C_D$  is maximum; that means drag is minimum. So, I know that value  $T$  by  $W$  as 1 by  $C_L$  by  $C_D$  max which corresponds to the  $C_L$  equal to  $C_D$  naught by  $K$ . Typically  $C_L$  by  $C_D$  max could be 15, one number I am writing. So, it could  $C$  in immediately  $T$  by  $W$  cruise is 1 by 15, but here you should understand this  $T$  by  $W$  cruise is 1 by 15, this is not equal to  $T$  by  $W$  takeoff. Why for 2 reason that for  $T$  by  $W$  takeoff, we need to have a different criteria, for  $T$  by  $W$  takeoff.

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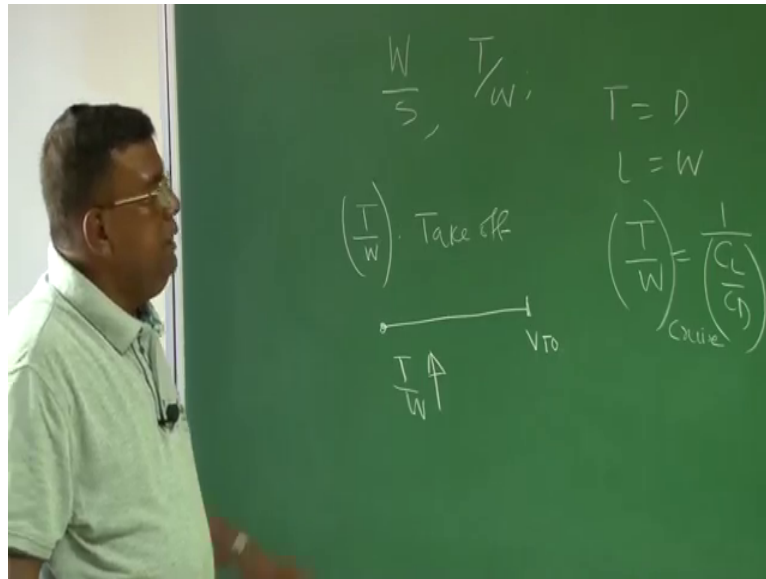
Handwritten mathematical notes on a green background. The notes include:

- $\frac{C_L}{C_D} = 15$
- $\left(\frac{T}{W}\right)_{\text{cruise}} = \frac{1}{15}$
- $T - D - W \sin \gamma = 0$
- $\left(\frac{T}{W}\right)_{\text{to}} = W \sin \gamma + \frac{1}{C_L/C_D}$
- $C_L = \sqrt{\frac{C_{D0}}{K}}$
- $\left(\frac{C_L}{C_D}\right)_{\text{max}}$

We have seen that if I write  $T$  minus  $D$  minus  $W$  sin gamma, and if I am to takeoff without any acceleration that is steady climbs small climb I am doing, then  $T$  by  $W$  is roughly equal to  $W$  sin gamma plus 1 by  $C_L$  by  $C_D$ , we have shown the roughly this is approximately.

So,  $T$  by  $W$  takeoff would be more governed by this part the climb angle gamma.  $T$  by  $W$  cruise will be more driven by  $C_L$  by  $C_D$  max right. So, if I am a designer, I immediately know what is by  $T$  by  $W$  cruise required because I know roughly it will be  $C_L$  by  $C_D$  10 or 15, whatever way I am designing. So, I know  $T$  by  $W$  cruise I required this much 1 by 15 and  $T$  by  $W$  takeoff how much is required one way do get it is if I am trying to climb at some angle 15 degree climb angle 20 degree climb angle.

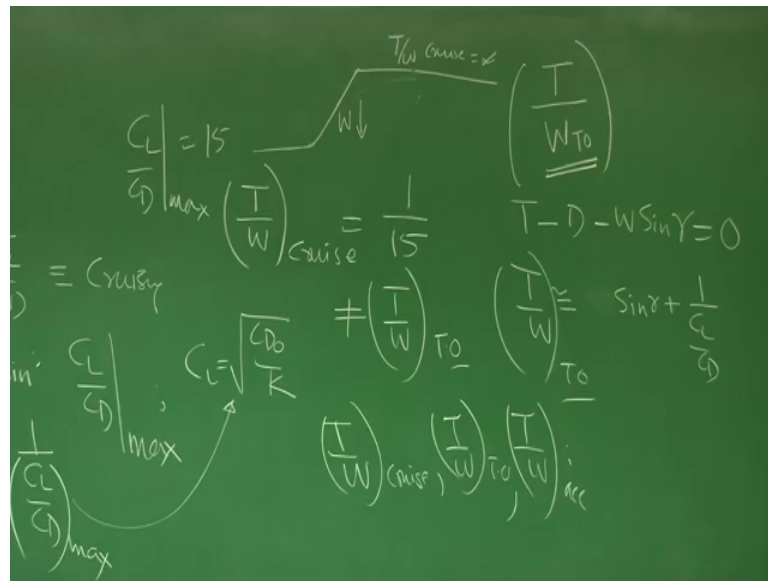
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So, I can easily see here of course, this  $W$  will not be here this  $W$  will not be here yes right directly proportional to  $\sin \gamma$ , but you also understand this  $T$  by  $W$  will play important roles during takeoff also. Takeoff means that phase that is from start to a speed where we takeoff has been achieved. If I want to shrink in this length before it gets the we takeoff speed I have to accelerate the airplane fast if I have to reduce this length; that means, if I have the really reduce this length then  $T$  by  $W$  has to be very high because  $T$  by  $W$  will decide how much active force is applied to accelerate the body or the airplane this case from 0 to we takeoff. So,  $T$  by  $W$  will play role here,  $T$  by  $W$  will play role here cruise what is the  $T$  by  $W$  require  $T$  by  $W$  for climb also without going into detail you know if you want to go for a turn high rate of turn  $T$  by  $W$  also will play a role there we will see as we revolve step by step. This is one we should be very clear.

When I am mentioning all this thing is the one point you should understand when you are designing an airplane we cannot a specify.

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Like  $T$  by  $W$  cruise, I want this much  $T$  by  $W$  takeoff I want this much,  $T$  by  $W$  some acceleration parameter I want this much. Because why, because notice that if  $T$  by  $W$  cruise is 15 and if I want to know what should be  $T$  by  $W$  where from I should start, then that it will be different than this value. Because I want  $T$  by  $W$  cruise some number now what is happening as it is going from here to here the weight is going on reducing because where is being consumed.

So, what is a better approaches, we if we know how much fuel is consumed then modify this weight here that is I have to increase this weight, and we talk in terms of  $T$  by  $W$  takeoff this is important. Every such ratios with weight will be converting back into  $W$  takeoff. That should be kept in your mind and the reason is very simple that if I say  $T$  by  $W$  cruise is some number I know that that  $W$  is not  $W$  takeoff, but finally, as a designer I want to know what is the  $W$  takeoff required that is where from I start. So, that is why always it is advisable you convert this number any number which is divided by  $W$  convert it to  $W$  takeoff by appropriately adjusting the weight most of the cases it is because of fuel consumption and that is not a very difficult task we will be doing that there is a procedure doing that. So, this was one thing I thought I must share with you before you start using them. Another important parameter you will find when I am talking about  $C_L$  by  $C_D$  max sometime we remember for power we talk about  $C_L$  3 by 2 by  $C_D$  max.

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Handwritten notes on a green chalkboard:

- Top left:  $R, E$
- Top right:  $C_L / C_D$
- Left side:
  - $C_L = \sqrt{\frac{C_{D0}}{K}}$  (min Drag)
  - $C_L = \sqrt{\frac{3C_{D0}}{K}}$  (min power)
- Right side:
  - $C_D = C_{D0} + K C_L^2$
  - $C_D = C_{D0} + K \frac{C_{D0}}{K}$  (MD)
  - $C_D = C_{D0} + K \frac{3C_{D0}}{K}$  (MP)
  - $= 4C_{D0}$
  - $C_{D, MD} = 2C_{D0}$  (circled)

Let us again go back to performance I am talking in terms of range and endurance that will be one of our important parameter performance parameter. So, if I go back you remember that  $C_L$  for minimum drag, the condition was  $C_L$  equal to  $C_D$  naught by  $K$ . And  $C_L$  for minimum power or equal to  $3 C_D$  naught by  $K$ . So, another purpose of writing all this thing is that to give enough time you go back and revise all those things. So, that you are prepared for application of this understanding in terms of synthesizing a design, synthesizing concept to get a conceptual design. If it is  $C_L$  minimum for minimum drag I mean  $C_L$  required for a minimum drag is  $C_D$  naught by  $K$  and you know that this is discussed call to  $C_L$  by  $C_D$  maximum and this corresponds to  $C_L$  3 by 2 by  $C_D$  maximum remember.

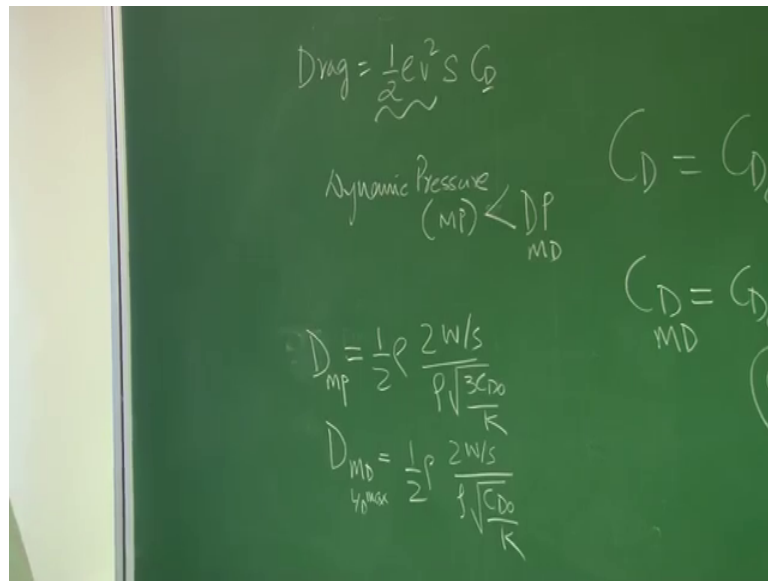
Now, let us see we will play around with this. So, if I write now  $C_D$  equal to  $C_D$  naught plus  $K C_L$  square. So, if I write now  $C_D$  for minimum drag is  $C_D$  naught plus  $K C_D$  for minimum drag is this one. So, this will become  $C_D$  naught by  $K$ . So, this will become  $2 C_D$  naught. So,  $C_D$  for minimum drag is to  $C_D$  naught this is nothing new for you. Similarly,  $C_D$  for minimum power will be  $C_D$  naught plus  $K$  in to  $C_L$  square means  $3 C_D$  naught by  $K$ . So, this is equal to  $4 C_D$  naught, please notice this that  $C_D$  for minimum power is twice the  $C_D$  naught and  $C_D$  for minimum power is  $4 C_D$  naught.

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So; obviously,  $V$  for a minimum power will be because after we flying lift equal to weight, that implies  $V$  equal to under root  $2 W$  by  $S \rho C L$ . So,  $V$  for minimum power will be what you know  $V$  for minimum power means  $C L$  is  $3 C D$  naught by  $K$ . So, I write it as under root  $2 W$  by  $S \rho$  under root  $3 C D$  naught by  $K$ . So, that will be your let me write here. So, that things are clear that will be  $2 W$  by  $S \rho$  under root  $3 C D$  naught by  $K$ . This is what is here right similarly  $V$  for minimum drag these are straight forward for you all know all this things this only will help you to go back and revise the nodes. So, that whenever I take some assumption you understand what I am doing, there is no objection here.

Now, let us say with this, if I now try to find out which case the drag will be more. Why we are asking this question we are trying to find out finally,  $C D$  is here

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And drag means when I talk about drag this is dynamic pressure that will half rho V square S into C D right. One thing is clear when I am flying at C L, is equal to under root 3 C D naught by K, which is a minimum power case to maintain lift equal to weight at same altitude. I will be flying slower because C L will be higher compared to C L required for minimum brackets right. So, the dynamic pressure for this case for minimum power case will be less, as for dynamic pressure is concerned I can write dynamic pressure corresponding to minimum power will be less compared to dynamic pressure for minimum drag.

No objection in this. Because the C L for a minimum power is higher as compare to C L for minimum drag. So, the speed for minimum power will be lesser compare to C L for minimum drag to maintain the same weight the lift equal to weight natural dynamic pressure which is half rho V square if both the things we have comparing at same altitude. So, dynamic pressure for minimum power will be less than dynamic pressure that minimum drag. So, drag minimum power will be half rho V square for V; I will write 2 W by S rho 3 C D naught by K. And drag for minimum drag case as is L by D maximum in this case L by D is maximum, that will be half rho 2 W by S rho into under root C D naught by K. If I ask a question in which case drag will be more, let us find out if we take the ratio between this 2 I can write D minimum power by D minimum drag.



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$$\text{Drag} = \frac{1}{2} \rho v^2 S C_D$$

Dynamic Pressure (MP)  $\leftarrow$   $\frac{DP}{MD}$

$$\frac{D_{MP}}{D_{MD}} = \frac{2}{\sqrt{3}}$$

$$D_{MP} = \frac{1}{2} \rho \frac{2W/S}{\sqrt{\frac{3C_{D0}}{K}}} S \cdot 4C_{D0}$$

$$D_{MD} = \frac{1}{2} \rho \frac{2W/S}{\sqrt{\frac{C_{D0}}{K}}} S \cdot 2C_{D0}$$

This will be equal to 2 by root 3. You see how it is happening I have miss some point here. So, drag minimum power will be half rho V square into S into C D. How much was C D for minimum power for the minimum power C D was 4 C D naught. We have shown that severally here I missed S into 2 C D naught right. Because we have seen C D for minimum power is 4 C D naught and C D for minimum drag is 2 C D naught we just we have shown that right.

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$$\frac{D_{MP}}{D_{MD}} = \frac{2}{\sqrt{3}} = \frac{2}{1.732} \approx 1.1547$$

15% more than  $(\frac{L}{D})_{max}$  case

$D \uparrow 15\%$ , MP case:  $\frac{C_L^{3/2}}{C_D}_{max}$

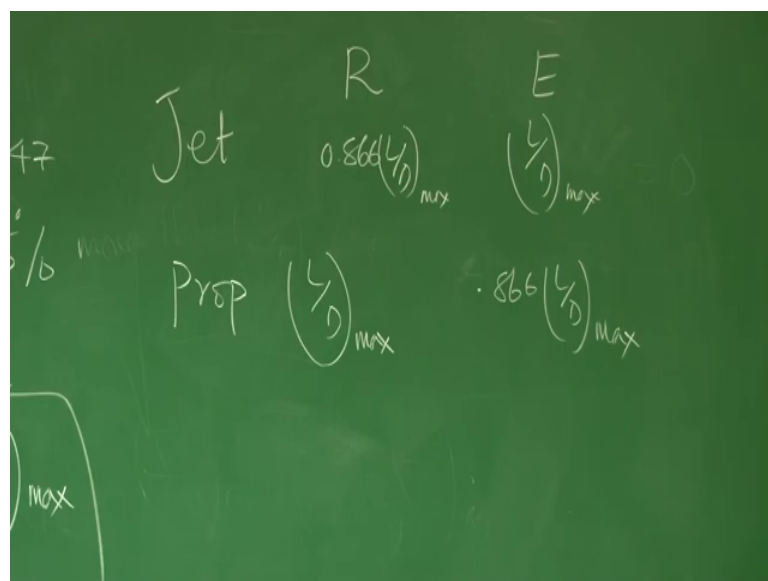
$$\frac{C_L}{C_D} = \frac{1}{1.15} = 0.866 \left(\frac{L}{D}\right)_{max}$$

So, now if I take the ratio we will find this 2 everything will get cancelled 4 and 2. So, the 2 remain there and root 3 root 3 will be there right. Do it yourself for this ratio will be roughly 2 by 1.732 and this is equal to 1.1547. So, as a designer we say drag during minimum power condition is 15 percent more than L by D maximum case right. You can see that drag during minimum power is 15 percent roughly more than the drag during L by D max case right. How that is important let us see.

Now designer interpretations, how a designer evaluate utilize this that is very important. If drag is increased by 15 percent, for minimum power case right, minimum power case here means, suppose this is a case were  $C_L$  3 by 2 by  $C_D$  is maximum with that we are going on and we know how  $C_L$  3 by 2 by  $C_D$  S come you can refer back your lecture. If that is true; that means,  $C_L$  by  $C_D$  if drag rises 15 percent then I can write  $C_L$  by  $C_D$  as 1 by 1.15 which is come from here the ratio of this 2 is 1.1547 which is equal to 0.866 L by D max.

This is part clear, what how a designer is utilizing this relationship, says drag during minimum power is 15 percent more than the drag if we are flying at minimum drag. So,  $C_L$  by  $C_D$  will be now modified, because  $C_D$  has increased by 15 percent and this 1 by 1.5. So, that gives 0.866 L by D max. With this understanding we will fix our initial design parameters for example.

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Which will be which will have relevance for range and endurance, if we recall if I take jet and it prop I am talking about range and endurance which is also called loiter, which is  $0.866 L$  by  $D$  max. And here it is  $L$  by  $D$  max and here it is  $L$  by  $D$  max, here it is  $0.866 L$  by  $D$  max for it is connect. If you see the expression for range for a jet driven airplane the condition for range maximum range is  $C L^3$  by  $2$  by  $C D$  should be maximum. So, which is like a minimum power conditions equivalently and you know for  $C L^3$  by  $2$  by  $C D$  if I take that we will have seen  $C L$  by  $C D$  should be  $0.8866$  of  $L$  by  $D$  max, but for endurance for a jet a airplane it will be maximum when  $L$  by  $D$  is max. So, I am putting is  $L$  by  $D$  max for a (Refer Time: 21:44) it is other way for maximum range the condition says it should fly such that  $L$  by  $D$  should be maximum. And for endurance it should be  $C L^3$  by  $2$  by  $C D$  that condition it gives  $0.86 L$  by  $D$  max.

You must be wondering I have not. So, for announce the books textbook I will be using. I will be doing that in the next class (Refer Time: 22:06) I will be follow it 2 3 books. So, mostly one book which I will be following initial part is reimar aircraft design by reimer which must be going on the most popular book, but I will formally give you the volume etcetera addition. And please understand why we are doing all those revision. Because for an aircraft designer what will be the weight of the airplane is a big question, how do I know how much is the weight luckily our situation is not situation of during right by those time with there were no historical data right.

See how great they were say I always say from right brothers. So, much of aircraft why we could not design a good civilian aircraft one of my friend told they had right brothers we have brothers per have they are not right they are wrong. And that is the my motivation for this lecture series that after this lecture at least we have initiated a process creating a right brothers among the wrong brothers and among the wrong brothers, I am one of those wrong brothers because I am also I have failed to design in civilian good aircraft. Main issue comes on the weight of the airplane. Say now the situation in little different whenever you think about the aircraft. Let us a civilian aircraft first question comes what sort of a range you are talking about, short range or long range aircraft. Then comes what sort of a number of passenger should talking about. And what sort of a speed you are talking about. Fortunately, whatever you think you will find some of that class is there. So, you have got lot of database.

Now, so it is very easy to select a baseline aircraft, for the mission what you are looking for closer to that. From there are you can easily get some number my aircraft will be this weight class right. When I am saying this please understand what I am trying to tell you is we will be using lot of historical data to get the initial numbers. For example, if this is  $W_{naught}$  gross weight of the airplane it will be composed of.

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$$W_0 = W_{crew} + W_{PL} + W_f + W_e$$

$$W_f = \left( \frac{W_f}{W_0} \right) W_0$$

$$W_e = \left( \frac{W_e}{W_0} \right) W_0$$

↓ MISSION REQUIREMENT

Now, you call it build up method, weight build up method. We will break it into different components. For example, weight of crew we know weight of payload we will be knowing, how many passenger or any other military payload. Then weight of fuel and you say empty weight. This is very important empty weight and if I now see here out of this 4 this is under my hand I know a priority this a priory I know, but  $W_f$  which is the fuel weight and  $W_e$  is the empty weight these are to be carefully find out. And there will be using lot of historical data. If you see now if I rearrange this I can write  $W_{naught}$  or it is say if I write  $W_f$  either function of  $W_f$  by  $W_{naught}$ , into  $W_{naught}$  and  $W_e$  as  $W_e$  by  $W_{naught}$  into  $W_{naught}$ .

What is the meaning of this that based on the historical data if I know this ratio  $W_f$  by  $W_{naught}$  based on the historical data, if I know  $W_e$  by  $W_{naught}$ , then my life will be simpler and the statistical data whatever is available they are being made available in this fashion? So, that becomes easier for us to use for to get initial gross weight. Another point you should understand  $W_f$  which is the fuel weight it will be direct function of

what sort of mission you have got you have a takeoff cruise you may accelerate dive you may again go up.

So, many things possible you may loiter. So, depending upon type of mission requirement, this is more driven by mission requirement, what is the mission of the airplane what are the missions what sort of operation is supposed to do what sort of manual supposed to do.

So, in the next class we will be starting from here. How do I get the initial weight estimates using historical data right, and also we will be talking about mission profiles.

Thank you very much.