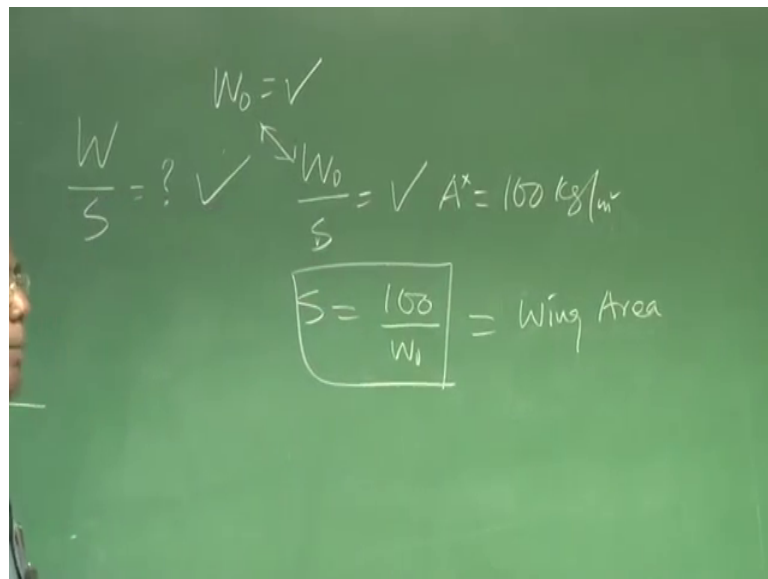


**Aircraft Design**  
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**Lecture – 33**  
**Stability Considerations**

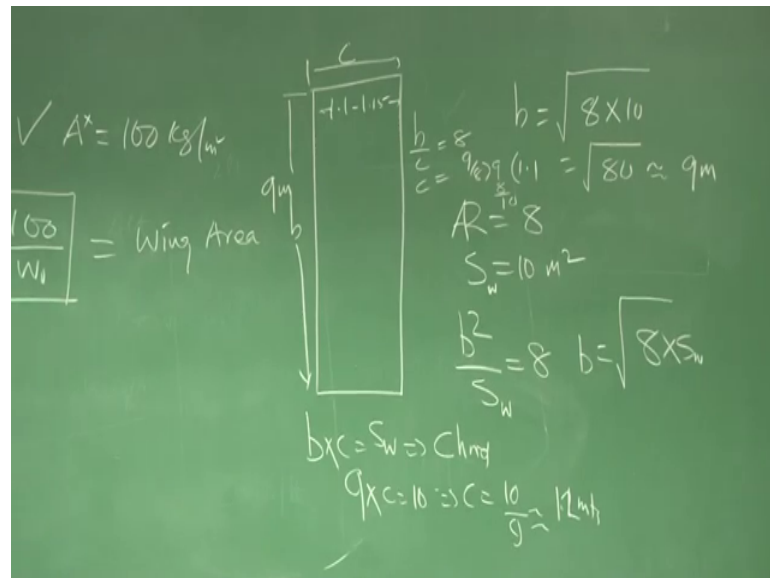
Good morning, we have gone through rough method to be  $W$  by  $S$ .

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What was the final aim, see we have already calculated  $W$  naught, we know how to calculate  $W$  naught by meeting all the mission requirement taking appropriate care about fuel consumption. So, if I have selected  $W$  by  $S$  some value then I naught I know  $W$  naught by at  $S$  is that value say  $A$  star let us say  $100 \text{ kg per meter square}$ . And since I already know  $W$  naught from here I get the value of  $S$ . So, once I get  $S$  - wing area which is wing area then now I have to layout that area we have assume some aspect ratio.

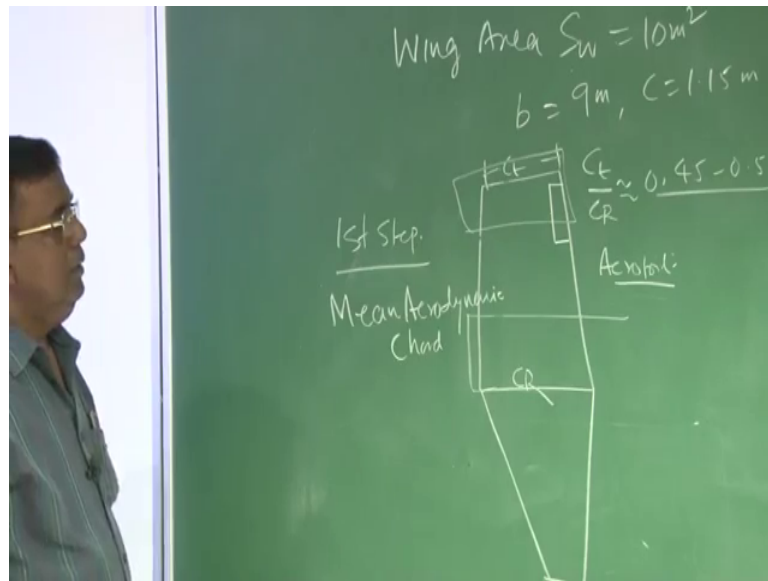
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So, we will say aspect ratio let say I have taken 8 and let us say wing area I am getting around 10 meter square, just a number. Then I know  $b^2$  by wing area is equal to 8. So,  $b$  equal to under root of 8 into wing area  $S_w$  or what can I write  $S_w$  here for clarity. So, from there I get a value of span, once I know span, I know  $b$  into  $c$  is equal to wing area, so I get the value of chord. So, I get enough idea, what will be the chord. If I take these number, what I get that  $b$  will be under root of 8 into 10 that is under root of 80, it would be around 9 meter roughly. Span is 9 meter and 9 into  $c$  is equal to 10 implies  $c$  is equal to 10 by 9, it is around 1.1 meter or 1.2 meter roughly 1.1 or 1.2 meter. So, this is 1.1 to 1.2 or 1.15 meter. So, this is my typical configuration.

You can always cross check that aspect ratio for rectangular wing will be also  $b$  by  $c$ ,  $b$  by  $c$  equal to 8,  $c$  equal to  $b$  by 8,  $b$  is 9 by 8, and 9 means 1 8 1 0 1.10 or something like that. Now, after you get idea about span and chord of the wing having area let us say which is 10 meter square. There are two important tasks you have.

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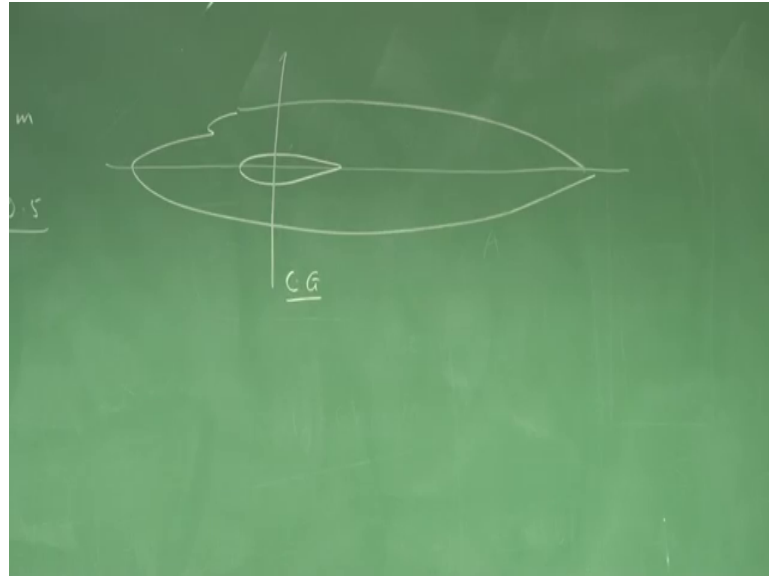
What are those important task I have got wing area if  $S_w$  and just for a case I am taken 10 meter square, I have  $b$  as 9 meter,  $c$  as 1.15 meter. So, what are the two tasks required, how should my wing look like that is what will be the taper ratio of the wing if it is a low speed wing. Typically you know taper ratio  $C_t$  which is  $t$   $C_t$  tip chord to root chord  $C_r$   $C_r$  generally 0.45 to 0.5 if I keep I am actually trying to come closer to elliptical lift distribution so that the induced drag is reduce right.

Then you have issue about aerofoil, you may like to keep aerofoil here and here different because you know that your aileron is here you do not want aileron to or this area to stall without having a warning or in indirectly you want this portion may stall earlier than this portion. So, accordingly you use aerofoil have different  $t$  by  $c$  different chamber or sometime you may give geometric twist also right at the tip I can give little bit of negative angle. So, that for a particular angle when this portion stalls this will not stall, so that your aileron is effective. So, all those details are required.

As a first step, we are talking about chord and span; the moment I put a taper ratio the question comes which one is the chord I would take, because chord is varying across the span. So, there again we have to come with the concept of mean aerodynamic chord which we have already explained in your earlier courses, but we will be revising this little bit and do a calculation for your understand. So, this is one aspect right. Second

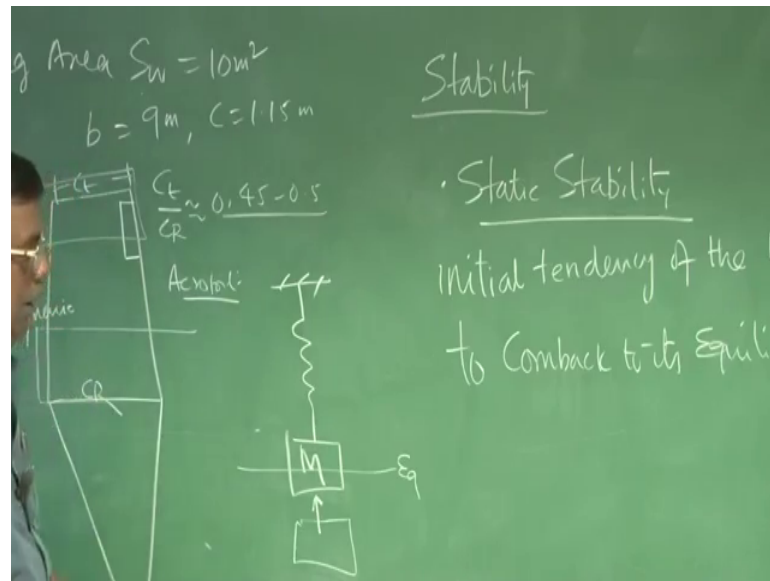
thing, you will also appreciate that after all wing is a sub system of an airplane, very important sub system of an airplane, its primary role is to generate lift.

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So, if I draw a fuselage (Refer Time: 06:26) it then the question is where do I put this wing, where means with respect to what with respect to the centre of gravity of the airplane. This is extremely important. How do I and where do I stall the wing, but then what are the requirements putting the wing if I know that then my answers will be simple and which we have done in our earlier course. At this, this issue we revise few things, so that you immediately you yourself know what to do and where to locate the wing.

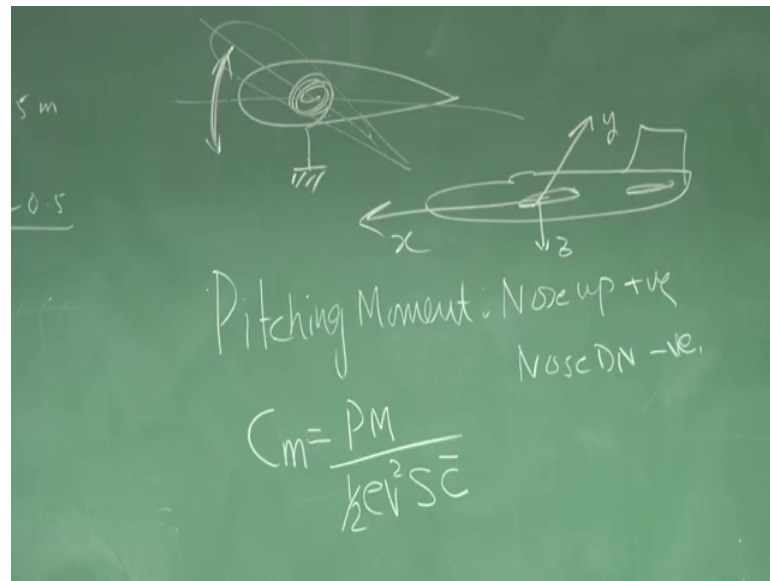
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And I am sure you are able to guess what I am going to talk now we are talking about stability because whatever take off, cruise, climb, descent, loiter whatever missions you are mentioning it has the inherent assumption that aircraft is statically and dynamically stable for general low speed aircraft. So, I thought I must talk about stability. And if you recall we when we are discussing stability, we define something called static stability. And how do you define static stability, it is the initial tendency of the body when of course disturbed from equilibrium, initial tendency of the body when disturbed from equilibrium. What I am saying initial tendency of the body to come back to its equilibrium.

Once this disturb from its equilibrium that is if I try to explain if I take a spring and a mass system, if this is the equilibrium and if I disturb it from equilibrium, I am saying here when you see immediately the spring force acts and it tries to push it back to the equilibrium. It is at initial tendency to push it back to the equilibrium. So, initial tendency condition is satisfied when the state is statically stable. For an aircraft, also I am trying to visualize through wing and what example we gave it was let us say wing and required spring.

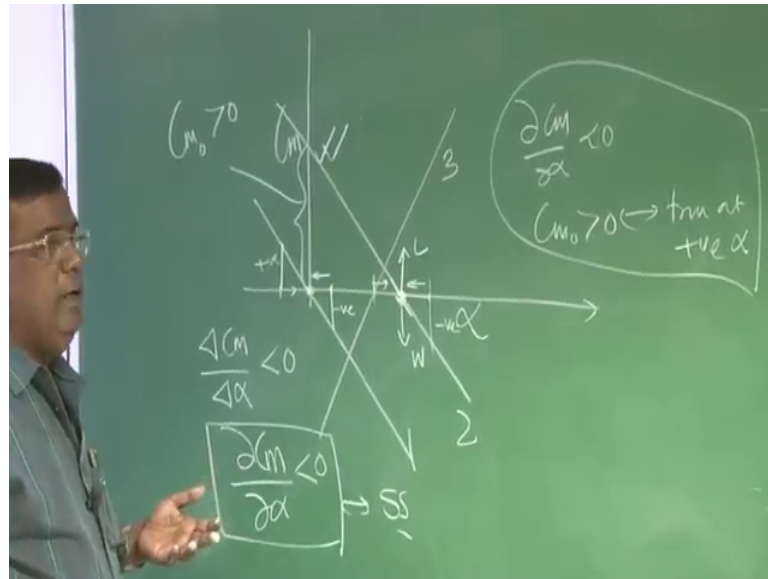
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So, if this is the equilibrium, if I disturb it about its equilibrium and release it will again try to come back to its equilibrium. So, it has static stability. And message was if you want to have static stability induce inside the aircraft, then you need to have some component which are aerodynamically waste and flounced which will give same action as a coil spring that is if angle of a deck is change from equilibrium the aircraft should automatically generate a nose down moment pitching moment. And since I am using pitching moment just for all of you pitching moment we defined as if I draw an aircraft this is just to revision so that you are ready this is x, this is y, this is z.

So, pitching moment is a angular motion about y-axis, this is. And the convention wise nose up we said positive and nose down we said negative is a convention. And we also defined pitching moment coefficient which is  $C_m$  that was pitching moment non-dimensionalized with dynamic pressure free stream dynamic pressure wing area and wing aerodynamic chord of the airplane. So, this was the understanding of static stability and which translated this into a meter form where we realized that.

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If I plot  $C_m$  pitching moment coefficient versus  $\alpha$  and if the variation is like this if the variation is like this, and one variation could be like this. So, 1, 2, 3 and we asked the question which one repeats static stability. So, for that if I try to recollect, how we analyze it was we have to think in terms of disturbance about equilibrium. So, if I take the first one the equilibrium is where net moment and forces are 0. So, this is the point. So, if I disturbed I want the equilibrium, let us say from here say disturbance  $\alpha$  as come, immediately this man will give a negative pitching moment.

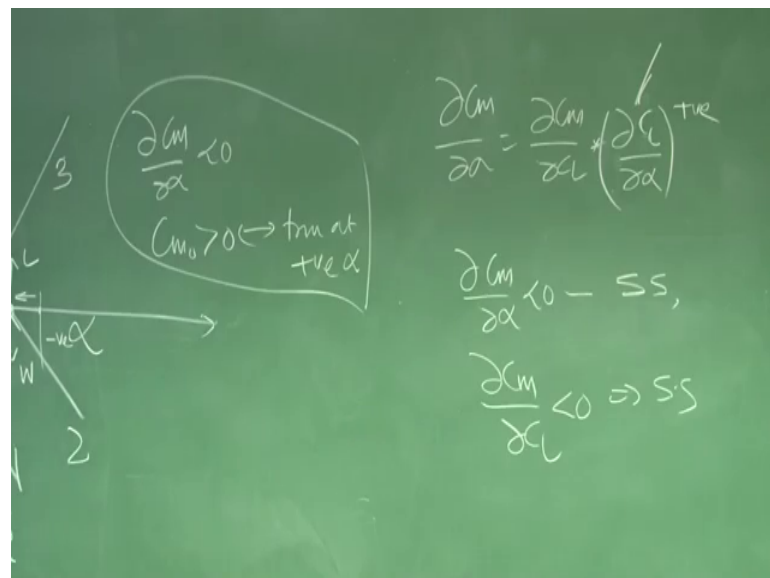
So, what is happening  $\alpha$  is and some angle came here. So, (Refer Time: 12:30) have increased so it reduce that come back to the same  $\alpha$  is 0  $\alpha$  1, 2 degrees it will touch generate a nose down pitching moment that is negative pitching moment. So, this will come back to this, reverse if it is reduced then it is positive pitching moment will be generated and it has always have initial tendency to come back to equilibrium. So, this variations  $C_m$  versus  $\alpha$  represents statically stable aircraft. Mathematically what we say that  $\Delta C_m$  by  $\Delta \alpha$ , the sign should be less than 0 or in neater form we write  $d C_m$  by  $d \alpha$  less than 0. This was the condition for static stability right.

Then we also are good what about the second line second line says again I check equilibrium point is here because here  $C_m$  is 0. Here I see if for some reason  $\alpha$  is increase from the equilibrium, it will generate negative pitching moment. So, in this case also it has a initial tendency to come back to equilibrium from this side also same thing

happens. So, say this equilibrium point is also having static stability the aircraft above this equilibrium point is also statically stable which is nothing newer because we have already realized  $\frac{dC_m}{d\alpha}$  should be less than 0 above that equilibrium point. So, above this equilibrium point  $\frac{dC_m}{d\alpha}$  the slope is negative. So, both are one and two are statically stable, but we preferred the second one why because we see that the second one the trim is at positive angle of attack. Finally, our aim is to generate lift and we should do balancing weight depending upon whether it is level flight or maneuvering flight we need to generate lift.

So, I will prefer to fly such that  $\alpha$  is positive. So, to ensure that  $\frac{dC_m}{d\alpha}$  less than 0 and also  $\alpha$  is positive, we note that this intercept which I said  $C_{m0}$  it should be greater than 0. So, we put a condition that for static stability  $\frac{dC_m}{d\alpha}$  less than 0 and  $C_{m0}$  greater than 0, this will ensure trim at positive  $\alpha$ . These things are clear to us. You can revise back to my course on aircraft stability and control, I think I have gone detail into it this is what is the understanding we will be having, we need to have to go for answering this question where do I locate my wing.

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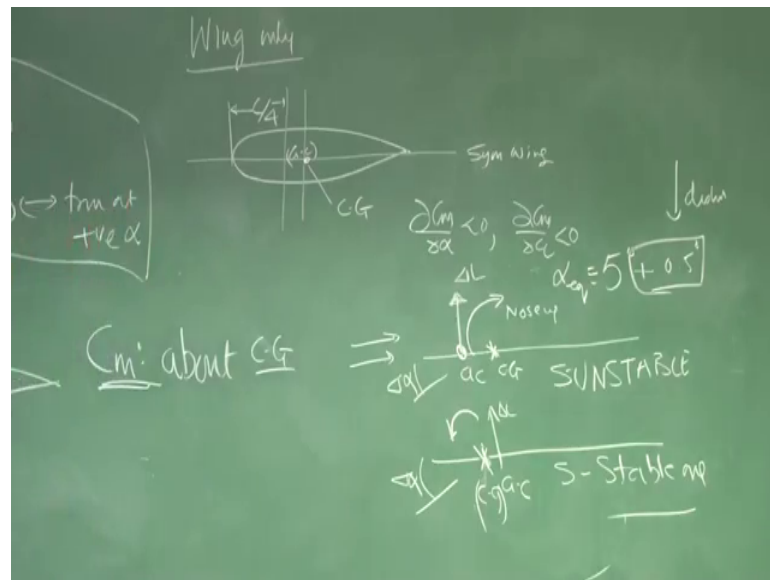


Also please remember one thing that once I write  $\frac{dC_m}{d\alpha}$ , I can write it as  $\frac{dC_m}{dC_L}$  into 1 by  $\frac{dC_L}{d\alpha}$  I can write  $\frac{dC_L}{d\alpha}$ . Assume we have using linear. So, once I said  $\frac{dC_m}{d\alpha}$  less than 0 is condition for static stability. Equivalently I can say  $\frac{dC_m}{dC_L}$  less than 0 for static stability, because this man is always positive



right that is a lift curve slope. So, either I think in terms of  $d C_m$  by  $d \alpha$  or  $d C_m$  by  $d C_L$ , I should be able to get a right answer. Now, when it comes to  $C_m$  naught, let us see our next condition is  $C_m$  naught before going to  $C_m$  naught, let us little work with  $C_m$  alpha.

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Let us say this is a wing symmetric wing, this is symmetric wing. And let us say this is the aerodynamic centre of the wing by now you know it will be roughly at quarter chord point correct. We are addressing that question or revising that question, how can I get this effect coil to spring effect, which will ensure static stability for my aircraft in flight. So, we are taking an example wing only that is disclosure to a tail list type of a airplane right only flying wing, I am taking a symmetric wing. Now, if I want to make it stable I need to have  $d C_m$  by  $d \alpha$  less than 0 or equivalently  $d C_m$  by  $d C_L$  less than.

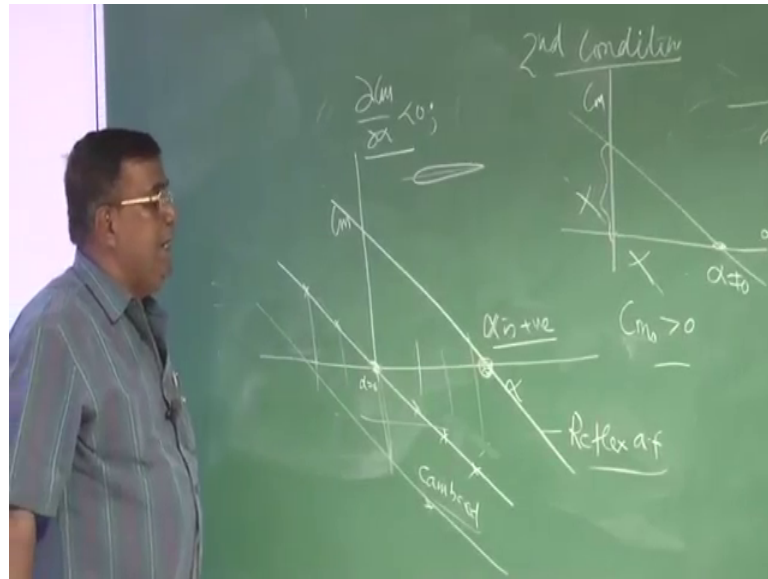
Now, this moment pitching moment when I am write it this is about pitch point in free space is moving. So, all this pitching moment are about CG. So, I need to know where is the CG because this  $C_m$  when I calculate which is non dimensionalized pitching moment that is about the center of gravity that matters when I flying the machine. So, let us see first case I put CG behind the aerodynamic centre of the wing what will happen. If I do that that is what I am doing if this is the ac, place the CG is behind aerodynamic centre. And in the aerodynamic centre is that fictitious point about which pitching moment is independent of angle of airline.

So, suppose it is like this I want to check whether this configuration is statically stable or not or in expanded way I want to check if by some disturbance the angle of a attack is increased, whether there is a spring type mechanism here or not which will try to take the aircraft nose down to the angle of attack increment is discouraged. So, it has initial tendency to come back to the equilibrium for example, meaning there by suppose it was flying at 5 degree which is equilibrium to the alpha and the disturbance came of let say 0.5 degree disturbance came. So, the aircraft if it is statistically stable, what it should do its aim should be to orient the aircraft such that its alpha equilibrium because 5 degree. So, what it should do it should generate a nose down moment, so that minus 0.5 is there and the this gentleman gets corrected or it has initial tendency to correct that.

So, let us see for positive delta alpha, we know we can draw this delta lift is represented at aerodynamic centre ideally should be perpendicular to velocity vector, but if it is small angle. So, I am drawing liberty taking liberty draw it like this. The message is if lift is here because ac is here, this will give a nose up moment right about CG. So, angle of attack further increase. So, it does not have initial tendency to generate a negative moment. So, this is statically unstable. So, what is the message that if aerodynamic centre is ahead of centre of gravity, then it will not be able to generate the restoring moment which is stabilizing in nature in particular static stability in context right.

So, what is the obvious message, message is if this is the aerodynamic centre then keep the CG ahead of aerodynamic centre. Now, see what happens, if I do that if there is a disturbance of positive delta alpha, there is a lift delta L which will give a nose down moment like this, yes indeed, it will try to discourage any increment in angle of attack which is the initial tendency to come back to the equilibrium. So, we say this is statically stable one.

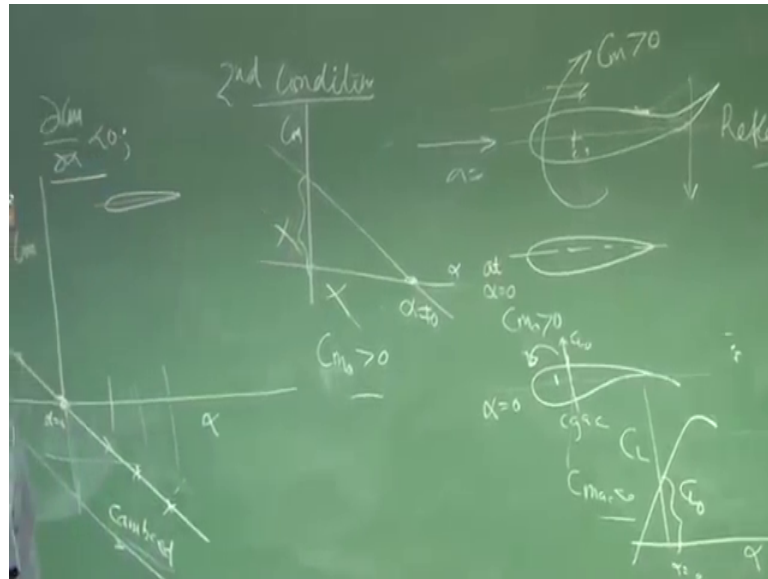
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So, what is the message? If I am designing a flying wing, if I am designing a flying wing, if I have to maintain  $\frac{dC_m}{d\alpha} < 0$ . So, I must ensure ac of the wing v behind CG of the flying wing right. Now, see if I try to plot  $C_m$  versus  $\alpha$  for this flying wing, which is symmetric aerofoil etcetera, etcetera then if I want to plot  $C_m$  versus  $\alpha$ , how should it look. If I want to see at  $\alpha$  equal to 0, what will be the  $C_m$ , it is symmetrical airfoil please understand. So, at  $\alpha$  equal to 0, I am taking of course, CG ahead CG is here and ac is here. At  $\alpha$  equal to 0, there are own way in lift because symmetric airfoil, so the pitching moment will be 0. So,  $C_m$  will be 0.

At  $\alpha$  equal to let say positive angle right what will happen at positive angle there will be a force here lift force which will give me nose down moment so that means, as angle of attack is positive, the pitching moment will be negative. So,  $\alpha$  positive, pitching moment will be negative like this. Similarly, if  $\alpha$  is negative  $\alpha$  comes like this then the lift force will be acting downward which will give me a nose up pitching moment. It sees like this. This is my ac, this is my CG. If negative angle of attack then the lift force will be downward and about the CG it will give a nose up moment. For negative, you will find positive pitching moment. If I draw for this airplane, it will look something like this  $C_m$  versus  $\alpha$  will look like this. And it indeed satisfied this condition  $\frac{dC_m}{d\alpha} < 0$  at equilibrium, equilibrium is here and this slope is negative.

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So, now the point is we also put a second condition, second condition was I should be able to ensure that  $\alpha \neq 0$ , there should be a trim point that is it should have positive  $\alpha$  so that it should produce lift. We are talking about symmetric aerofoil flying wing as an example we are taking right, so that can said is not satisfied here because here trim is there  $\alpha = 0$ . So, for this symmetric wing flying wing, it will not be able to generate lift at trim. So, I need this should be I want to fly it flying wing so that the variation is something like this. Instead of this, I do not want this slope is negative here, but it should have a  $C_{m_{ac}} > 0$  (Refer Time: 25:18) that condition.

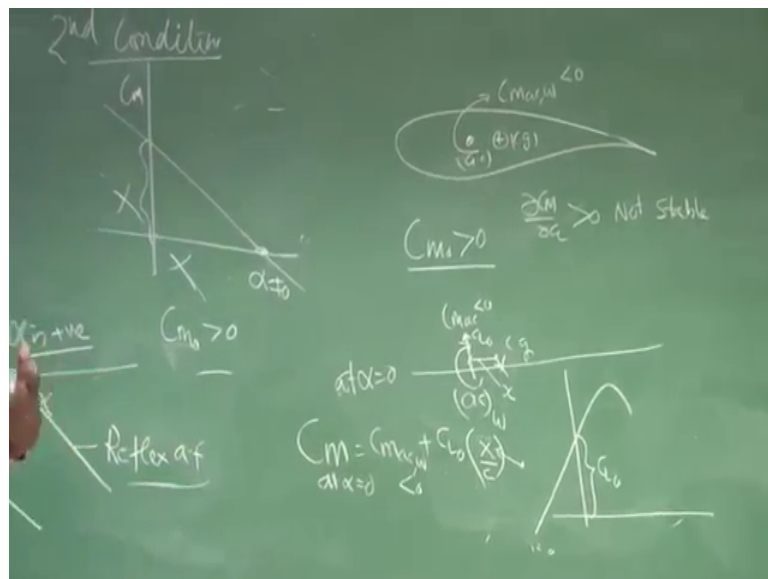
So, what can I do in these aerofoil wing. So, that at  $\alpha = 0$   $C_{m_{ac}} > 0$ , for symmetric it is not possible. Second thing comes to my mind if I make it cambered, and then again here it is CG, here it is ac at  $\alpha = 0$  what happens at  $\alpha = 0$ , you know that  $C_L$  versus  $\alpha$  goes like this for a cambered. So, for  $\alpha = 0$ , there will be a positive lift  $C_L$  naught. So, here there will be a  $C_L$  naught, so that will give a negative moment about CG. Also you know for a cambered aerofoil at aerodynamic centre, we have got  $C_{m_{ac}}$  which is also negative may minus 0.02 to minus 0.05

So, what is happening if I am putting a cambered aerofoil for a flying wing at  $\alpha = 0$ , the whole  $C_m$  at  $\alpha = 0$  will become negative, this is cambered.

Again I am not able to trim it at positive angle of attack, yes, I can maintain the static stability by ensuring that the ac is behind CG that is possible, but I want something like this. So, I want  $C_m$  at  $\alpha = 0$  greater than 0. So, what is the way, if I am using a symmetric wing instead of making it cambered like this, I do reverse. I make this wing like this, which is reverse of cambered and this is called reflex wing which is when I say reverse of camber in a very limited sense right, they should have this it is this.

If I do this aerofoil then what happens at  $\alpha = 0$ , if you see there will be 4 C L which will give about C G a C m which is greater than 0 is not it, which were that possible here and here it was negative. So, if I put a reflex aerofoil and ensure that ac is behind CG then I can create variation of C m versus alpha like this. So, now, I have a trim point, where alpha is positive and I can ensure lift equal to weight by choosing appropriate angle or appropriate dynamic pressure at this alpha, this was just to revise you. And we will also see that if you go back to my earlier lecture, how to use cambered aerofoil so that C m naught positive is obtained.

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For example, if I am looking for  $C_m$  naught positive for a minute camber, how do I look it. Let us say this is my camber aerofoil, and you know this is ac and here you have C m ac of the wing, which is negative on the time. If I locate this ac ahead of CG, please understand once I locating ac of the wing ahead of CG of the wing in this case, there no tail here is no fuselage, this man is statically unstable. But this configuration will help

you in getting  $C_m$  naught greater than 0, this will say  $d C_m$  by  $d C_L$  is greater than 0 for not stable statically.

But  $C_m$  naught greater than 0 possible because if you see that if this is the ac of the wing where you have got a  $C_m$  ac which is less than 0 and CG is here. So, at alpha equal to 0, you know there will be because cambered aerofoil at alpha equal to 0, there will be  $C_L$  naught there will be  $C_L$  naught here. So,  $C_m$  at alpha equal to 0 will be equal to  $C_m$  ac is already there ac wing now plus  $C_L$  naught into this distance  $x$  bar which is nothing but this distance divided by  $C$ , if I say this distance is  $X$ , I say  $X$  by  $C$ .

And this I am measuring this is positive. So, what do you say  $C_L$  naught into  $x$  by  $C$  plus  $C_m$  ac this will my  $C_m$  at alpha equal to 0 that will make location of  $X$  how much gap is there that will be used you see to nullify this  $C_m$  ac which is always negative for a cambered aerofoil, because we want  $C_m$  naught a positive. So, whatever negative  $C_m$  ac wing is there, we will try to see it to the extent possible we marginalize this by locating appropriately the ac of the wing because I know ac of the wing is ahead of CG, so statically unstable. If I have flying a flying wing I will simply use a what you call reflex type aerofoil for a flying wing which will be statically stable.

Now, I can trim it also. If I want to fly a cambered type of this then of course, you need to use some sort of a control system because I understand that unstable does not mean uncontrollable. So, we are talking about that. We are trying to jump from here to an aircraft within a statement that ok for  $C_m$  naught I will put ac of the wing ahead of CG, so that  $C_m$  ac is neutralized. However from the stability point of view for the whole aircraft, I will be using tail horizontal tail stabilizer right to see that overall the aircraft becomes statically stable and that is where this you should complimentary role provided to aircraft by wing and the tail wing takes care of lift and tail takes care of primarily the stability. And that is why the horizontal wing is called horizontal stabilizer, and vertical tail is called vertical stabilizer. So, next class we will be starting from here right.

Thank you very much.