

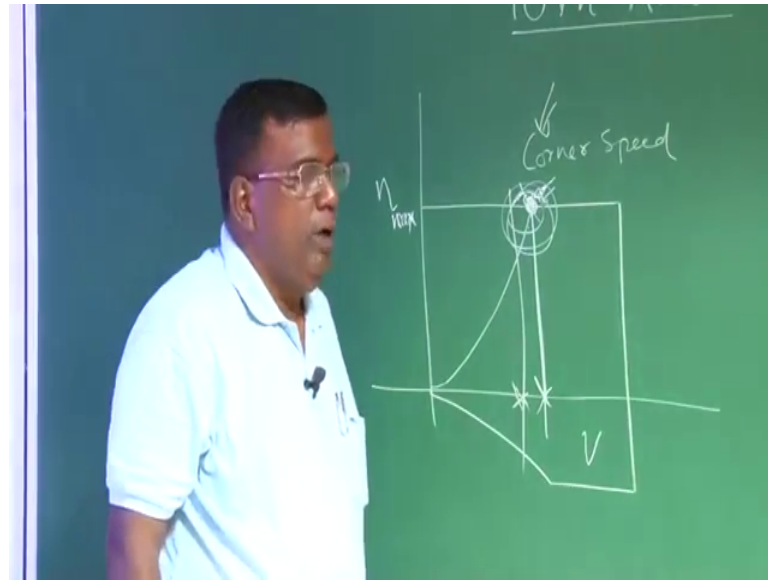
**Aircraft Design**  
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**Lecture – 26**  
**Wing Loading: Maneuver, Climb and Glide**

Good morning friends. We are continuing our lecture on wing loading. We are trying to explore; what sort of or what magnitude of wind loading we should have to perform various mission requirements. You very well understand that lift; amount of lift will be proportional to the wing area. Amount of lift will also be proportional to the speed with which the wing is relatively flying with atmosphere. Apart from densities and angle of attack, etcetera when we try to ensure it has requisite velocity or speed, we think in terms of thrust to weight ratio. After all, thrust is a force which will give you the acceleration and hence the speed well you try to see how much lift it can generate keeping other parameters constant for a given weight.

We think in terms of what is the area that is why we are discussing about ratio between weight and the wing area that is wing loading. We have completed first exposure for wing loading primarily for cruise and loiter although, we have spoken something about takeoff, but as I promised; we will be talking about takeoff. What is the wing loading requirement for takeoff in little more detail because we need to also understand high lift devices which can increase  $C_L$  max little more and the selection of such highly lift devices will play an important role in designing overall aircraft performance in that continuation we were also talking about turn rates and we discuss loosely about instantaneous turn rates right 10 years turn rates and sustained turn rates.

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Whenever you are turning; how do you turn? I am moving like this, if I want to turn, I have to bank and I have to use that lift force to generate that centripetal acceleration to get a particular rate of turn, whenever you are talking about turn rates you please revise at  $V$   $n$  diagram load factor and speed diagram it is something like this and this point is the corner speed that corresponds to a speed where  $C L_{max}$   $n$  is  $n_{max}$  right. That is the speed required, if you want to take advantage of large load factor as well as at a particular speed if you do not maintain this speed, then you will not get this  $n_{max}$  or you will not be able to generate the type of acceleration you want right.

So, it is important keep back of this in your mind. Right now what happens in instantaneous turn rate in instantaneous turn rate; we are talking about I am going like this I am turning, but I am not bothered about whether the speed is reducing or whether it is losing the altitude. So, it can do like this, but in sustained turn rate, we want the speed should remain constant, right? Why it is important? Come here, if I want to take advantage of corner speed, then I need to maintain the corner speed. Suppose I am turning and I am not touching the throttle and all the drag will increase as I am going to take a instantaneous turn. So, it will reduce the speed.

The moment it is the speed you are not you are somewhere here. So, you are not able to take advantage of corner speed and when you are for a fight; dogfight or chasing an

aircraft and going for such scenario. Suppose you are taking a turn and your speed is reducing and losing the altitude, then you are in a disadvantaged position, right.

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$\frac{W}{S} = \frac{q C_{L_{max}}}{n v}$

$L = nW$

$\frac{1}{2} \rho v^2 C_{L_{max}} = nW$

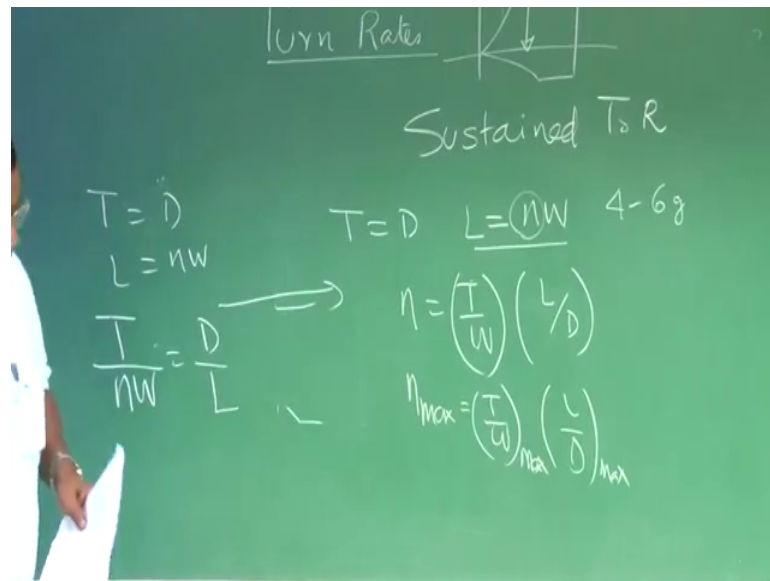
$C_{L_{max}} = 0.6 \text{ to } 0.8$   
 Single TE flap

$C_{L_{max}} \text{ (Complex)} = 1.0 \text{ to } 1.5$

So, sustained turn rate becomes important when you are talking about a fight in the air may be a dogfight if I recall if our instantaneous turn rate we have seen that  $W$  by  $S$  is  $q C_{L_{max}} / n$  and that we derived because lift equal to  $nW$  and lift is  $\frac{1}{2} \rho v^2 C_{L_{max}}$  whatever possible;  $C_{L_{max}}$  and from there I can find  $W$  by  $S$  as this, but a word of caution this  $C_{L_{max}}$  is not that  $C_{L_{max}}$  when you are going for takeoff or landing, right.

So, just to give you an idea for normal aircraft it is you are not using any flaps when you are doing a turn because it demands lot of increment in the drag right, but yes, for fighter; the special fighter, where flaps are also deployed; flaps are also deployed for a few fighter airplane to get some  $C_{L_{max}}$  more than the conventional  $C_{L_{max}}$  and to have some idea the  $C_{L_{max}}$  could be you can take around 0.6 to 0.8 with single trailing edge flap for combat of course, for combat and the  $C_{L_{max}}$  for calculation purpose for a complex airplane it could be one point 0 to 1.5, but this sort of a maneuver demands huge  $T$  by  $W$ .

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So, from here we want to come to sustained turn rates; sustained turn rate, we understand; if I had to turn the same time I have to ensure that I do not lose altitude nor there is a reduction in the speed; that means, the thrust has to play an important role during the sustained turn rate this is important for a combat scenario and if I now try to see; what does it mean? You know you have to generate that much of lift. So, that you get a particular load factor and this could be four to 6 G or 8 G depending upon what you are doing.

So, from here I can write  $n$  equal to  $T$  by  $W$  into  $L$  by  $D$ ; if you are stick all to  $D$  lift equal to  $n w$ . So, I can write  $T$  by  $n w$  equal to  $D$  by  $L$  and which is I can directly see that  $n$  equal to  $T$  by  $W$  into  $L$  by  $D$ . Now if you are trying to maneuver. So, that  $n$  equal to  $n_{\max}$  because you are at corner speed somewhere here this is corresponding to  $n_{\max}$  and this is a corresponding speed; so, then  $n_{\max}$  equal to  $T$  by  $W_{\max}$  and  $L$  by  $D_{\max}$  and once I write  $n_{\max}$ .

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$$V_{max} = \left(\frac{T}{W}\right)_{max} \left(\frac{L}{D}\right)_{max}$$

$$C_L = \sqrt{\frac{C_{D0}}{K}} \quad K = \frac{1}{\pi A Re}$$

$$C_L = \sqrt{\pi A Re C_{D0}}$$

$$\frac{W}{s} = \frac{q_{\infty} C_L}{n} = \frac{q_{\infty} \sqrt{\pi A Re C_{D0}}}{n}$$

$$\frac{L}{D} = \frac{C_L}{C_D}$$

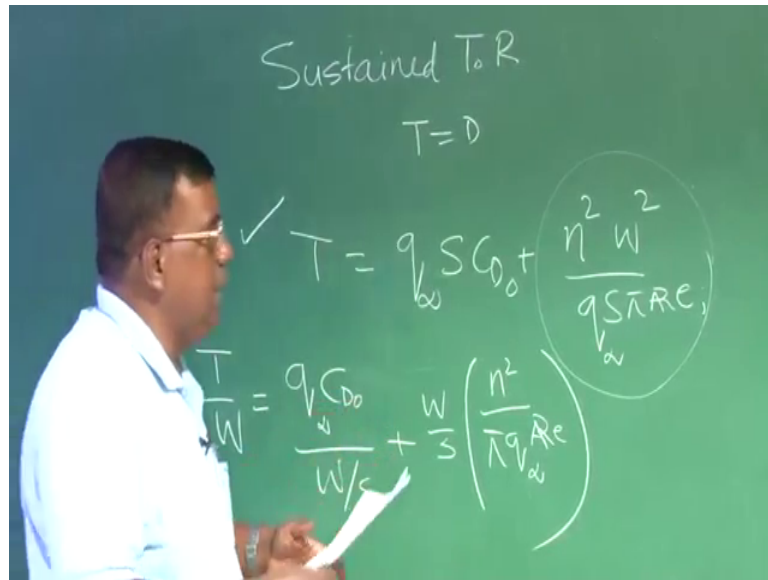
Equal to T by W max into L by D max and for L by D max we know L by D max there are all familiar this coherently C L by C T max and that fixes the C L value as under root C D naught by K.

So, C L equal to under root C D naught by K and K equal to 1 by pi aspect ratio e. So, your C L becomes under root of pi aspect ratio e into C D naught. Please understand, we are talking about combat, but relationship which I am writing here as drag polar therefore, subsonic speeds right and those combats are generally performed at a speed lower than the sonic speed maybe around 280 meter per second worst case 300 meter per second you have to really go for a wind tunnel testing and get those drag polar correctly.

So, these are the mismatch and w by s equal to into C L by n. So, that will make use this under root pi aspect ratio is C D naught by n here if it is the free stream dynamic pressure and you will see that as you require larger n right your wing loading requirement goes down which is true because going down means what means you need larger area right that is why the larger lift will come, but then your thrust limitation will come larger area means larger drag. So, to be too demanding for T by W max right, so, you will find that typical this value come very low and you have to give a halt to that whether really it is possible or not or I have to non-linearly increase the T W value by using some local supercharger afterburner like that or I do not mind spending money let it be inefficient, but at least it does the performance.

So, all those questions will be asked when will be actually trying to synthesize these concepts right there is another way of looking into it that is more realistic and it is being followed also.

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So, when I say sustained turn rate I am assuming thrust equal to drag, but I do not want the speed to go down because then I will not be able to fly at a prescribed corner speed and if I do not fly prescribed corner speed I will not be able to add take advantage of  $n_{max}$  and  $C_{L_{max}}$  or  $n_{max}$  well maximum load factor. So, if I do this then I right thrust equal to if you infinity  $S C_D$  naught plus  $n^2 w^2$  by  $q_{\infty} S \pi$  aspect ratio  $e$  and you know; what from it has come your expert this part you can easily derive because you know lift equal to  $n w$  and this this part of drag is  $K C_L^2$  over  $K$  is equal to 1 by  $\pi$  aspect ratio  $P$ .

So, you just substitute you will get this expression thrust equal to  $q_{\infty} S C_D$  naught plus  $n^2 w^2$  by  $q_{\infty} S \pi$  aspect ratio  $e$  I divided left hand side and right hand side by  $w$ . So, I get  $T/W$  equal to  $q_{\infty} C_D$  naught by  $W/c$  plus  $W/S$  into  $n^2 \pi q_{\infty}$  aspect ratio  $e$ . This expression I can further rearrange to write  $W$  by  $S$ .

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$$\frac{\pm \sqrt{\left(\frac{T}{W}\right)^2 - \left(\frac{4n^2 C_{D0}}{\pi A e}\right)}}{\frac{2n^2}{q \pi A e}}$$

$$\frac{2n C_{D0}}{\pi A e} \quad n=2 \quad \frac{2 \times 2 \times 0.02}{3.14 \times 8 \times 0.6}$$

$\alpha \rightarrow \text{Higher}$   
 $e = 30\%$

Equal to T by W plus minus under root T by W square minus four n square C D naught by pi aspect ratio e divided by 2 n square by q infinity pi aspect ratio e for the youngsters you please yourselves to derive this expression from here if you see; it is in a quadratic in W by S right.

So, you can find out W by S and these are 2 roots what is important here you could see that whatever may be wing loading whatever may be wing loading to make a real sense of this expression you have to select T by W such that it is greater than equal to 2 n C D naught by pi aspect ratio e under root right these square minus four ratio should be greater than equal to 0 from here. This gives you a condition whatever may be irrespective of wing loading you have to ensure that T by W is greater than equal to 2 n and that you C D naught by pi aspect ratio e. Just have a feel for this number you can always see if n is 2 then this value is 2 into 2 typical value of C D naught let us say 0.02 divided by pi 3.14 aspect ratio e and let say 0.6 I am taking value of e.

So, that will give me a rough idea of what is the T by W minimum is required irrespective of wing loading right or a given value of load factor that is why in connection it says whatever wing loading you are doing to your friend you have to ensure that T by W satisfy this condition for different values of n. So, when you are designing you need to check this and a design stage you should take some realistic value of C D naught aspect ratio and e and that is where the synthesis come from statistical

data right and also if you are flying at an angle of attack little higher little higher then whatever e value you have taken for other calculation reduce it by thirty percent that is also a good guideline right.

When you will be doing an exercise you will be using these concepts where wherever it is required, right; so, that you will know that how we are going to synthesize all those understanding, but unless you have this understanding. Since this is become extremely difficult right that is why I am spending time on this one of my friend complain you are going on; giving a lecture when you are going to synthesize and follow me the answer is very simple before you go for any Olympic competition you have to do your training at home perfect right you have to sweat it during training revisit climb rate or climb and glide rate give you that title.

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The image shows a green chalkboard with handwritten mathematical equations. At the top right, the title "Climb and Glide" is written. Below it, "Climb gradient" is written. On the left side, the following equations are written:

$$\frac{T}{W} - \frac{D}{W} = G$$

$$\frac{D}{W} = \frac{T}{W} - G$$

$$\frac{D}{W} = \frac{q S C_D}{W} + \frac{q S C_L}{W A R e}$$

On the right side, the following equations are written:

$$T - D - W \sin \gamma = 0$$

$$\sin \gamma = \frac{T - D}{W} = G$$

When you where climbing there are 2 things that comes to our mind one is what is the climb angle right another is what is the one is climb angle another is rate of climb suppose it is climbing at a speed v then we will know v sin gamma is the rate of climb we will be sharing with you different values recommendations by aviation authorities may be f a or e sau or D G C a once you are clear to this and before you come for a design will go for one lecture on all those specifications right that is important because unless you follow those regulatory specifications you will not get certificate or certificate of airworthiness or certificate of safety you not get right.



So, we will visit that once these fundamentals are clear there is another term being use will find this climb gradient typically to see if I write for climb it is T minus D and in sin gamma equal to 0 for the steady climb and you have sin gamma equal T minus D by w which is referred to as climb gradient now you understand what is the meaning of sin gamma where gamma is the flight path angle right now if I use this expression I can write T by W minus D by W equal to G because G is T minus D by W. So, I am not doing anything right and then I; D by W is equal to T by W minus G from here comes here.

So, then I can write D by w this is equal to q infinity s C D naught plus q infinity s in to C L square by pi aspect ratio e which I can write as. So, D by w is this which I can write D by W equal to further q C D naught by W by S because there will be w here n is the w here D W D means drag half rho v square S C D naught half rho v square s into K C L square, but divided by W is here.

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$$\frac{D}{W} = \frac{q C_{D0}}{W/S} + \frac{W}{S} \cdot \frac{1}{q \pi A e}$$

$$\frac{W}{S} = \frac{\left[ \left( \frac{T}{W} \right) - G \right] \pm \sqrt{\left[ \left( \frac{T}{W} \right) - G \right]^2 - \frac{4 C_{D0}}{\pi A e}}}{2 / (q \pi A e)}$$

So, that is why D by w I can right now as Q C D naught by W by S W by S into 1 by q infinity pi aspect ratio e I repeatedly I tell young friends please derive this expression if somewhere I am committing a mistake right in the forum.

So, D by W I have expressed using W is S. Now I go back to this equation and substitute D by W expression here and do some rearrangement and find out an expression W by S

as  $T$  by  $W$  minus  $G$  plus minus under root  $T$  by  $W$  minus  $G$  square minus  $4 C D$  naught by  $\pi$  aspect ratio  $e$  derived by  $2 q$  infinity  $\pi$  aspect ratio  $e$ . Here you see this expression again if this gentleman has to have a realistic value of  $w$  by  $s$  so; that means, then this discriminant has to be greater than or equal to 0 otherwise the value will become unreal imaginary.

So, what you do what is the condition you get which a designer will try to snatch as an very important information.

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$$\frac{T}{W} \geq G + 2 \sqrt{\frac{C_{D0}}{\pi A e}}$$

$$G = \frac{T-D}{W} = \sin \gamma$$

$$\gamma = 10^\circ$$

$$G = \sin 10^\circ \approx \frac{10}{57} \approx \frac{1}{6} \quad (1/6) \cdot 10 = 1.6$$

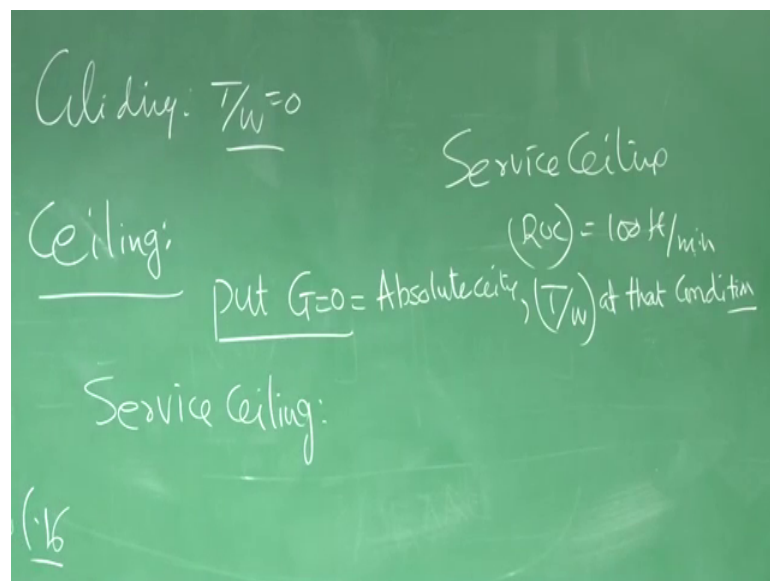
So, it will tell you that  $T$  by  $W$  greater than equal to  $G$  plus 2 under root  $C D$  naught with  $\pi$  aspect ratio  $e$ , right. What is the message you are getting here. Do not forget, you are talking about climb right climb means  $G$  is what  $G$  is  $T$  minus  $D$  by  $w$  right. So, for a given value of  $G$  whatever climb rate you want you put that number here put the aerodynamics here the way we have to ensure that  $T$  by  $W$  should satisfy this condition.

For example if you want to extrapolate what will be  $T$  by  $W$  for climb angle says this is nothing, but  $\sin \gamma$  let us say  $\gamma$  you want 10 degree then  $G$  value is  $\sin$  of 10 and let us say that value approximately is 10 by 57 or it is 1 by 6 roughly. So, that is 0.1 around 0.1; 10 by 57.3 means roughly 1 by 6 1 by 67.1; 6.16 roughly value. So, that value; you have to put it here. Put the aerodynamic derivatives and their values here  $C D$  naught 0.02 aspect ratio is 8 or 8 or 7  $e$  may be 0.7  $\pi$  you know. So, we will get a

condition what is the  $T$  by  $W$  required to have a climb rate given by the value of  $G$  as simple as that.

If I want to extrapolate the information about gliding once we are gliding means there is no thrust. So, in that expression whatever I have written put  $T$  by  $W$  equal to 0. So, that will give you what sort of  $W$  is  $S$  required for gliding from this expression itself.

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So, for gliding you just put  $T$  by  $W$  equal to 0 because no thrust during gliding. So, you can use that earlier expression to get what is  $W$  by  $S$  and for ceiling maximum ceiling you know that ceiling is that altitude at which rate of climb maximum is 0, but we will generally work with service ceiling for service ceiling the rate of climb maximum is hundred feet per minute.

So, you convert it into appropriate unit and then put  $G$  equal to 0, but at absolute ceiling the rate of climb will be 0 put  $G$  equal to 0 for absolute ceiling and get the expression by putting  $T$  by  $W$  at that condition because it is that all altitudes what is the thrust; what is the weight. So, that you put that expression along with  $G$  equal to 0 for absolute ceiling because  $G = 0$  mean rate of climb is 0, but if you want to go for service ceiling same expression you can use let me we write that expression so that you will be able to follow it.

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$$\frac{W}{S} = \left[ \frac{T}{W} - G \right] + \sqrt{\left[ \frac{T}{W} - G \right]^2 - \frac{4C_{D0}}{\pi A e}}$$

$\tan \gamma = \frac{V_v}{V_h}$   
 $\sin \gamma = \frac{V_v}{V} = G$   
 $V_v = ROC$   
 $100 \text{ ft/min} = V_v$   
 $G = \frac{\text{Vertical Speed}}{\text{H-speed}}$   
 Cliding:  $\frac{T}{W} = 0$ , Ceiling,  $ROC = 100 \text{ ft/min}$  (Service Ceiling)

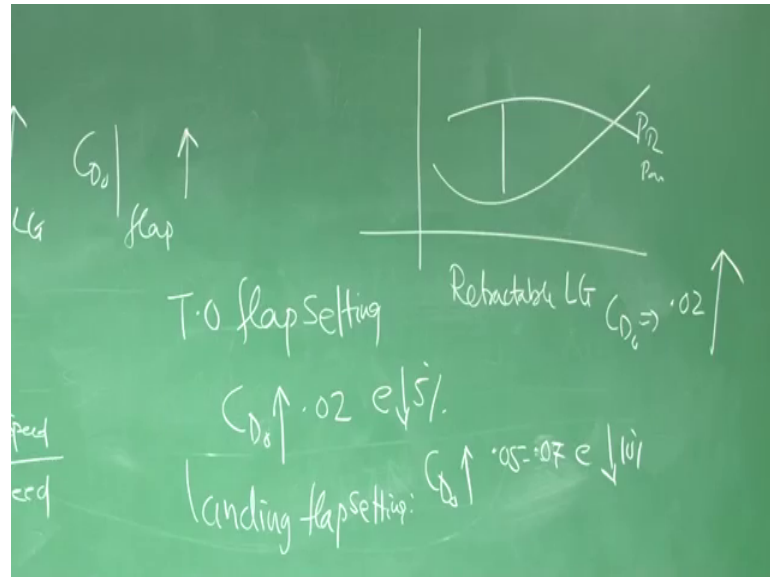
So, I repeat; if I want to find out what is the wing loading for gliding I can use this expression because for gliding  $T$  by  $W$  is 0. So, I am not applying any thrust we can use the expression of  $W$  by  $S$  here by putting  $T$  by  $W$  0 in this expression, right. Now if you want to go for ceiling; they say rate of climb is 100 feet per minute which I am talking about service ceiling then I have to find out the equivalent value of  $G$ ;  $G$  is what?  $G$  is ratio of vertical speed by horizontal speed see this is if you see that; this is the  $V$  and this is  $\gamma$  this is  $V \sin \gamma$  and  $\gamma$  is  $\tan \gamma$  is vertical velocity by horizontal velocity and also you could say  $\sin \gamma$  equal to vertical velocity, but total velocity and what is your  $G$ ;  $G$  is nothing but this what is  $V_v$ ?  $V_v$  is rate of climb.

So, what do you do if I am going for a service ceiling with hundred feet per minute I convert that hundred feet per minute to vertical velocity that is the rate of climb? So, and I know what speed I am going divide it I get value of  $\sin \gamma$  which is the value of  $G$  I put it here and what is the  $T$  by  $W$  for that local condition I put it here and I get  $W$  by  $S$  for service ceiling right of whatever magnitude do you want correct because remember service ceiling is that altitude at which rate of climb becomes hundred feet per minute. So, that altitude part is taken care by  $T$  by  $W$  local conditions also  $C_D$  naught because this number will be different. So, all these things will be covered into this expression.

You have to carefully use it and get the right wing loading values also another thing you must understand this  $C_D$  naught plays important role and from a specification point of

view I thought I will tell you little more before I end when you are designing a set twin engine aircraft right it is possible that one engine may fail, but through rudder you will be able to control the yawing moment, but from safety point of view there is a prescribed rate of climb even with one engine failed right there is a prescribed rate of climb with the landing gear out because as a landing gear is out there will be increment in the drag.

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So, that will change the rate of climb because you know rate of climb is this is the power required and whatever power available this is excess power.

So, as the landing gear is out the drag increases what is that made the drag increase the  $C_{D0}$  value was landing here out will increase when you are climbing if I put in the flap then this landing gear by  $C_{D0}$  with flap they will increase I am comparing this increment with respect to clean aircraft. So, when you are finding this wind loading and all we have to very carefully take the value of  $C_{D0}$  right and for that I can tell you for takeoff flap setting these are the tentative number right  $C_{D0}$  gets increased by 0.02 and  $e$  generally goes by down by 5 percent and landing the flap setting is maximum right because you want to also increase drag. So, that time will find will not be surprised if  $C_{D0}$  is increased order magnitude of 0.05 to 0.07 and  $e$  reduces by 10 percent.

So, these are 0.05 to 0.07; we have some number the designer should know. So, that in the conceptual design he takes care of this may be little conservative does not matter. Typically retractable landing gear retractable landing gear is when you are putting the landing gear out suppose I have a clean aircraft C D naught let say 0.02 and once I am putting the landing gear out the C D naught may get increased by 0.02. These are substantial actually, right.

So, this these are things which we need to keep back of our mind before we try to configure these lectures has to tell you give you an idea about what a designer should look for what is trying to get few basic parameters and how they are sensitive to some system which are unavoidable you cannot avoid a landing gear for aircraft mostly, but when you will be actually solving a problem for a specific airplane we will see the exactly how much it is increased right maybe we will be using standard data which possible I will use data for Hansa 3 aircraft whatever data available. So, those things will come there will be a total case study that we need to prepare ourselves with this understanding right.

Thank you very much.