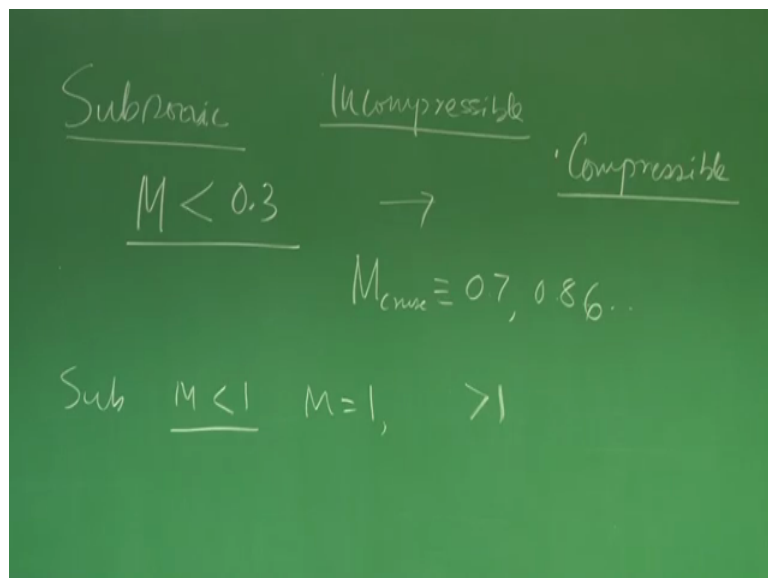


Aircraft Design
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Lecture - 14
Wing Design: Aerofoil

Good morning friends, we are again back as you have been noticing that we are continuing our understanding about aerofoil. I am trying to add some features which are directly designers feature.

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So, far when I was discussing our attention was focused to subsonic flow, aircraft for subsonic regime. Subsonic means we categorize that Mach number less than 0.3 to be more precised what we were discussing was primarily incompressible flow right, but still lower than 0.3, fairly you are assuming that density at point remain constant there in change in density, the medium was interacting with the body.

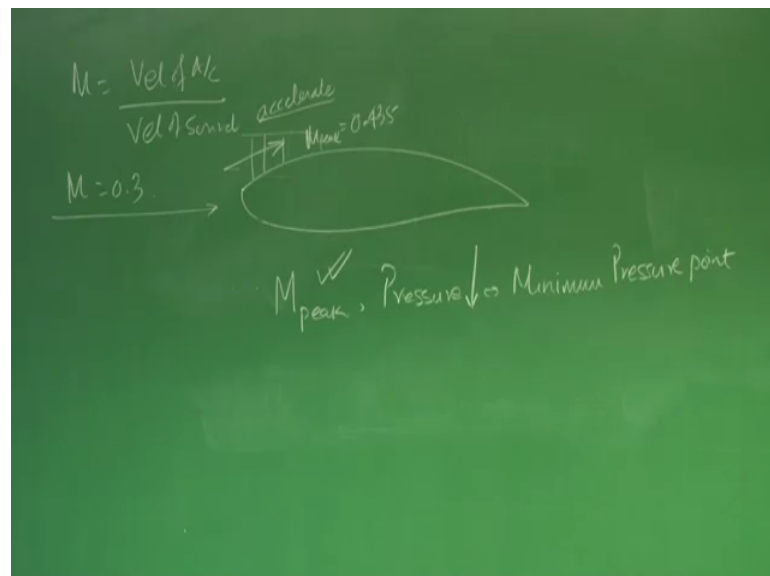
But as you know we always aspired to increase the speed, the problem with or the challenges when we increase speed is so far what assumption we had incompressible, may not be true as all of us understand air is actually compressible, but for all practical purpose we have categorized as incompressible flow. The moment you try to cross Mach 0.3 roughly we need to be bothered about compressible features which I need to know when I design or select an aerofoil. For example, grouse Mach number could be order of

0.7, 0.8 like that I will love to go for 0.86 who stops me going further or who cautions me if you are going further you have to do something extra.

We all know that as I increase Mach number, there is a formation of shockwave that results in drag, that results in flow of separation. So, immediately you have to bother about the engine power, bother about the structural weight, you have to bother about vibration associated with shockwaves, flows operations therefore, for in subsonic airplane or high subsonic airplane what we look for to avoid formation of shock or delay that that should be in our mind. When I am writing here subsonic airplane typically by definitions subsonic you know it should be Mach number less than one at Mach one on and around one we call it transonic and greater than one we call supersonic. But you have seen here subsonic and then subsequently I would a Mach less than 0.3 what I am trying to tell in in a recilistic manner that we are talking about an airplane design, but flow is incompressible, but as I go for high subsonic that flow incompressible assumption will not work.

So, what this is going to mean for selection of aerofoil what I should know to select appropriate aerofoil that I will be discussing in short now.

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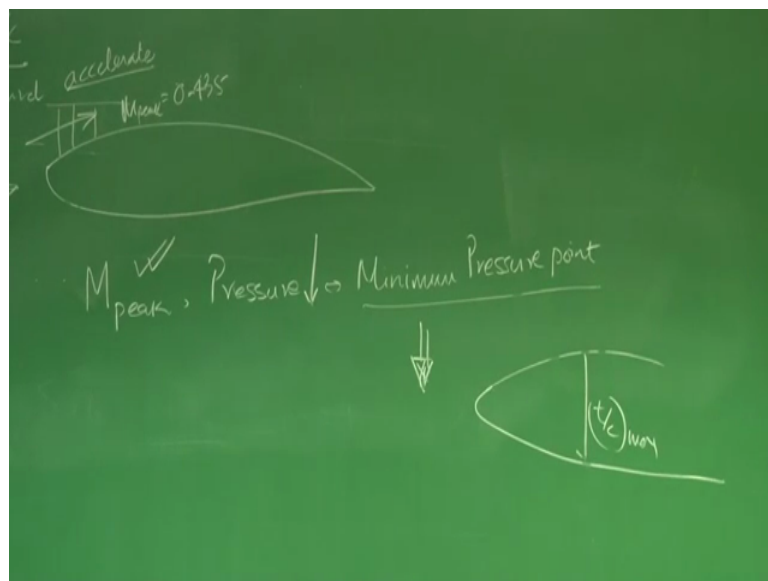


Let us take an example you will find today's lecture there will be material which I have taken from Anderson's book of airplane performance. So, you can go through airplane performance on this topic, and you must read I have been always telling it is a design

course you need to read you to see the data you have to Google for data, Google for definitions then only you can really get a feel for design.

So, let us take case here and let us say Mach number is 0.3, you all know Mach number is within, but velocity of aircraft way shift velocity of sound at that altitude right. It is 0.3 and now you know because of this contour the flow will accelerate and at some point the flow will accelerate to maximum speed let us say that is M_{peak} . There say this is 0.435 question is first of all as the speed is increasing from 0.3 to 0.435, the pressure at M_{peak} will pressure will the static pressure will go down, and wherever I get maximum speed the corresponding pressure in the minimum pressure, pressure point this directly comes from Bernoulli's because throughout the flow the total pressure stagnation pressure remain constant. So, it gets converted into kinetic energy or the potential energy that is one of a conversion goes on.

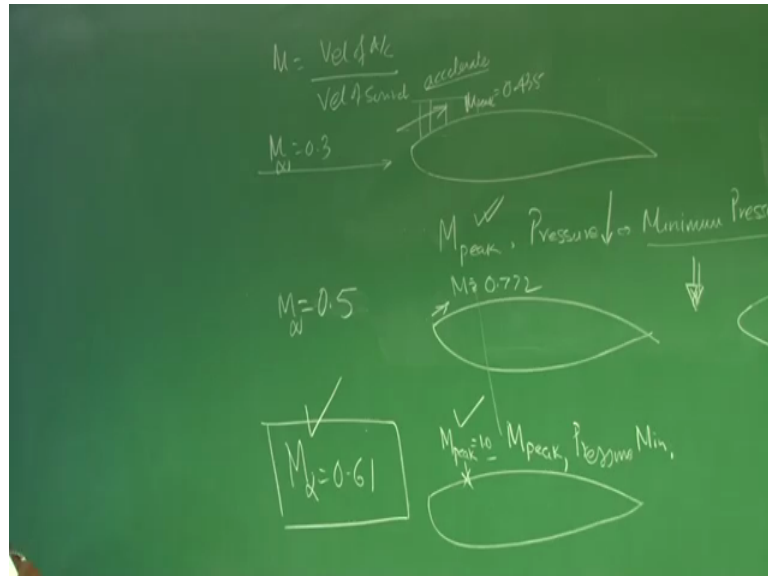
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But here please understand this minimum pressure point if you intuitively think that it should happen where t/c is maximum then that will be wrong the intuition fails here because the minimum pressure point will not just depend upon local t/c , it will depend upon the whole flow conditions. So, keep this back of your mind yes velocity maximum or M_{peak} will correspond to minimum pressure point; however, it is not that it will always opera at where t/c is maximum, but generally that is what the mistake being done and that is why you have seen when we define NACA aerofoil series, there is a

normal creature driven guidance that yes do tell the minimum pressure point is at 0.4 times chord which is not necessarily that u by c maximum location right.

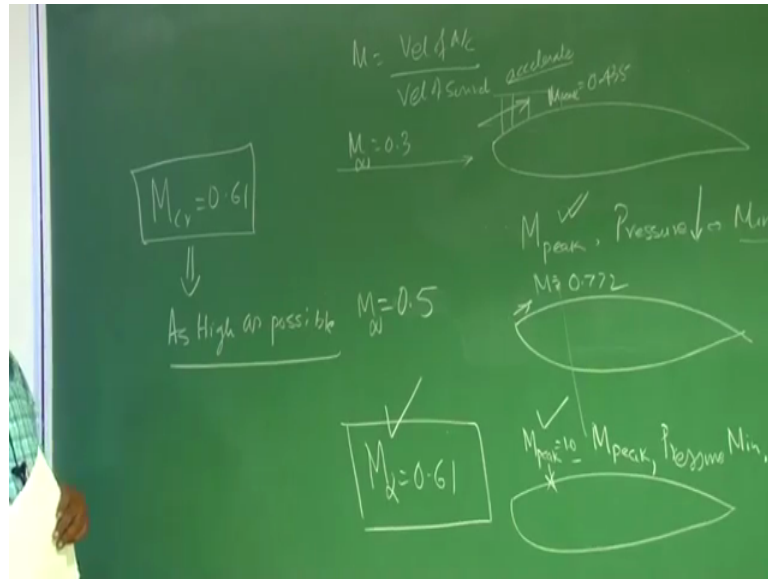
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Now, similar a same airfoil and let us say I increase mark to 0.772, when I am saying M_∞ I mean M_∞ and a free stream Mach number 0.772. So, on the same aerofoil at a minimum pressure point the flow will accelerate at minimum pressure point let us say this becomes 0.772 let me correct this let us say add from 0.3 I am going to 0.5, I repeat from 0.3 I increase the free stream Mach number 0.5 and let us say M_{peak} ; that means, the pressure minimum the speed is Mach equal to 0.772.

Now, I do further I increase M_∞ 0.61 and same aerofoil, find at minimum pressure point M_{peak} is 1.0. So, the critical Mach number is defined like this, what is that free stream Mach number for which for the first time any point in the aerofoil has reached Mach one. So, if you see if I keep this three data I know it is Mach M_∞ 0.61 add this Mach number for the first time M_{peak} has become 1.0.

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So, my critical Mach number is 0.61 right. What is so important about critical Mach number? You understand the moment we talk about local Mach number going to one. So, there will be change in the flow structure we know the shock wave interaction will start coming up the moment there is a shock wave generation or some sort of interaction of shock wave involved in air there will be dissipation of energy and it may lead to flow separation as well. So, there is a rise in the drag.

So, if I want to really design a high speed or high subsonic airplane, I should ensure that M_{cr} should be as high as possible for high subsonic airplane. So, that is the importance of M_{cr} and when you selector aerofoil suppose you want to design an airplane or design a wing for a given airplane where you want the speed to be around 0.6 or 0.7, then this question this criteria this requirement become predominant in selecting the aerofoil. So, I should look for an aerofoil which has larger M_{cr} ; second stage I should look for a wing which has larger M_{cr} ; when I am saying wing to aerofoil or aerofoil to wing I am telling that not only airfoil shape, but something you can do with the wing to increase the M_{cr} of critical Mach number.; with this understanding we will now see what explicitly it means.

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First question will come how do I determine M critical? Let us have a closer look on M critical, I will be writing few equations these are important if you want to develop a feel for numbers some understanding of that should help you. Whenever I am talking about M critical we use a point that the speed accelerates to a peak speed a peak Mach number where the pressure is minimum.

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So, you talking about pressure in a non-dimensional form we talk about pressure positions which is P minus P infinity by half rho v square. P is the static pressure and P

infinity than ambient pressure that gives you a differential pressure if C_p is negative means you have a (Refer Time: 12:30) we all understand that right.

The whole game is on critical Mach number and our whole game is on minimum pressure point, because that is where the velocity will be maximum that is where the Mach number will maximum to will check at minimum pressure point whether mark has become one or not, that is right we are using this C_p . This I can further simplify to write P_∞ by q_∞ to P_∞ by P_∞ minus 1 and of course, you know what is q_∞ is dynamic pressure of V square.

Now, I write q_∞ is equal to half, $\rho_\infty V_\infty^2$ which is half, ρ_∞ by γP_∞ you see why this has been written and multiplied by γP_∞ into V_∞^2 . What I have done half $\rho_\infty V_\infty^2$ which dynamic pressure, I have multiplied by γP_∞ and divided by γP_∞ and we know that we know that a is equal to the speed of sound the speed of sound, and we express as $\gamma P_\infty / \rho_\infty$ under root and if I do this with a further clarification a_∞ free stream will be V_∞ / ρ_∞ . So, I can further write q_∞ as $\gamma / 2 P_\infty M_\infty^2$ right you see from here, this a_∞ is $\gamma P_\infty / \rho_\infty$. So, this becomes $1 / \gamma$ by a_∞ comes here. So, that becomes M_∞^2 and γ is here P_∞ here remains that 2 is here fantastic good.

We also know from isentropic relationship, where P_0 is the total pressure P is the static pressure and how conversion goes on with Mach number is given by this isentropic relationship. In this relation you know M is Mach number, γ is ratio of specific heats and values typically 1.4, these things all you know and is a isentropic relationship the assumption is here the total pressure remain constant in a subsonic domain right total means P_0 , as a like a tank that pressure is there for the conversion will happen depending upon become some kinetic energy gets converted to this pressure energy, but the total P_0 remain constant and this relationship is given by this relationship which is based on isentropic assumptions.

Then you can write P / P_0 , but P_0 / P_∞ it will be $1 + \gamma / 2 M_\infty^2$ I am using the relationship on the P I am replacing by P_∞ .

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The image shows handwritten equations on a green chalkboard. The top equation is:

$$C_p = \frac{2}{\gamma M_\infty^2} \left\{ \frac{1 + \frac{1}{2}(\gamma-1)M_\infty^2}{1 + \frac{1}{2}(\gamma-1)M^2} \right\}^{\frac{\gamma}{\gamma-1}} - 1$$

The middle equation is:

$$C_{p,c} = \frac{2}{\gamma M_\infty^2} \left\{ \frac{2 + (\gamma-1)M_\infty^2}{\gamma+1} \right\}^{\frac{\gamma}{\gamma-1}} - 1$$

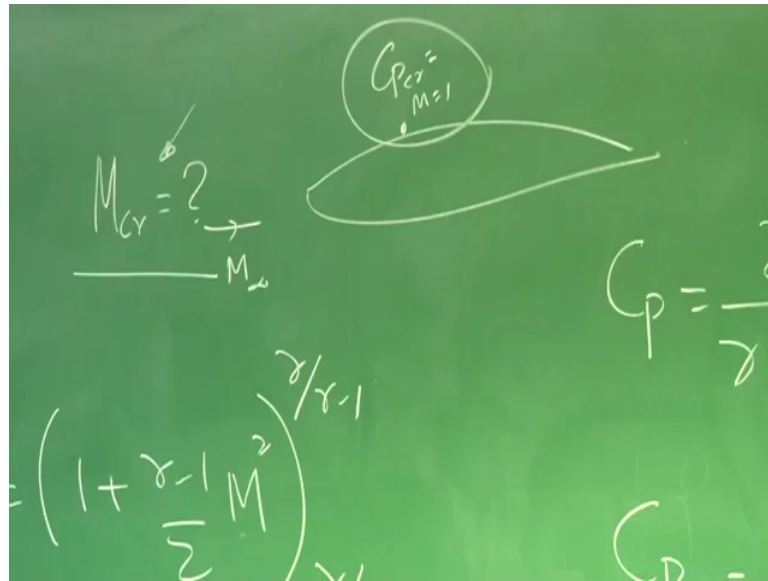
Below these, it is noted that:

$$C_{p,c} = f(M_\infty)$$

Where the ambient condition is pristine conditions, using these 2 relationships we will get C_p is 0 on to 2 by gamma M_∞ square, 1 plus gamma minus 1 M_∞ square by 1 plus half gamma minus 1 M_∞ square to the for gamma by gamma minus 1 minus 1. Do not get carried away by all this expression these are you have to see once for all to understand what is the C_p critical or is M critical how do you as a designer, how do you extract information out of it.

If this is the pressure coefficient then C_p critical if I have to ask you how do find out what was C_p critical? C_p critical is that pressure coefficient which corresponds to M critical and what is M critical? M critical was the free stream Mach number for which for the first time there is a Mach one on the aerofoil.

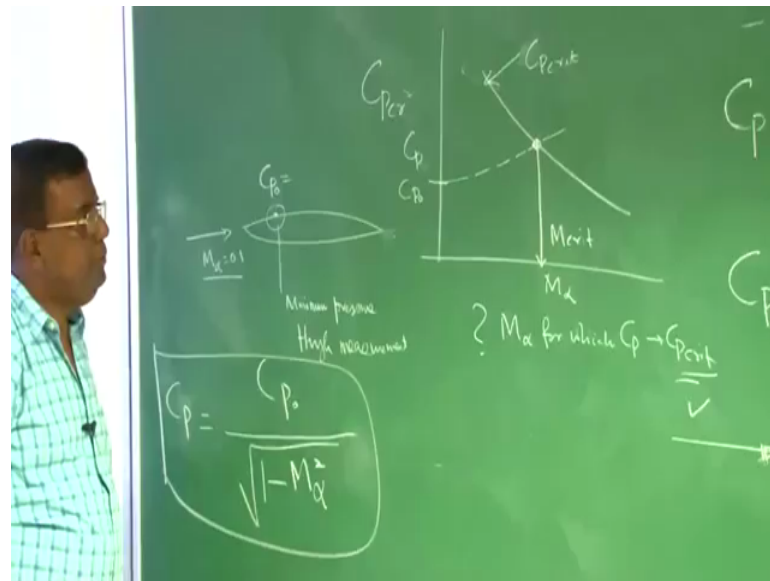
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So, wherever Mach one has achieved that is. So, for first time Mach one has that sit here. So, what is that C P critical mean what is the value of C P here that we are talking about for a given M infinity; this should be clear.

So, if I want to find out C P critical from here; that means, I am assuming on the free stream is M infinity, but this M has become local mark has become one for the first time it is also an clear? If that is so, then just have to put one here and I get expression of C P critical as $2 \gamma M_\infty^2 + \gamma - 1$ by $\gamma + 1$ minus 1. This expression c can directly c in Anderson book interaction to flight. So, what we are seeing we are seeing C P critical is function of M infinity is very very important observation.

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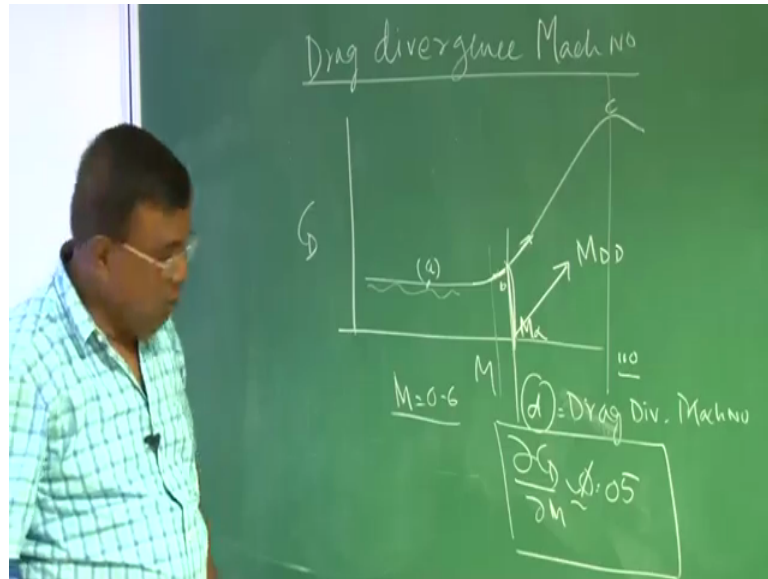


And you will find that we are realized what is C_p critical and you could see that C_p critical varies with M infinity and you have explained why.

Now, we want to find what is that Mach number, what is that M infinity for which C_p become C_p critical, so that I know what is M critical right. This is C_p take an aerofoil do some experiment and find out what is the value of C_p naught specially when I am at C_p naught we are talking about very low speed, unless that point is C_p naught here how do calculate C_p naught? To find for a given slow speed like a point one Mach number find the minimum pressure through measurement.

And then once I know C_p naught I can give a correction divided by 1 by infinity M infinity square to find out how this C_p is changing with Mach number, and if I do this this will follow this sort of variation and this is the point because this is the locus of M critical or this is the locus of C_p critical for different Mach number, but these points will give me M critical as simple as that. I will try to solve one example so that you know how to at exactly find it out. This part was necessary so that you take we all take M critical very seriously; if you are thinking of designing an airplane high speed high subsonic the M critical will play very important role along with another Mach number what we call drag divergence Mach number drag divergence Mach number.

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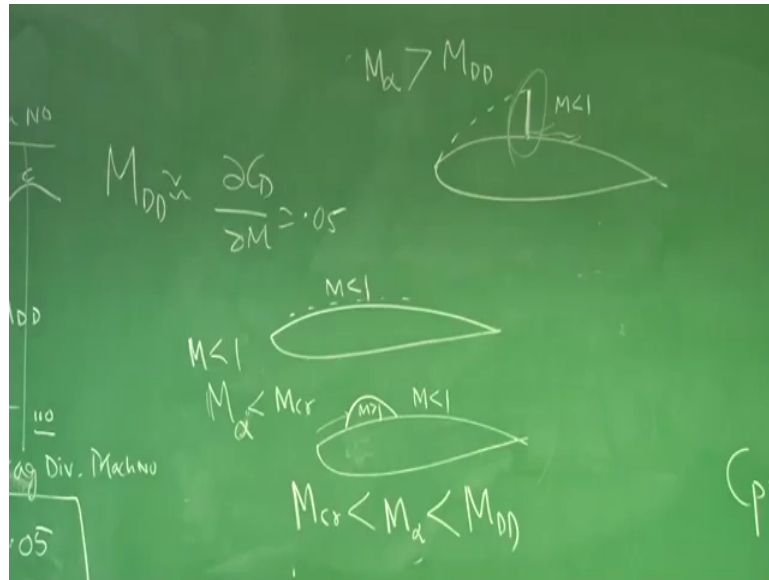
So, after M critical will be now revising our self what is drag divergence Mach number, I am sure you people have done this in your (Refer Time: 22:06) course, but it is important that we a proper revision so that we are ready to use this understanding in a curved design. Specially selecting aerofoil and wing plan for you are well aware the C_D versus Mach number is goes like this right up to point a there is point b and there is a point c the same. This up to point a on around what I am presenting here typically let us say around 0.6, all there are done make efficient they remain constant they do not show much dependence on Mach number.

But beyond that we find there is a dependence at Mach number, this number may be 0.7 something like that and around one we all know that there is a peak rise in the drag coefficient which we all understand because now shock formation is there and which is a having a distributive property. So, rise increases and drag increases, but this rise after certain point very sharp rises there to see like this. So, there is a point c or d I call, there is point d which is called drag divergent Mach number that is Mach numbers.

This is typically that Mach number from this plot you can get it where dC_D/dM is typically around 0.05, this is a some thought of a thumb rule ideally you should do a winter testing and you could find out where there is a sharp rise, but most people agree this is a good thumb rule wherever dC_D/dM is around 0.05, you call that Mach number corresponding Mach number is M_{DD} . Why there is important? If we cross

beyond M_{DD} the drag rise will be very very sharp. So, what is important for us we select an aerofoil select and we so that $M_{critical}$ should be large M_{DD} also should be large. So, that I can fly with more aerodynamic efficiency with drag being lower at high speeds there is a purpose right that is why we are going through this understanding.

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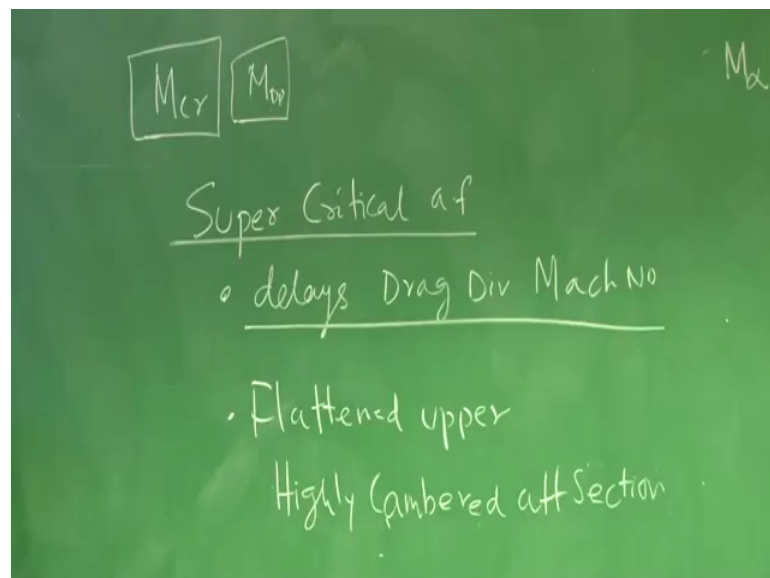
Third part which will come; so, let me write M_{DD} is that M where $d C_D$ by $d M$ locally around 0.05. You know in aerodynamics perception or the way he tries to visualize we will translate all of these through three diagrams, he say if this is the aerofoil and M is fairly less than 1 M locally $M < 1$, this is I write $M_{\infty} < M_{critical}$ because there are no shock wave generation at any point. that is at any point M is not reaching locally mark one so, always less than one. So, say M_{∞} is less than $M_{critical}$.

If second condition where $M_{critical}$ is more than M_{∞} , but less than M_{DD} drag divergence, then M locally where the velocity accelerates to a local condition where for the first time Mach number equal to one. So, there is a envelop where M is greater than 1 and after that M is less than 1 this is the condition if you are flying little 1 above critical Mach number. But if we are having third condition where M_{∞} is greater than M_{DD} , then we will have definitely we will have formation of a shockwave and this will result in lot of drag will flow separation this shock will interact with

boundary layer very flow separation, and which is not desirable for any efficient flight performance right.

So, these are the 2 different views from my designers perspective I define tell me how to select an airfoil where M_{DD} is large tell me why how to select and wing M_{DD} is large, tell me how to select an aerofoil for $M_{critical}$ is large, tell me how to select an aerofoil where M_{DD} is large designer looks for that, here he look for database look for some empirical relationship, but what to look for he only knows one you understand these things and a back end of his understanding right, that is why I am touching upon this. There is another aspects of handling critical Mach number or delaying rise in drag you know that is called sweep.

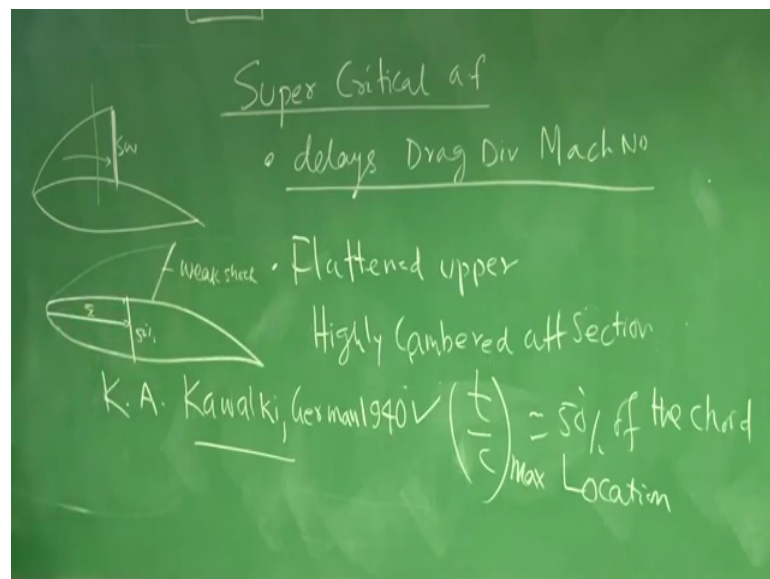
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Before you come to sweep just see what sort of the evolution happened just to handle $M_{critical}$ and $M_{drag divergence}$. Why there is the formation of shock wave here because we have designed the contour in such a way the flow accelerates and very fast it goes to the Mach number 1 depending upon free stream condition. So, that is decided by the contour here. If this contour is redesigned, I can delay the formation of Mach number which is here it could be somewhere here someone here right I can always delay that. So, that was the understanding were super critical aerofoil came into existence. Primarily this delays drag divergence Mach number very very important because he wanted to fly the high subsonic right most of airplane dream liner or all those they are inspired by this.

In Google you will find that of course, that customizer of fail, but is inspired by this understanding; what is that? Special about supercritical airfoil as far as construction is concerned because we understand that we do not want to accelerate the flow so far that it quickly it comes to Mach 1 I want to delay it. So, what is the best way we do it do not make it so contoured, so curved on the top make it flat and at the end we make it sharp. So, that was the philosophy, philosophy was flattened upper highly cambered aft section that is.

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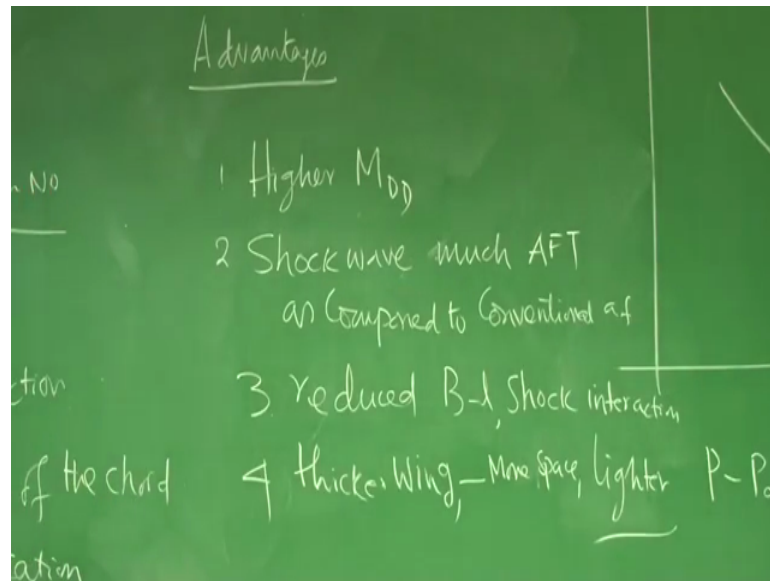


If I see normal aerofoil something like this and here it is to see here there is curvature first curvature. So, when you are flying over critical Mach number which may be very close to the drag divergence number there is the shock wave this point. In fact, it mean somewhere here as well.

But when you are flattened it then what is happening this will happen somewhere here and that to it will be a weak shock that is how we are taking that advantage of less drag and delayed formation of shock fly increasing drag, divergence Mach number. The person who should get credit number who gets credit if I am not wrong is Kawalki German 1940 on that please Google and check that I M not done in a mistake on spelling in all in 1940 this concept of supercritical airfoil where the location of t by c max was around 50 percent of the chord please understand it is a location of that is the t b c maximum is almost around 50 percent of the chord if I measure this distance right. If I

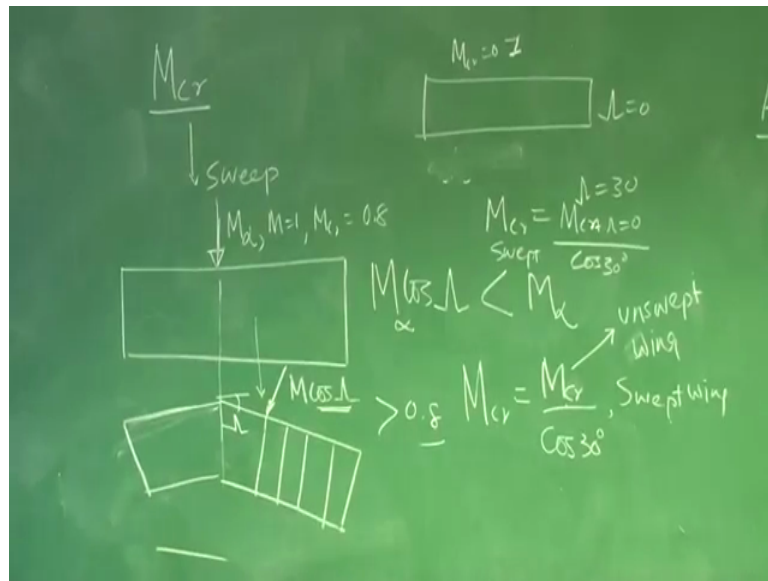
try to share t by c distribution of a supercritical aerofoil as compared to the conventional aerofoil you say t by c distribution for supercritical aerofoil will be 10 to 15 percent more than a conventional aerofoil. But here I am talking about the location of t by c max that is location is delayed so that this shock which was coming early it will come later and also it will become weak shock this is the philosophy of that.

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And there are design advantages number one goes higher guide diverges Mach number, number 2 is shockwave which is have seen shockwave much aft much aft as compared to conventional aerofoil. Third one reduce naturally shock wave is less lesser in strength so reduce boundary layer shock interaction. So, less a drag and fourth is thicker wing many designer very happy about it thicker wing. So, the more space and wing becomes lighter also it is important to understand thicker wing is easier to in force rather a thin wing.

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We have been talking about critical Mach number and drag divergence Mach number. If I again come back to critical Mach number I can increase critical Mach number on the popular way through sweep right you all know what is sweep this is the wing and flow is coming like this, if I now make it something like this. So, the aerofoils are like this now, this is important please understand the aerofoils are now stack like this. So, now, also if this is the Mach seen by the wing the normal component which primarily decide the pressure distribution lift and drag that becomes $M \cos \lambda$, λ is the sweep angle and since $M \cos \lambda$ is less than M infinity.

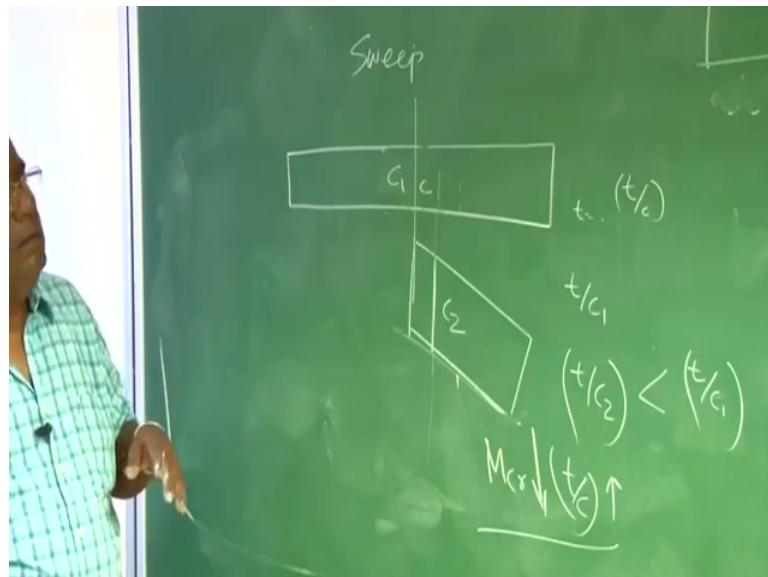
So, naturally you know that it will automatically increase the critical Mach number, right for example, if critical Mach number here the M infinity for which locally Mach equal to one happened that is M critical let us say that is 0.8, now with the sweep because it is now $\cos \lambda$ has come I can go more than 0.8. So, my critical Mach number will increased because of the sweep angle, but nothing comes free you understand seeing the normal component has reduce the lift also will reduce right. So, those details will see in stability and control part of design aspects.

There is a another way of also visualizing roughly we can say M critical will be whatever M infinity was there divided by coughts of 30 degrees for swept wing for a swept wing, this M this is M critical for a un swept wing this is a rough life is not. So, simple the moment you put a sieve the flow become three dimensional if you sweep is very large

there will be formation thickening of boundary layer at the tip that will cause stall. So, these are little bit of involved thing, but roughly as an initial stage if we know the M critical for a rectangular wing configuration if it is 0.7, and if we are giving a sweep angle of 30 degree sweep roughly we assume that M critical for the swept, for the swept wing that this is sweep is equal to 0. So, for sweep is equal to 30 degree this will be M critical sweep equal to 0 divided by cos of 30 degree that is all is very very rough, but it gives you an idea.

There is a another way of understanding what is the sweep doing I repeat when you give sweep there are many three dimensional flow issues we are neglecting those things we are just telling at the preliminary stage, how do I generate the initial conceptual number.

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There is another way you can see what effect of sweep let us say this is the wing rectangular wing, now this is the c and there will be a thickness right. So, it has particular t by c what happens if I do it like this now? Flow still coming like this now what is happening? C apparently becomes this because the flow is coming like this.

So, if this is C_1 and this is C_2 initially it was t by C_1 second case becomes t by C_2 . So, which one is less initially the wing was like this now I have tilted it like this. So, the c becomes this c has increased. So, C_2 is more than C_1 . So, these gentleman is less than un swept wing for same area and since t by c , but this swept wing is less it will also increase the critical Mach number because we know as t by c increases critical Mach

number also reduces right because more and more flattening more and more thicker wing it will actually very fast right. So, we know that M critical when decreases if t by c is increases.

So, here t by c has reduced. So, M critical will increase that is also one of the easier way of checking the cross effect for example, if I give a sweep we simply see how much t by c reduction is there see the chart for this must t by c reduction how much in critical increment will be there. So, that is how we were going to use the small small informations because please understand when I design an aircraft nothing is final right you are selecting a wing you do not know what is the sweep wept you selecting a tail you do not know what is the engine. So, these understandings are important to accelerate the process of synthesis, that is what is understanding extremely important and I strongly recommend whenever I refer some book and all or I say Google it do it then only you will enjoy this course right.

Thank you very much.