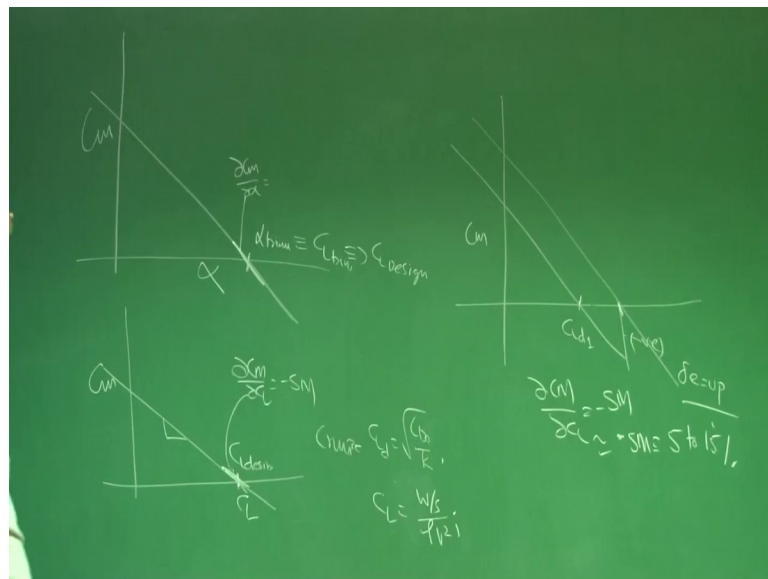


**Aircraft Design**  
**Prof. A.K Ghosh**  
**Department of Aerospace Engineering**  
**Indian Institute of Technology, Kanpur**

**Lecture – 35**  
**Wing and Tail Contribution to Longitudinal Static Stability**

Good morning friends. In the last class, we were discussing about del volume ratio primarily to ensure that the airplane has adequate static stability. We also realize that there are distinct roles for wing and tail wing as a primary responsibility of producing lift to balance weight or live for maneuver. However, the horizontal tail, its primary responsibility is to produce enough restoring moment, so that it is statically stable and of course, dynamically also stable.

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In that connection we realize that if I plot  $C_m$  versus  $\alpha$ , then if this is my  $\alpha$  trim, it is which corresponds to  $C_L$  trim which say  $C_L$  design. I can visualize like this and this slope at trim will say  $d C_m$  by  $d \alpha$ . I can visualize it through  $C_m$  versus  $C_L$ , where this is directly  $C_L$  design and the slope here at the equilibrium is minus static margin.

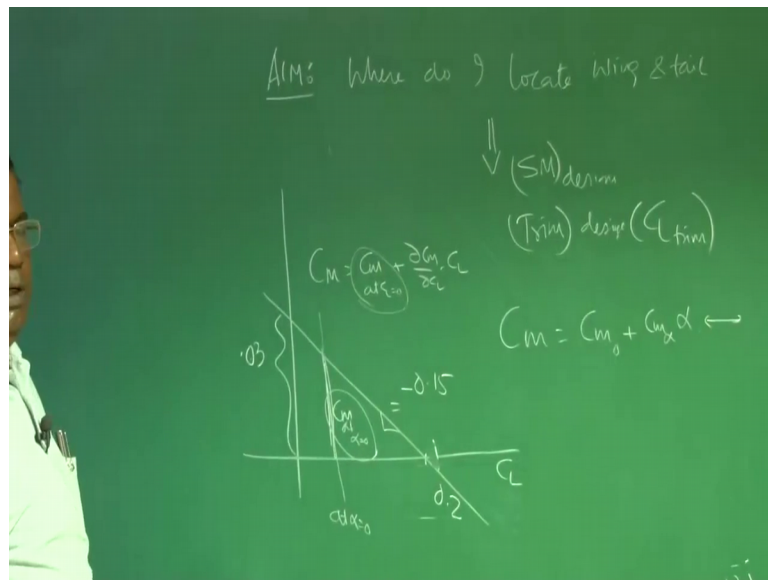
So, if I know what  $C_L$  design is, I need to look for which for example for the cruise  $C_L$  design would be under root  $C_{d1}$  naught by  $k$ . It could be  $2 w$  by it could be  $w$  by  $s$  by  $\rho v$  square depending upon the situation. So, once I know what  $C_L$  is, I am going to fly. Please understand when I write  $C_L$  design and if I know for most of my operation this is

what I am aiming for. So, I will always design the aircraft in such a way that I should be able to trim the airplane without deflecting any elevator, right because you would understand that if basically design my plane here for the  $C_l$  design, one if I want to fly at this  $C_l$ , then I have to give elevator deflection, so that this negative moment is countered. So, we have to give it an elevator up, right.

The moment you try to trim an airplane with elevator, up or elevator down you are giving drag penalty. So, if most of the time I am going to fly the airplane up particular  $C_l$  design, I should ensure that  $C_m$  versus  $C_l$  follows this trend without applying the elevator where I know that the slope  $dC_m$  by  $dC_l$  should be minus static margin and let say we design the static margin, we keep around 5 to 15 percent depending upon what level of confidence I have got on those aerodynamic estimations.

Now, the question, second question you are asking that we realize that  $dC_m$  by the alpha of the whole aircraft primary responsibility of tail and the  $C_l$  design, this is the primary responsibility of the wing, but then the question came where do I locate wing and how do I locate tail in combination.

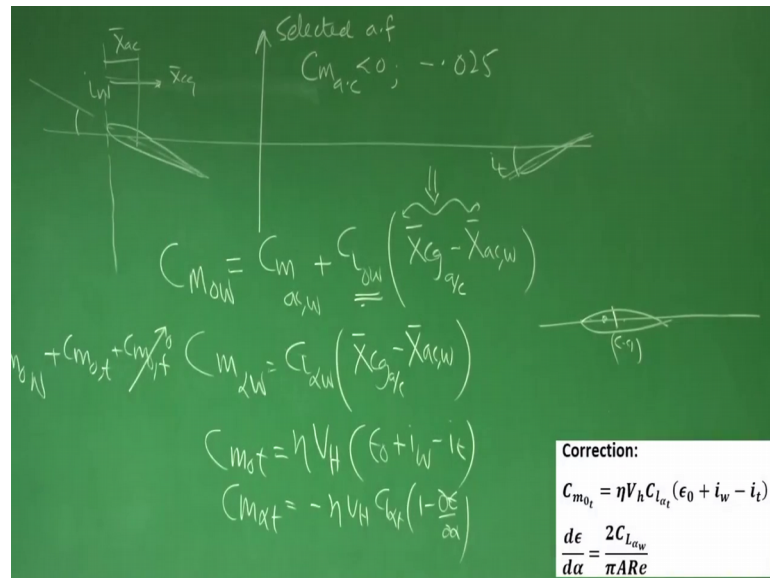
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So, both this condition that  $dC_m$  by  $d\alpha$  less than 0 and the static margin within 5 to 10 percent is satisfied that is our aim and this treatment will be purely on a conceptual stage. We use a little bit of expressions.

So, aim is where do I locate wing and tail, so that it has static margin design and trim design as per the design which I mean Cl trim, right. That is our aim.

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Now, let us go back to our aircraft stability in different graphs which we have done and before I use anything, let me recall if this is the wing. We define something called wing setting angle  $i_w$  and similarly, we define something called tail setting angle which is  $i_t$ . As per the convention, this is positive and  $i_t$  anything down is negative, right.

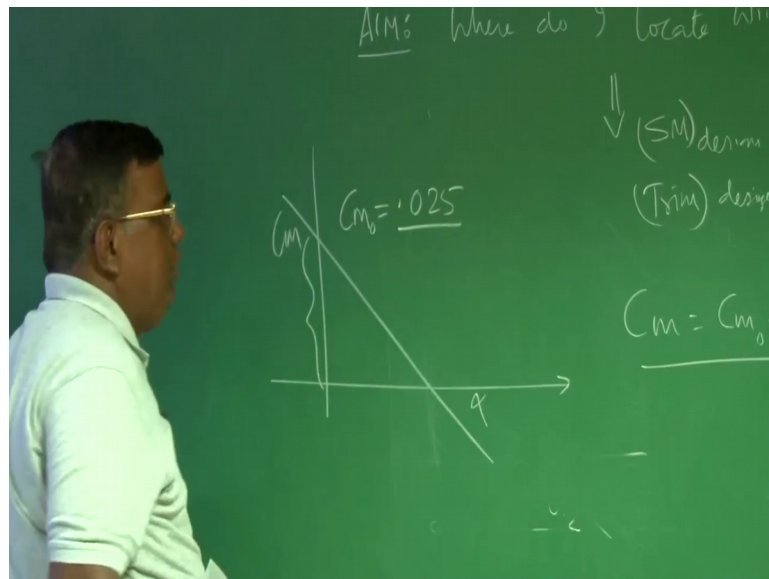
Then, if I measure reference distances from wing leading edge, then somewhere it is  $x_{cg}$  to more precise let say this is  $x_{cg}$  and somewhere it will be  $x_{ac}$ . Whenever we are writing  $x_{cg}$  and  $x_{ac}$ , all the linear dimensions are non dimensionalized by linear order record. All these things we know and if you recall in the our aim is if I want to fly at a particular  $C_l$ , let say 0.2 and static margins, I require 15 percent the slope here is minus 0.15 which essentially means this value should be 0.03, right to make life simpler because this graph is based on  $C_m$ . This is called  $C_m$  at  $C_l$  equal to 0 plus  $dC_m$  by  $dC_l$  into  $C_l$ , right. At a conceptual stage, you may find little difficulty in visualizing through  $C_m$  at  $C_l$  equal to 0. It is better as an approximation. You would try to visualize like this  $C_m$  equal to  $C_m$  naught  $C_m$  alpha into alpha, right.

So, if you see  $C_m$  versus  $C_l$ , this xenon that  $C_m$  at  $C_l$  equal to 0 is 0.03. You can always find out at alpha equal to 0, it was a cambered aerofoil. If it is symmetric aerofoil, then

this point is  $C_m$  at  $C_i$  equal to 0 as well as  $C_m$  as well aerofoil going to 0. Please understand at a conceptual stage, we are talking about the expansion of  $C_m$  assuming everything to be linear. So, if this is 0.03, you know always find out what is the corresponding value of  $C_m$  at  $\alpha$  equal to 0.

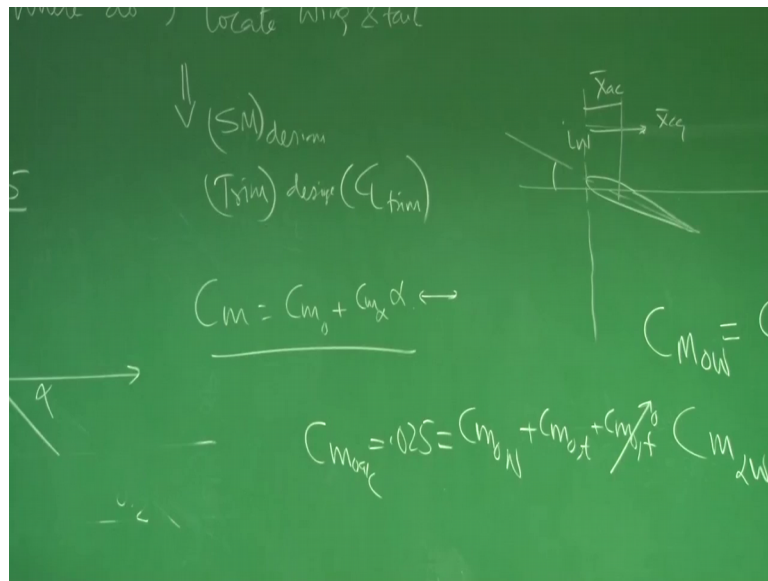
In fact, I would request at a conceptual stage you better use this. There will be some differences, but this will give you a good conceptual configuration and once you configure it, then you go for detailing what is  $C_m$   $cl$  equal to 0, all such things you should modify and if I follow this which is  $C_m$  equal to  $C_m$  naught plus  $C_m$  alpha into alpha, where you recall that  $C_m$  naught wing is equal to  $C_m$  assuming plus  $C_l$  naught wing into  $x_{cg}$  bar minus  $x_{ac}$  wing bar and  $C_m$  alpha wing is nothing, but  $C_l$  alpha wing into  $x_{cg}$  bar minus  $x_{ac}$  wing bar. There  $C_g$  is now aircraft cgs of the aircraft. This is the wing contribution towards the  $C_m$  naught, ok.

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So, let me erase this, so that you are not confused. So, we write it like this.  $C_m$  at alpha unless say this is  $C_m$  naught. Let say I require something called 0.025.

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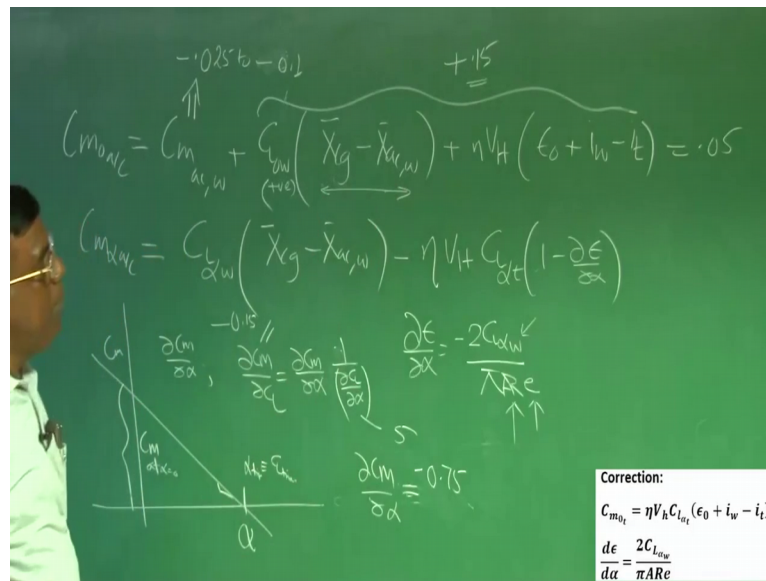


This example if I request  $C_m$  naught to 0.025, then what for a designer it means  $C_m$  naught of the aircraft which should be called 0.025 which should be called  $C_m$  naught because of wing plus  $C_m$  naught because of tail plus  $C_m$  naught because of fuselage, right. At the conceptual stage we will say this gentleman is 0. I make my life simpler if I want to see  $C_m$  naught of the aircraft. I need to know what  $C_m$  naught of the wing is and  $C_m$  naught of the wing is  $C_{mac}$  wing and these value I know because I had selected an aerofoil. The moment I select an aerofoil, cambered aerofoil, I know  $C_{mac}$  is less than 0.

Let see that values minus 0.045 now  $C_l$  naught wing also I know because I have selected an aerofoil, but now it depends on me how do I locate  $c$  of the wing reserve  $C_g$  of the airplane, right. The whole problem is you may have initially some rough idea about the  $C_g$  location because you know what are the layouts in different systems longitudinally and generally it will be around 40 to 42 percent. You can see historical data. Now, the question is how do I put the wings? I put it something like this, but AC is the head or AC is behind the problem is as you shift the wing, the  $C_g$  also will change.

So, lot of iteration goes on. So, this is an area which needs iteration and as far as  $C_m$  naught of the tail is concerned, that is if you recall is given as  $\eta v h$  into  $\epsilon$  naught plus  $i w$  minus  $i t$  and  $C_m$  alpha tail given as minus  $\eta v h c_l$  alpha tail into  $1$  minus  $d$  epsilon by  $d$  alpha. So, if I write this expression here, then it will be clear how to manage these expressions.

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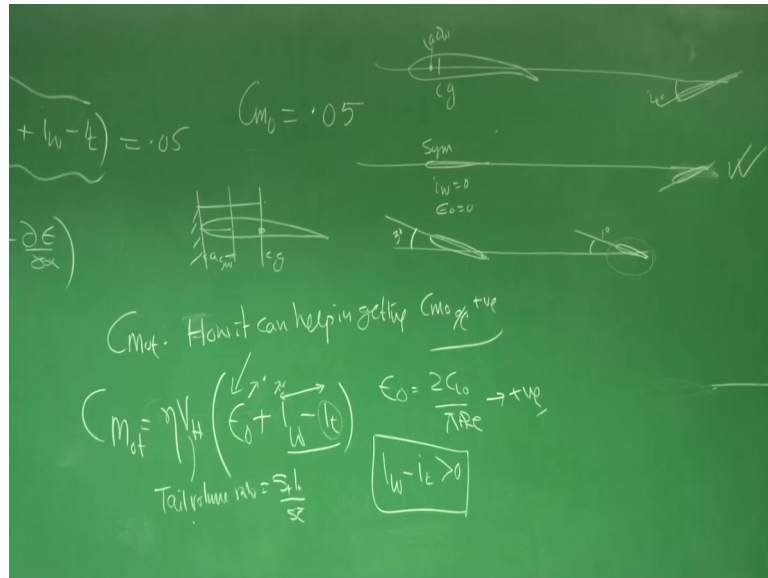
So, let me write  $C_{m_{airc}}$  of the aircraft equal to the wing which is  $C_{m_{a/w}}$  plus  $C_{L_{a/w}}$  into  $\bar{x}_{cg}$  minus  $\bar{x}_{ac,w}$ , right. There  $C_{m_{airc}}$  is plus  $\eta V_H (\epsilon_0 + i_w - i_t)$  and  $C_{m_{airc}}$  is roughly equal to  $C_{L_{a/w}}$  of the way that is  $C_{L_{\alpha w}}$  into  $\bar{x}_{cg}$  minus  $\bar{x}_{ac,w}$  minus  $\eta V_H C_{L_{\alpha t}} (1 - \frac{\partial \epsilon}{\partial \alpha})$ , the first order approximation I can write as  $2 C_{L_{\alpha w}} / \pi A R e$ .

You have already chosen aspect ratio  $e$  as well as  $C_{L_{\alpha w}}$ . This value may come around 0.3, sometime 0.4 pretty large. So, you have to locate the tail those will be in the final design. It does not change the focus here. What is our aim? We are if I am this is my  $\alpha$  trim and this is the  $C_m$  at  $\alpha$  called 0. Then, I need to ensure that I lay the wings and tail in such a way not only get this slope here, the slope is  $C_{L_{\alpha w}}$ , but we have started with  $dC_m / dC_L$ . So, you can always know  $dC_m / dC_L$  is  $dC_m / d\alpha$  into  $1 / dC_L / d\alpha$ , right.

So, if we are designing for minus 15 percent, so these values minus 0.15 and  $C_{L_{\alpha w}}$  which is around 5. So, you know that  $dC_m / d\alpha$  will be equal to minus 0.75. So, this is very easy to translate, correct. So, I am talking in the  $C_m$  verses  $\alpha$ . So, I know the  $\alpha$  trim which I know from  $C_L$  3, right. So, I need to have  $C_m$  at  $\alpha$  equal to 0.



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Let say I started with some 0.025. Let us take a number and let me take some number let say  $C_{m_0}$  naught is coming out to be 5. Take a fresh number. So, that means this whole gentleman should be called 0.05, right. That  $C_{m_0}$  naught is for the aircraft. Please understand I have working with  $C_m$  versus  $\alpha$ . So,  $C_m$  at  $\alpha$  equal to 0 is 0.05 which will not be same as  $C_m$  at  $C_l$  equal to 0, but we know that from whatever  $C_m$  at  $C_l$  equal to 0 is required, we can approximately find out what  $C_m$  at  $\alpha$  equal to 0 is because we know at  $\alpha$  equal to 0, there is some  $C_n$ . So, we have shown that.

So, let us work in this fashion that the conceptual stage it is simpler way of handling. It does not make much of an error. So, if you see this expression if I put the cambered aerofoil, this man will be afforded of minus 0.025 to minus 0.08. If you take all minus 0.1, if we take very highly cambered aerofoil value will shift towards minus 0.1. So, if you want  $C_{m_0}$  Ac positive, so you have to nullify this. That means, if this is minus 0.1 let me take an extreme example. If this is minus 0.1 and if you require 0.05, that means the contribution from here should be minus 0.15, right. It should be positive contribution, but this is minus 0.1 and this is this contribution if it will become plus 0.15 in the total will be 0.05.

Now, the question is how do I get such a huge component plus using these two expressions, what are the option you have got? We know that  $C_l$  naught into this man is positive notions. This will give a positive contribution as long as this difference is

positive and this difference positive means if you see our diagram from here where this is, but this is  $C_g$  somewhere here it is  $A_c$ .

If this  $x_{cg} - a_c$  has to be positive, that means  $a_c$  of the wing should be ahead of  $C_g$  of them right, but more you earn, more you put  $a_c$  of the wing ahead of  $C_g$ , the  $C_m$ . Alpha of the aircraft what happens if I put  $a_c$  of the wing ahead of  $C_g$ ? See here then this becomes positive as this being positive is helping in getting  $C_m$  naught positive, but the moment these become positive, this first term which is coming from wing that becomes positive and  $C_m$  alpha of the wing positive means it is destabilizing, right.

So, this sort of a conflict starts again if I go back here. I saw no I will not take the  $a_c$  of the wing. So, further ahead of  $C_g$  a little bit rest I will compensate from tail. So,  $C_m$  naught tail I am trying to see how it can help in getting  $C_m$  naught aircraft positive. The same if I see the  $C_m$  naught tail expression, it is  $\eta v h$  into epsilon naught plus  $i_w$  minus it, right. You know it is tail volume ratio. This is typically is  $s_{tail} l_{tail}$  by  $s_{cbar}$  and  $l_t$  is the distance between  $x_{ray}$  of the tail and  $C_g$  of the airplane, all these things we know, but look here if it is a cambered aerofoil epsilon naught for cambered aerofoil epsilon naught will be  $2 C_l$  naught by  $\pi$  aspect ratio  $e$  approximately, but this gentleman will be a positive number, right. If it is more cambered, then this epsilon naught will be more positive, but more cambered means  $C_{mac}$  is also becoming large negative and then, you have  $i_w$  minus  $i_t$ . The message is as long as  $u^3 i_w$  minus  $i_t$  greater than 0, it will give you positive  $C_m$  naught. Isn't it and you can scale it up by increasing the value of  $v h$ . So, if I increase tail volume ratio and if I increase this difference, then  $C_m$  naught the aircraft will become more and more positive.

So, what is the meaning in terms of design meaning thereby if I have a wing here and tail here, I can increase  $C_m$  naught contribution to aircraft by let say first case, I put  $i_w$  equal to 0 and let say it is the symmetry. So, epsilon naught equal to 0. So, in this expression if you see this man goes to 0, so only way to get  $C_m$  naught positive from the tail is make  $i_t$  negative, right. If I make  $i_t$  negative, then this man will become positive. So, the solution is you do this. So, how I am laying the wing, no wing setting angle. However, there is some tail setting angle also. It means that I can also do like this see I put tail setting angle. Let us say there is 3 degree, right. I do not give negative tail setting angle; I give positive tail setting angle, but this angle is 1 degree.



You see this, then also  $i w$  minus  $i t$  is becoming positive. I can generate your  $C_m$  naught positive. This is important. You will realize that such configuration where  $i t$  is positive will help in reducing induce drag at the tail because of tail which I have discussed in our performance course, but mostly you will find this is a preferred configuration and to be more precise, the preferred configuration is if you are not giving a setting angle, if this is  $C_g$ , you put the cambered aerofoil wing a c little bit of ahead of  $C_g$  of the airplane and will give some setting angle will be minus 1 to minus 6 degrees. 6 is a higher side.

How much I should put a c ahead of cg that we will know once we try to see that. The moment I put ac of the wing ahead of  $C_g$ , the stability gets affected. So, I need to ensure the static margin is what I am designing for. So, I must ensure this tail area, wing area tail location is good enough to give me the neutral point, where I get static margin of around desired static margin.

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The image shows handwritten mathematical derivations on a green chalkboard. The equations are as follows:

$$C_{m_{airc}} = C_{m_{ac,w}} + C_{L_{\alpha,w}} (\bar{x}_g - \bar{x}_{ac,w}) + \eta V_H (\epsilon_0 + i_w - \epsilon) = -0.05$$

$$C_{m_{airc}} = C_{L_{\alpha,w}} (\bar{x}_g - \bar{x}_{ac,w}) - \eta V_H C_{L_{\alpha,t}} \left(1 - \frac{\partial \epsilon}{\partial \alpha}\right)$$

$$\bar{x}_{np} = \bar{x}_{ac,w} - \frac{C_{m_{airc}}}{C_{L_{\alpha,w}}} + \eta V_H \frac{C_{L_{\alpha,t}}}{C_{L_{\alpha,w}}} \left(1 - \frac{\partial \epsilon}{\partial \alpha}\right)$$

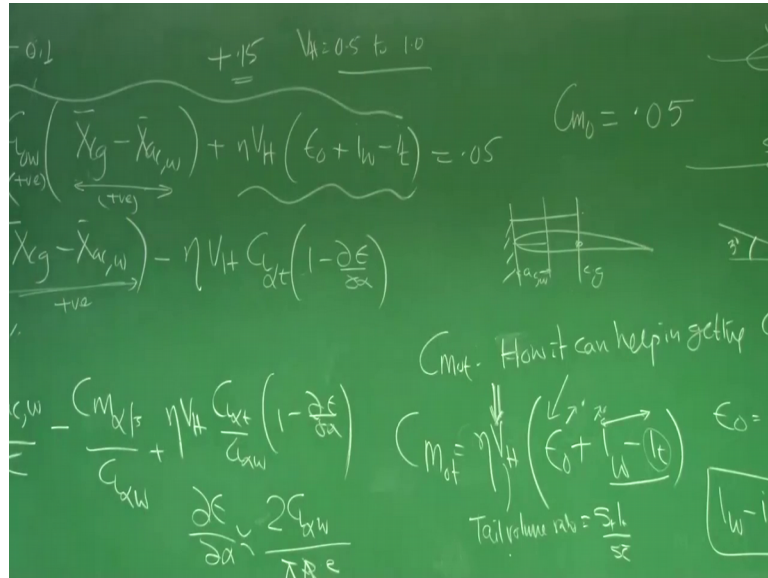
Additional notes on the board include:

- $C_{m_0} = C_{m_{airc}} = -0.05$
- $\frac{\partial C_{m_0}}{\partial \alpha} = \bar{x}_g - \bar{x}_{np} = -SM = -0.15$
- $\frac{\partial \epsilon}{\partial \alpha} = \frac{2C_{L_{\alpha,w}}}{\pi A_w e}$

If we recall the expression for neutral point, neutral point bar was  $x_{ac}$  wing by  $c$  minus  $C_m$  alpha fuselage by  $C_l$  alpha wing plus  $\eta v h C_l$  alpha tail by  $C_l$  alpha wing  $1$  minus  $d$  epsilon by  $d$  alpha, where  $d$  epsilon by  $d$  alpha is approximately  $C_l$  alpha wing by  $\pi$  aspect ratio of  $e$ .

So, now you have got these three things or in fact, you see one and two is sufficient. You have to iterate between one and two with a variation of wing setting angle, tail setting angle, then tail volume ratio is very important.

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You will start with  $v_h$ , start with point 5, go up to 1.0 mega iteration, so that the condition to be satisfied is  $C_m$  naught equal to  $C_m$  naught design. Let say typical value 0.05 and  $dC_m$  by  $dc_l$  which is  $x_{cg}$  bar minus  $x_{np}$  bar that should be minus static margin which will be minus let say 0.15 if I am doing 15 percent static margin design aircraft.

So, what is it I iterate 1 and 2 and try to satisfy these two conditions. Then, I will get various combinations with  $i_w = 0$  without  $i_w$  symmetric aerofoil cambered aerofoil tail setting angle, all those data will be generated. This is the theory behind it. In the next class, once you know this theory, we will now tell a designer how fix number, right. The  $v_a$  should be 0.5 to 0.8. What is the historical trend and immediately you should if I say  $v_h$  0.5 to 0.8, you should immediately say yes what has been done is satisfied. These two conditions, then those numbers will have a meaning to you. That is why I thought I will revise this quickly and prepare you, so that tomorrow when I discuss how a designer picks those number for example, designer will say I will take  $v_h$  around 0.6 or 0.8 to start with, then he will ask a question how do know it tail momentum term because  $C_g$  is not clearly known.

So, all those numbers, 70 percent of the fuselage line what will be the fuselage line, all those things are no based on statistical results. So, those thing I will be discussing tomorrow, so that a conceptual stage you can configure it and then, finally you do a quick accurate calculations. So, that was the purpose of this lecture today to warm you up that now we are going to use this through a designers perspective.

Thank you very much.