

Aircraft Design
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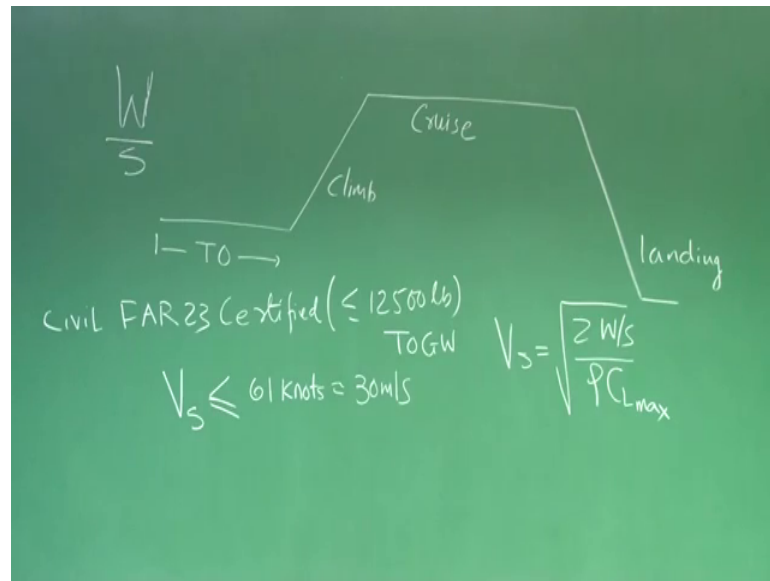
Lecture – 25
Wing Loading

Good morning friends. We will be talking about wing loading today. Till this time I was trying to have a relook on thrust to weight ratio required for different machines and the approach had been without having much of a formula or a data sheet; how can I get initial feel for these numbers using law of physics. Similar approach will also have on wing loading and knowing very well for a particular mission operations mission we need to have a cumulative effect of thrust towards ratio as well as wing loading and a better design will ensure that we have optimally used both this thrust loading and wing loading smartly as and when required.

We have options to vary thrust loading by controlling the thrust we may not have that much of control to alter wing loading volume flight although; we know because there will be fuel consumption wing loading will go on reducing at the weight of the whole aircraft will go on reducing. Similarly when you drop some stores from the aircraft, the wing loading also will decrease; you also know for a general aircraft; when I am taking off and when I am landing the wing loading are different typically passenger aircraft, if we say where we are not dropping anything out of aircraft still; the wing loading during takeoff and wing loading during landing will be different because substantial amount of fuel consumption will happen during the mission requirement.

Even during cruise; the wing loading during start off cruise and wing loading during end of cruise will also change because they are change in weight because of fuel consumption for a designer. As I am telling all the time that whatever wing loading, we are talking about for a particular mission. They should be translated back to wing loading take off; that means, if there is a particular wing loading required during cruise and you know weight has reduced. So, you have to correct the weight assuming it is in the take off conditions. Essentially, it means the amount of fuel which is consumed that has to be added to the local weight during the cruise and divide by the wing area to get the wing loading during takeoff as simple as that and if I try to see wing loading.

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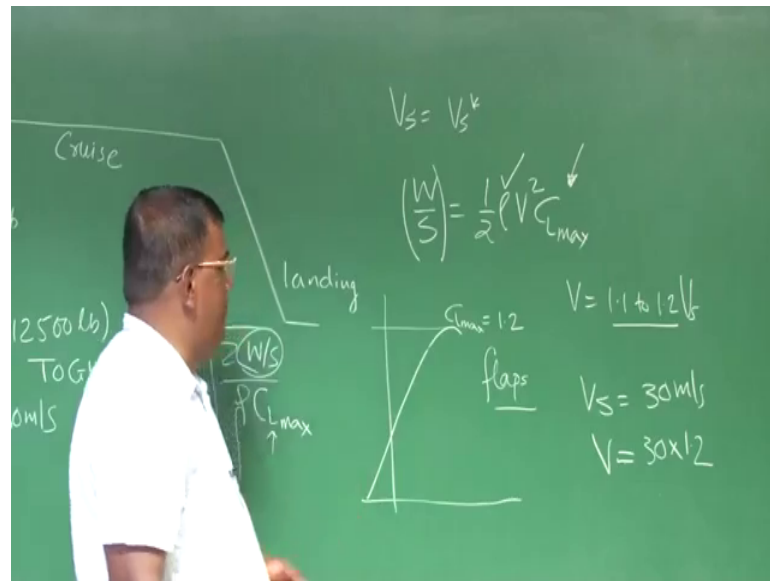


W by S and try to see basic maneuvers are the mission. Here I know, this is basically warm up and take off; this is climb, this is cruise and this is landing.

We also know whenever you designing an aircraft from safety and standardization point of view; regulatory authority prescribes different conditions. For example, if you see your civil; if we have; now it is easy. There is a lot of standardization is going on to the ensure their globally we talk same thing, but I am using whole specification for a far 23 certified. The aircraft under 12500 pound take off; gross weight the V stall is prescribed as less than equal to 61 knots which is roughly 30 meter per second.

You could see that we are talking about V stall and the takeoff and landing and either 10-20 percent more than V stall speeds which you know already, but when I try to connect V stall and W by S; I know this is $2W/S$ by ρC_L or C_L max.

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When I talk about V stall, once you have chosen an airplane particular wing tail and fuselage combination you have a $C L_{max}$ fixed, we say this is $C L_{max}$ and let us say this equal to 1.2. You can increase $C L_{max}$ locally by using flaps which are nothing, but highly devices and you can take this $C L_{max}$ from 1.2 to around 5; what is the implication of that if you increase $C L_{max}$; a V stall is going to reduce and that will not only help you in reducing this take off distance; it will also reduce the amount of thrust required because we have to accelerate from here to that V take off speed.

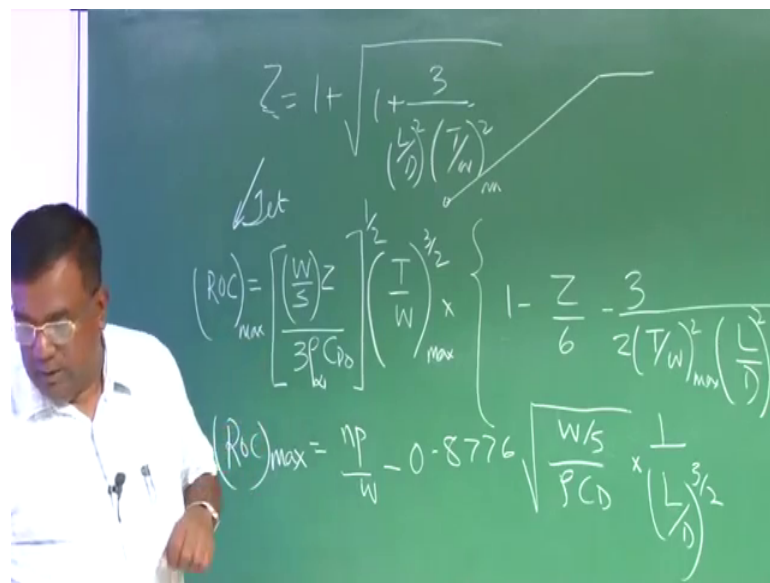
So, that demands for a given altitude take off that demands a particular W by S . So, $1 W$ by S selection should be once you prescribe $V S$ equal to $V S_{star}$; then you know W by S required for that is half rho V square into $C L_{max}$ and that V could be 1.1 to 1.2 V_{star} some where you will find it may be 1.15 or 1.3 during all those numbers are as per the regular to the prescriptions. The important thing is once I know at what altitude I am going to take off because you can take off from sea level also from a high altitude.

Then you already known V is nothing, but 1.1 to 1.2 times V stall and what $C L_{max}$; we are going to fly; that will decide what sort of a wing loading you require. For example, a V stall limitation; where you say V stall cannot be more than 30 meter per seconds then roughly you know V take off will be 30 into let us say 1.2. So, that you put here density of air put here put the $C L_{max}$ value; if it is just as plane wing may be around 1.1 to 1.2; if you are using different types of flaps; that will increase the value of $C L_{max}$. So, you

know; what is the W by S required for maintaining the take off conditions. This is one way you are getting an idea of wing loading required for a particular prescribed take off.

But one advantage you have you see here if you want to manipulate V S in turn you want to manipulate we take off we always play around with C_L max value by using flaps by giving different-different deflections on the flaps when you come to climb and let us say somebody gives you an expression rate of climb is equal to or rate of climb maximum is equal to W by S into Z divided by $3 \rho \infty C_D$ naught.

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Let me write this T by W max square into L by D square Z and rate of climb maximum this is for jet for a popular it is NP by W minus 0.8776 under root W by S ρ C_D naught by L by D 3 by 2 and of course, here the Z equal to 1 plus under root 1 plus 3 L by D square into T by W max square something like this. If I were a designer you see this expression; I am definite a designer; will not get really motivated when he is sticking in terms of a conceptual design, right and whatever interaction, I am having with you; I am trying to see that for developing basic understanding, we should avoid this expression at a conceptual stage and what will be our approach let me discuss that

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$$(R/C) = \left(\frac{TV - DV}{W} \right)$$

$$R/C = \left(\frac{T}{W} \right) \cdot V - \frac{q_{\infty} C_D V}{W/S}$$

$$= V \left\{ \frac{T}{W} - \frac{1}{2} \frac{\rho V^2 S C_D}{W/S} \right\}$$

$$V \approx \sqrt{\frac{2W/S}{\rho C_L}}$$

Let us say, we were talking at rate of climb; let me talk about rate of climb; what is the rate of climb? We understand from basics fundamental a $T V$ minus $D V$ by W and we know very well is the excess power divide by the weight and you are assuming climb at a constant speed right at a small angle. So, now, if I see this; so, we write this as T by W into V . We understand, we are trying to see; how wing loading is going to affect rate of climb; that is our basic understanding minus $q_{\infty} C_D V$ by W by S , right. Drag is half $\rho V^2 S C_D$ that is $q_{\infty} C_D V$ and this again further write as V into T by W minus half $\rho V^2 S C_D$ by W by S where V equal to roughly V equal to roughly $2 W$ by $S \rho C_L$ as we understand when I am climbing then this is the lift. Lift is going to balance only the $W \cos \gamma$ component.

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The image shows handwritten mathematical derivations for the rate of climb (R/C) on a green background. The equations are as follows:

$$R/C = \sqrt{\frac{2W/S}{\rho C_L}} \left\{ \frac{T}{W} - \frac{1}{C_L} \frac{C_D}{C_L} \right\}$$

$$(R/C) = \left(\frac{TV - DV}{W} \right)$$

$$\sqrt{\frac{2W/S}{\rho C_L}} \left\{ \frac{T}{W} - \frac{1}{2} \frac{2W/S}{\rho C_L} \frac{C_D}{W/S} \right\} R/C = \left(\frac{T}{W} \right) \cdot V - \frac{\rho C_D V}{W/S}$$

$$R/C = \sqrt{\frac{2W/S}{\rho C_L}} \left\{ \frac{T}{W} - \frac{1}{C_L} \frac{C_D}{C_L} \right\} = V \left\{ \frac{T}{W} - \frac{1}{2} \frac{\rho V^2 S C_D}{W/S} \right\}$$

So, there will be cos gamma here, but we assuming small climb if that is true then what do I get then I get rate of climb see rate of climb equal to under root 2 W by S by rho C L T by W minus 1 by C L by C D. So, rate of climb, I can write as V into t by W minus half rho V square S C D by W by S and now I can write this as under root 2 W by S rho C L into T by W minus half rho per V square. I write 2 W by S by rho C L into S into C D by W by S, right. Half rho C square for V square 2 W by rho C L then S C D by W by S and this gives me 2 W by S rho C L into T by W minus; rho-rho gets canceled, 2 gets canceled, W by S gets canceled and S should not be here, but W by S is here that we only wrote S. So, this will be minus one by C L by C D; please check these expressions, right.

So, for a designer, he will be very happy with this sort of an expression; if at all he wants to derive an expression, but these are all based on very simple understanding then from this what message he gets instead of rate of climb will increase if you are flying at a W by S higher which is true. W by S higher means wing area is relatively less. So, drag will be less. So, for a given thrust, you will have a more speed. So, rate of climb should increase because the rate of climb is nothing, but V sin gamma. So, everything even if same; if the wing area reduces then the speed increase will be more. So, W by S increasing means wing area reducing drag is reducing. So, rate of climb will increase then also here you see that T by W is increased rate of climb will increase then also when have larger C L by C D this value will diminish. So, again rate of climb will increase.

So, this gives a clear cut understanding to the designer what is the combined effect of t by W and W by S when I am talking about rate of climb. So, it does not go into. So, many big-big equations from there he tries to evolve the conceptual design.

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$$\text{Prop-a/c. } R_{\max}; C_L = \sqrt{\frac{C_{D0}}{K}}; \left(\frac{C_L}{S}\right)_{\max}$$

$$L = W$$

$$\frac{1}{2} \rho V^2 S C_L = W$$

$$\frac{1}{2} \rho V^2 S \sqrt{\frac{C_{D0}}{K}} = W$$

$$\frac{W}{S} = \frac{1}{2} \rho V^2 \sqrt{\pi AR e C_{D0}} \quad K = \frac{1}{\pi AR e}$$

$$\boxed{\frac{W}{S} = q_{\infty} \alpha \sqrt{\pi AR e C_{D0}}}$$

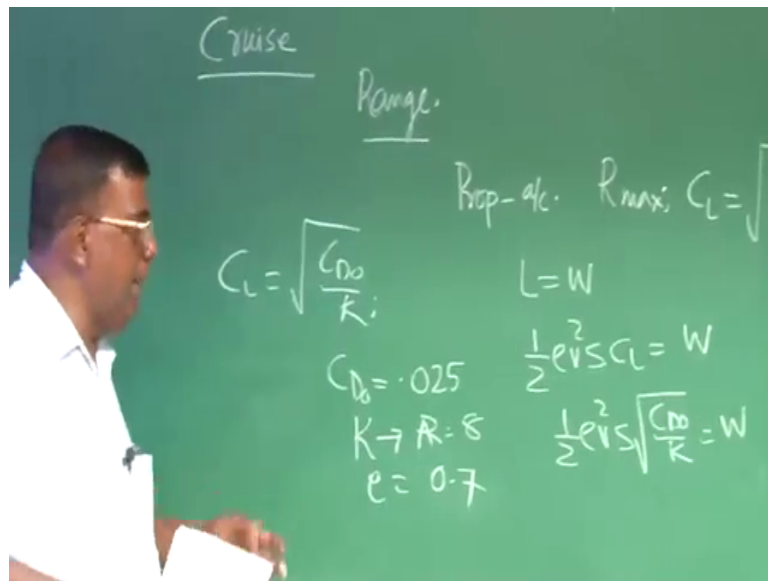
So, from rate of climb now we will try to understand the cruise. This is indirectly you are also revising before we try to utilize this for our aircraft configuration selection at a conceptual stage.

We remember for cruise primarily we talk about range because if you see the mission requirement will not; would be for 20 minutes, 30 minutes; the maximum climb rate is the range that it reminds the mission requirements. Now looking for wing loading to meet range requirement and if I take propeller given aircraft and we know that for range maximum condition was C_L should be equal to C_D naught by H which essentially meant C_L by C_D with maximum what does it mean in terms of wing loading that is all important.

When I am talking about range maximum, I am talking about cruise; that means, lift is equal to weight. So, I write half rho V square $S C_L$ equal to weight and for C_L , I write half rho V square $S C_L$ is C_D naught by K equal to weight. So, W by S equal to half rho V square under root pi aspect ratio $E C_D$ naught because we know K equal to 1 by pi aspect ratio E we talking about low subsonic speeds and these are the estimates I am trying to find for my conceptual design, right.

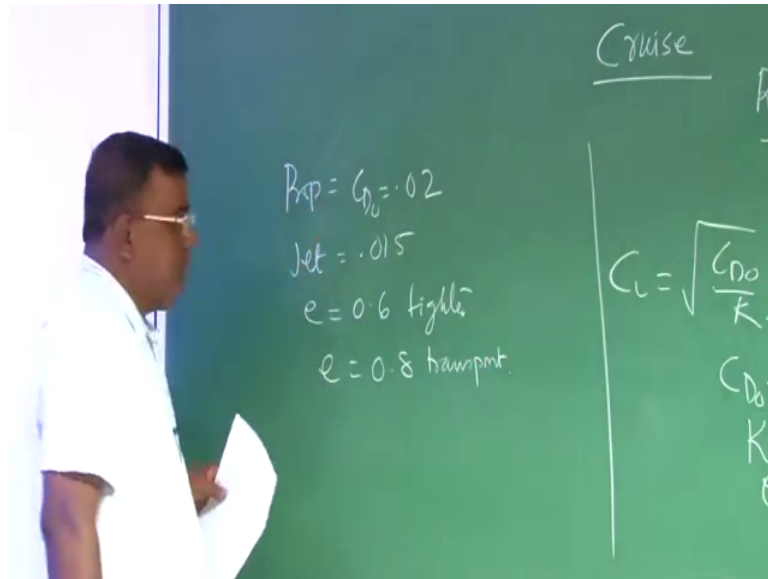
So, that tells me W by S equal to q root of π aspect ratio $E C D$ naught. Now at a conceptual design stage; how do I know this numbers now aircraft is already, but you could see that q infinite means the dynamic pressure? So, naturally when we are conceptualizing an aircraft you try to know what altitude I should be flying or cruising mostly if it is a turbo prop or jet engine it is around 30000 feet; the turbo prop where the engines are most efficient and also you have an idea about what should have cruise speed you want to fly

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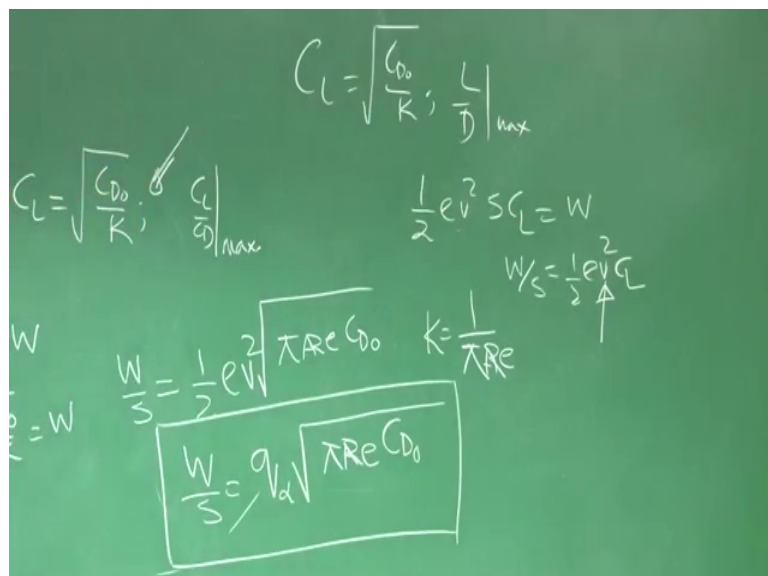
Because if you see that once you are talking about range maximum; your C_L is fixed C_L equal to under root $C D$ naught by K . So, once I am cruising at a particular altitude, I have also a fix speed with which I should because C_L is fixed and typical value of $C D$ naught at a conceptual stage; you can take around 0.0 to 5 and K . The aspect ratio; we can take around 8 and E is better you take less point seven this is what my numbers.

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But especially from recommendations you will find for a propeller driven aircraft. They are telling C D naught you take as 0.02 and for jet it is 0.015 E; 0.6 for fighter and E 0.8 for transport. So, that gives the fair idea of what sort of the range we were talking about, but you will be surprised you will find that if you want to fly at this C L for which your range will be maximized this may not satisfy your requirement for the speed in a sense; it may happen that speed is too low.

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So, then you do not have a option, but you have to select the speed at which you are flying and try to get a W by S value, but you should be very careful that this expression of W by S as assumed the C L equal to C D naught by K right for example, if you are not able to fly at C L equal to C D naught by k; that means, you are not flying at L by D maximum. Then you have no other option, but to get W by S from this half rho square S C L equal to W. So, W by S will be half rho V square C L and if we have chosen a speed at a given altitude which is a requirement and then you have to find out what sort of C L you are going to fly since we do not have a idea of W by S we see historic data same class of airplane you find C L may be 0.2 or 0.3. So, accordingly pick that number, right.

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LOITER

$$\text{Jet } \frac{W}{S} = q \sqrt{\frac{A R E C_{D0}}{\rho}} ; C_L = \sqrt{\frac{C_{D0}}{K} \rho V^2 E_{\max}}$$

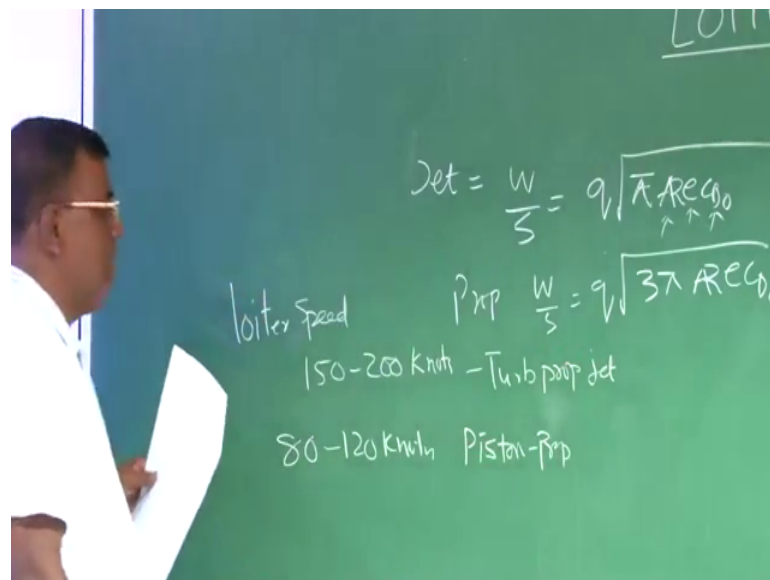
$$\text{Prop } \frac{W}{S} = q \sqrt{\frac{3 A R E C_{D0}}{\rho}} ; C_L = \sqrt{\frac{3 C_{D0}}{K} \rho V^2 E_{\max}}$$

But basically when you are talking about range maximum this is the way you pick all those things will come later just realize that then we talk about W by S loiter. We will talk about later. I must mention at this point that mostly we design transport airplane for more for range loiter; loiter in 20-30 minutes, but still we need to have an idea about loiter specifications and see what is the wing loading require to maintain that loiter now how do I do that you know for jet W by S you can easily see it will be q pi aspect ratio E C D naught and for propeller W by S will be q 3 pi aspect ratio V C D naught. How does it happen because for a jet airplane if you are going for endurance maximum that time the C L requirement is C L equal to C D naught by K this is for jet E max endurance max you can revise your lecture and this is for a propeller C L equal is under root 3 C D

naught by K this is for propeller V max if this C L you know how to find out W by S by equating lift equal to H.

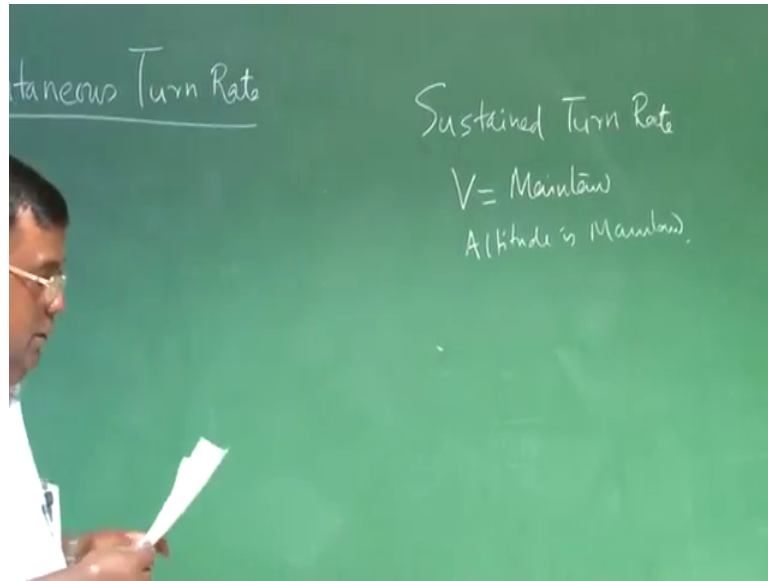
So, you are now having expressions for W by S again story same this is pi what is aspect ratio what is typical value of u t a victim value of C D naught initially I have to take and get the feed for W by S generally it is best to assume loiter speed around 150 to 200 knots. 1 knot is roughly 0.5 meter per second and for and 80 to 120 knots for piston prop and this is for turbo prop jet.

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These are guidelines say typically larger speed now for some time, we will be talking about turn rates that one turning is possible it turns like this and maintains the altitude and speed right another is it turns, but it loses the altitude.

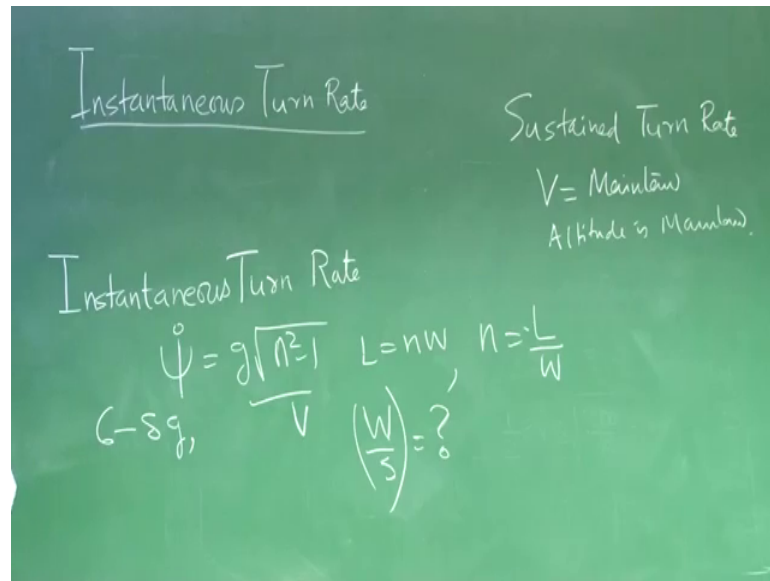
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So, that is what is we talking about is instantaneous turn rates and it not take condition it should maintain the speed or altitude and you can understand in contrast instantaneous turn rate is sustained turn rate where the condition is that I am doing a turn same time the speed should be maintained V maintained as well as altitude is also maintained.

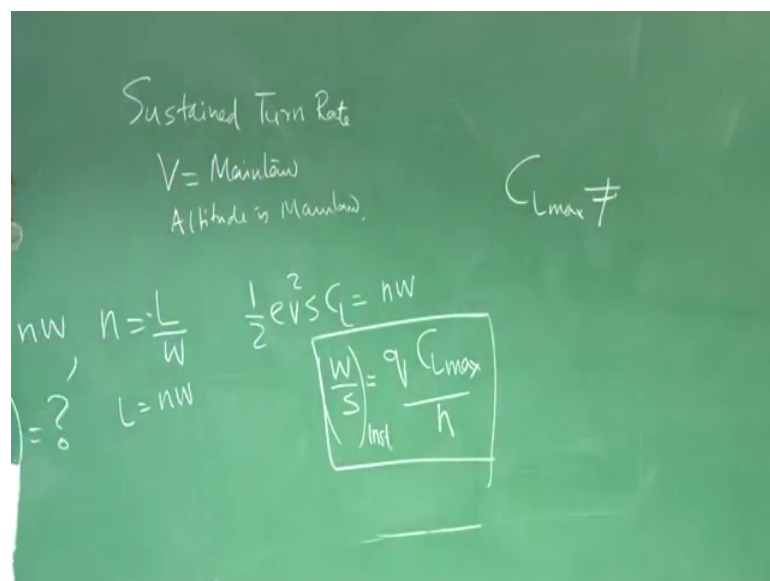
So, you have to really play with the thrust; obviously, you understand this is more demanding. So, if you want to go for maximum turn rate, you prefer instantaneous turn rate or indirectly, I can say; you can get higher turn rate. If you are going for instantaneous turn rate because for sustained turn rate some part of energy will be utilized in maintaining V and the altitude right. So, we will just talk about first instantaneous turn rate where you know this expression under root $n^2 - 1$ by V and where lift equal to $n w$ or the load factor n equal to lift by weight ratio where n equal to ratio, but lift and the weight.

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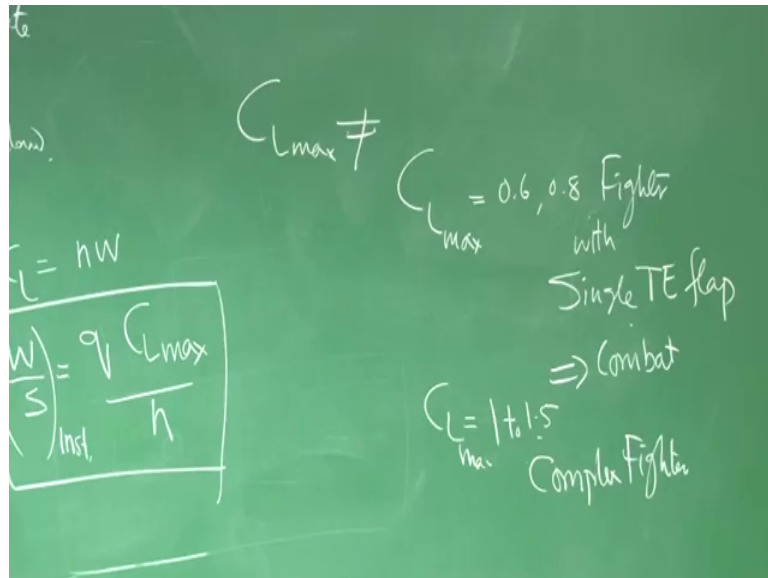
Typically sustained turn rates are lower than instantaneous turn rate for reason much explained earlier for instantaneous turn rate value I could see anything 6 to 8 g accelerations, but if I want to really find out, what is the wing loading required for instantaneous turn rate; how do I find out simplest way to do that is if you know; what is the n required from this expression that is at what speed at what turn rate I want to move that will correspond to particular value of n right and then you know lift equal to n w put that value of n and half rho V square S C L equal to n w.

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So, C_L ; I know or W by S I know as $q C_L$ max by n is this clear what is the turn rate you require at what speed I require it gives me what is the value of n required for these value of n , I come here and find out what is the W by S required for maintaining instantaneous turn rate at a particular dynamic pressure.

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This is given by this, but please be careful this C_L max is not as high as C_L max when you talking about takeoff or landing right, but this value typically will be C_L max for all practical purpose. It will fight 0.6 to 0.8 for fighter with single trailing edge flap during combat. This is important during combat; you never would like to use all these things during normal flight plus lot of penalty increase.

Similarly, C_L max could be 1 to 1.5 for complex fighter airplane complex fighter aircraft, but when I write this please understand where at high speed may be on 300 meter per second. If we are doing a combat then there is lot of drag penalty and lot of excess thrust you require, but if an airplane is meant for that then you have to have this much of wing loading. So, we are now seeing that what sort of wing loading is required for different machine requirement you have talked about instantaneous turn rate we have also mentioned about sustained turn rate.

In next class, we will again revisit instantaneous turn rate and sustained turn rate. Now we see them together and see why sustained turn rate is important why instantaneous turn rate is important right.

Thank you very much.