Aircraft Design
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Lecture – 10
Estimation of Fuel for a Mission

Good morning friends. We have been discussing about the issues or a type of alertness we need to have when you use brigade relationship to compute range, endurance for a jet airplane for propeller driven airplane. We have also talked about lift to drag ratio. These all have been done to get an initial fill for numbers, will have one session where will be talking about how to use historical data to get an estimate of initial estimate of L by D for the airplane which you are going to design. Please understand that. So, far last 2 lectures we are preparing our self to calculate or to estimate the initial amount of fuel required for a particular machine requirement.

So, I thought today I will take one example from book Raymer, you know aircraft design book by Raymer, which I am be following now for some time. And there is a one example given in that book and I am trying to share that example. So, that if you have that book which you must have if you are really interested in design and you can read the book and these 2 things will complement each other, but do not forget one thing whatever be given the book. And what whatever I am telling they have been validated through limited experimental data limited statistical data.

The same time the technology is are changing very fast. So, how good will be those data whatever available last 10 years or 10 years ago is a big question. So, best way to handle this is understand fundamentally what you are doing, and try to put your insight it. And that is the whole purpose of this level one aircraft design course. So, will now quickly take it the example, I repeat this is the example given in raymers book what is the machine requirement given there that crew weight is 800 pound.

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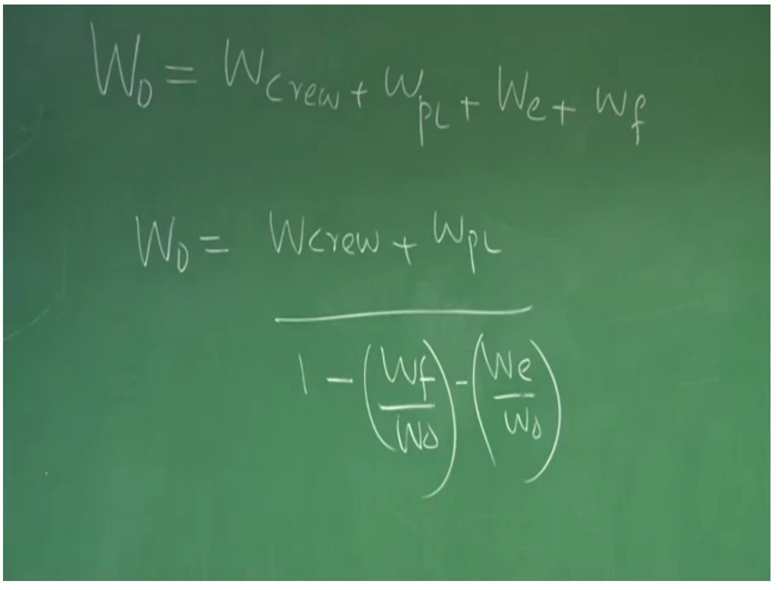

Remember we want to measure or estimate edit we want to estimate what is the initial gross weight and you say that is equal to W crew, plus W payload, plus W empty, plus W fuel right. We segregated like this right and then we have shown I can express W naught equal to W crew plus W payload by 1 minus W f by W naught minus W e by W naught, we have also discussed how to get W e that the empty weight fraction using that historical data, but last 2 lectures we are working towards a procedure. So, that I can find W f by W naught that is how much fuel is required to get an idea of that.

So, now we have done everything required for that, to get an estimate initial estimate of W f by W naught. We will try to see how far we have understood what is the method through an example.

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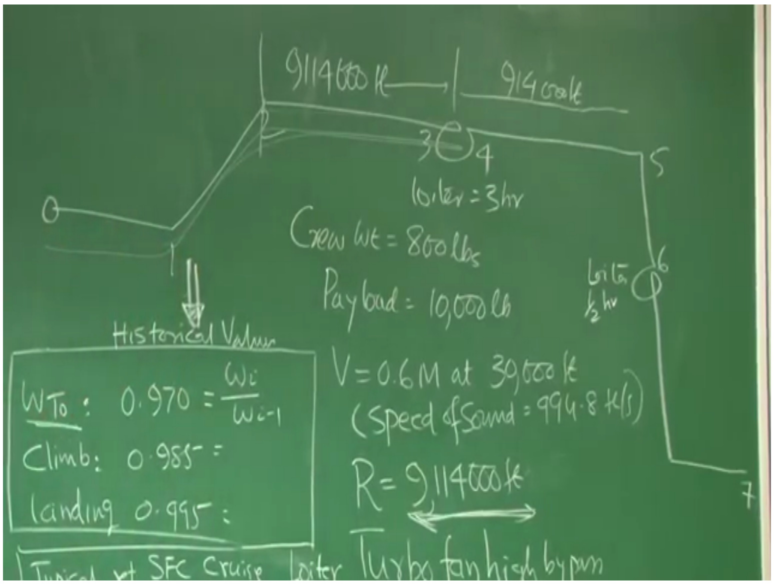

The example is crew weight is 800 pound payload, avernicus may be 10000 pound. We want to fly at Mach 0.6 at an altitude 30000 feet, will find when where you are using a jet driven engine, the optimal altitude is around 30000 feet tropopause. All those details will come as we grow just to see how to compute fuel consumption we are taking some number. And remember when I was talking about Mach number I need to know what is the speed of sound at 30000 feet, and that is 994.8 feet per second. And the aim was you take off from one point go to range of 9 1 1 4 triple 0 feet may be around 1500 nautical miles.

So, I goal take off climb go to that 911400 feet, but then I have taken come back. If that is my machine requirement which is like a transport airplane I have am designing or a fighter airplane am designing or a comet airplane am designing. One could be I take off go do the operation and again come back to the base if that is the situation then I have to add this range as in additional range right. So, the airplane has gone from here climb did it the operation now again it is coming back here. So, I am adding this range twice 91400 and am adding again 91400 that is understanding. It is not necessary that I have to do it every time I may just come from here and land here possible right. Some distance not on the same point, but this example is telling I am taking off climbing way for a range and I am again coming back to the base; that means, I have to cover that much range as well that is why although R is designed for this feet this examples, I am taking additional range this assuming that operation is happening like that.

It depends upon the user how do define range from the machine requirement point of view is just a number more importantly, how do I get what are the fuel consumption what is the fuel consumption.

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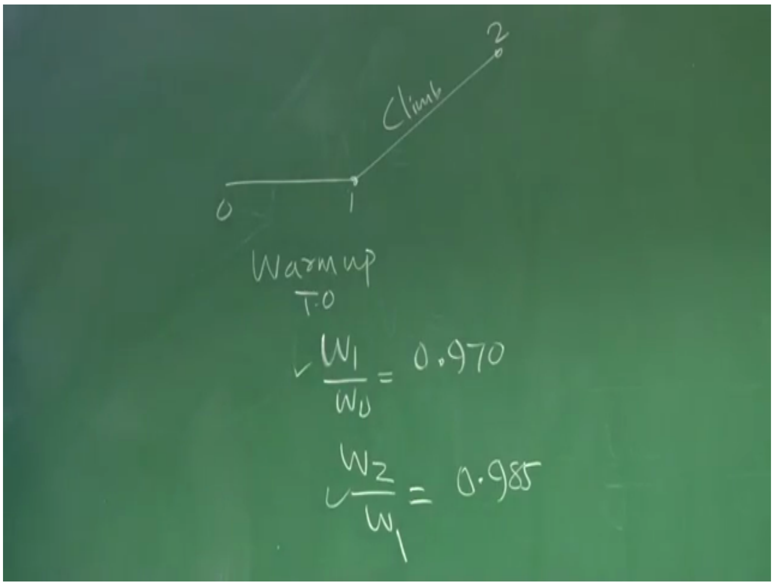

So, now you have 0 to 1, this is we call warm up and takeoff segment and if you see the historical data you have to look here.

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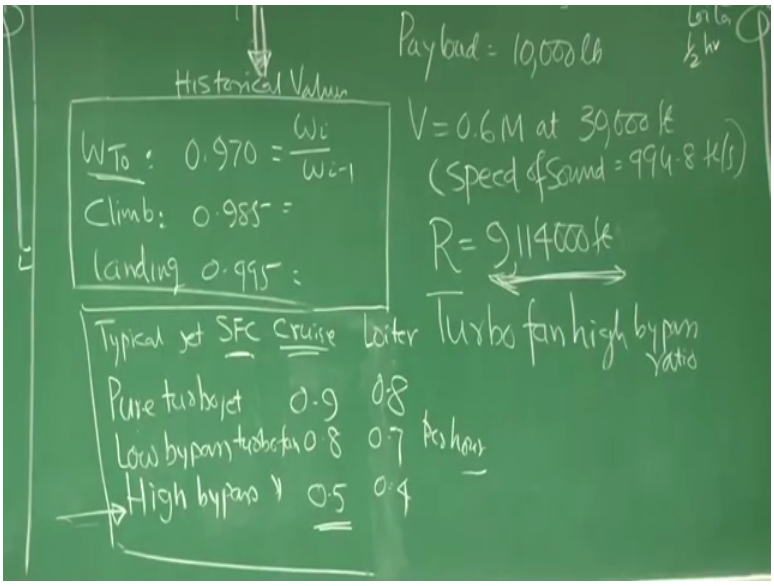

You have to look here historical values that based on historical data different type of airplane, but please understand, when I am designing a machine requirement of this class

I must select a base line airplane that is some airplane will available which will have capability closer to this. So, I pick that airplane.

To get the initial values historical value it has been recorded that generally W take off that is first 0 to 1, 0 to 1, W 1 by W naught this is typically 0.970, this is a fraction right. And every initial design stage we are taking this value. Then similarly from 1 to 2, which is climb this is W 2 by W 1 again based on historical value that is 0.985. These are initial estimates. So, once you freeze the plane I start evaluating in a refiner manner this numbers will get modified, but these are good enough initial estimates based on historical data. So, I know W 1 by W naught is this W 2 by W 1 is this.

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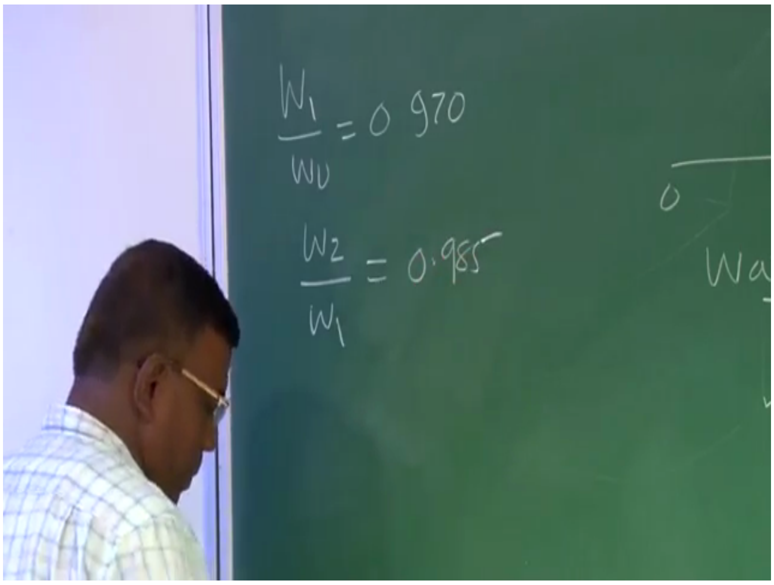

So, let me write it here, W 1 by W naught is 0.970, W 2 by W 1 is 0.985 and let me erase this.

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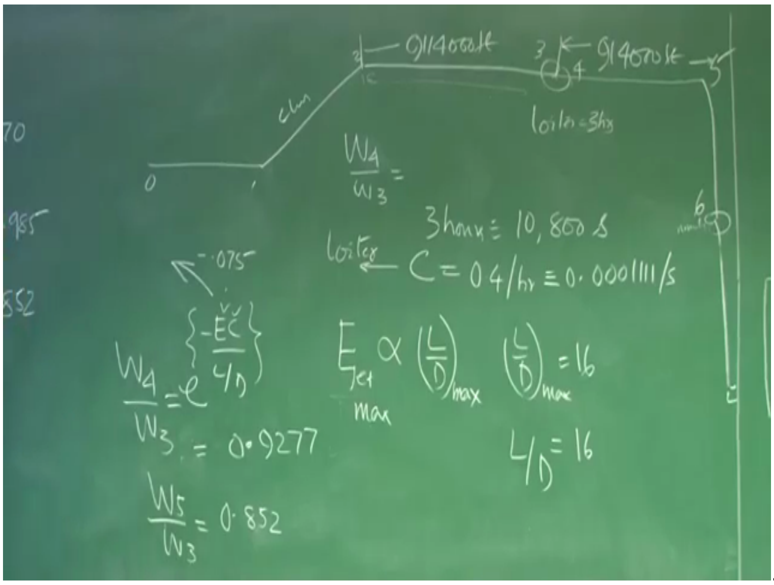

So, I have handled 0 to 1, warm up and take off then climb, this is 2 then I am going for first range which is around 9114 triple 0 feet. So, upto this if I put a 0.3 and 0.4 here this is representing a loiter let us say loiter in that example is for 3 hours, then it goes again comes back again loiter for 30 minutes, and land this is the machine requirement.

So, if I modify this diagram, I write here loiter 3 hours and loiter here also for half hour half an hour or 30 minutes then land this is clear. So, what am doing I am warming up take off climb cruise loiter, again cruise, this come like this or again repeated like this warm up take off climb go to the range loiter for 3 hours, again I come back. This is the same that is why I have added the range once the see 91400 and then again 91400 because that is type of machine I am doing.

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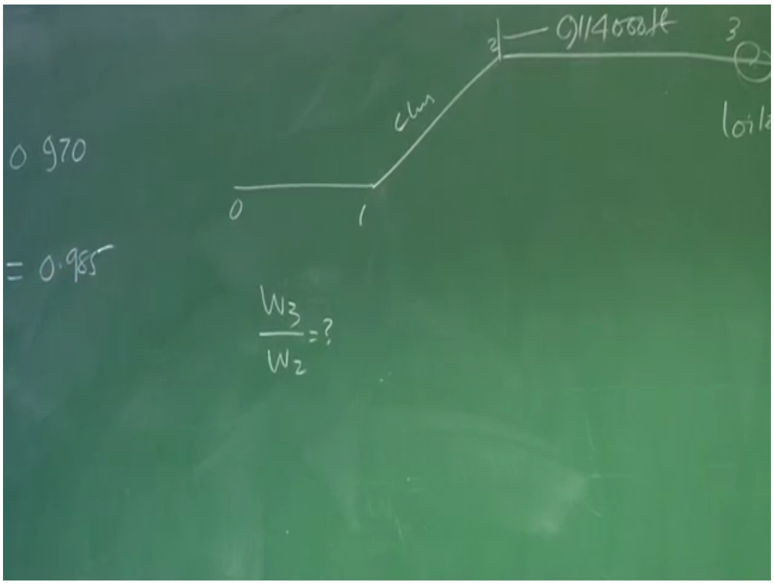

Now, what is W 3 by W 2. Why we are interested in W 3 by W 2. W 2 by W 1 or W 1 by W naught I am trying to see what is the weight W 1 after it has done the operation from 0 to 1. So, that will give me the fuel consumption because W naught minus W 1 will be the fuel consume at that point. So, W 3 by W 2 will be what is the fuel consumed during this operation once I know W 2.

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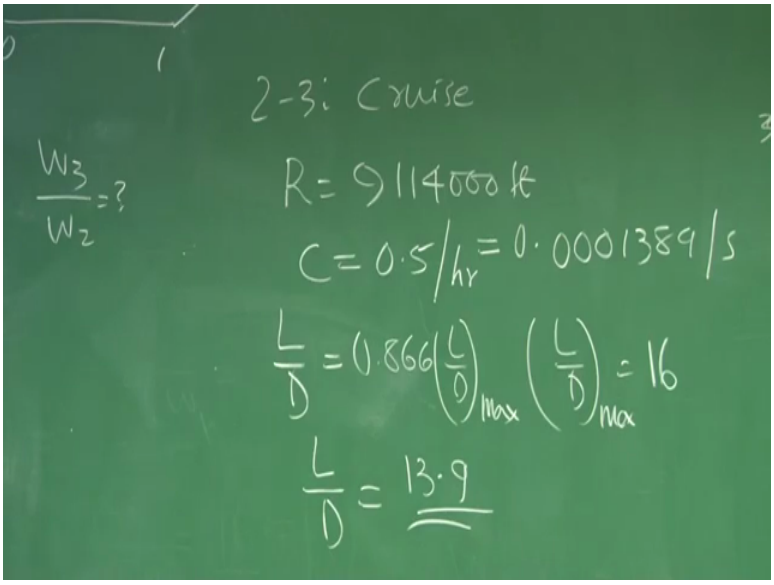

We know this is 2 to 3 is cruise. So, I can write R equal to 911 4 0 0 0 feet this is requirement and C we have seen typical jet s f C cruise, we have decided to take turbo fan high by pulse ration why not low by pulse ratio why high by pulse ratio will be discussed later, just now we are assuming that this use you know then how to calculate the full fraction.

So, I again come to the turbo jet or typical s f C for jet you could see pure turbojet then the for cruise I should take the value of C or s f s as 0.9, if it is loitering then it is 0.8, but we are interested in high by pulse ratio turbofan. So, I will take 0.5 for the cruise and 0.4 the loiter these at again based on historical data. So, at a 0.5 and if you see in the table it is given per hour. So, this is per hour. So, we need to be very care full when we use such tables see the unit, but we have to converted it into per second because we are working in SPS system. So, if I do that that becomes 0.0001389 per second 0 point. Now the question comes what L by D, I should take. As I told you know for jet engine if I going to maximize range it should fly at 0.866 of L by D max, and yesterday I gave one example will also have one session on L by D max for different airplane you have seen that the value of L by D max is somewhere between 16 to 20 dream liner and all a 3 8 y is approaching 20, but it is from 16 15 16 17 18 around that it will lie generally for this type of airplane. So, for test case we will take L by D max as 16 will also demonstrate how to take this L by D max using historical data, may be tomorrow or next class, but 16 is not a bad number yesterday we have seen that.

So, in that case then L by D which will be used for the range will be around 13.9, this is important 0.866 L by D.

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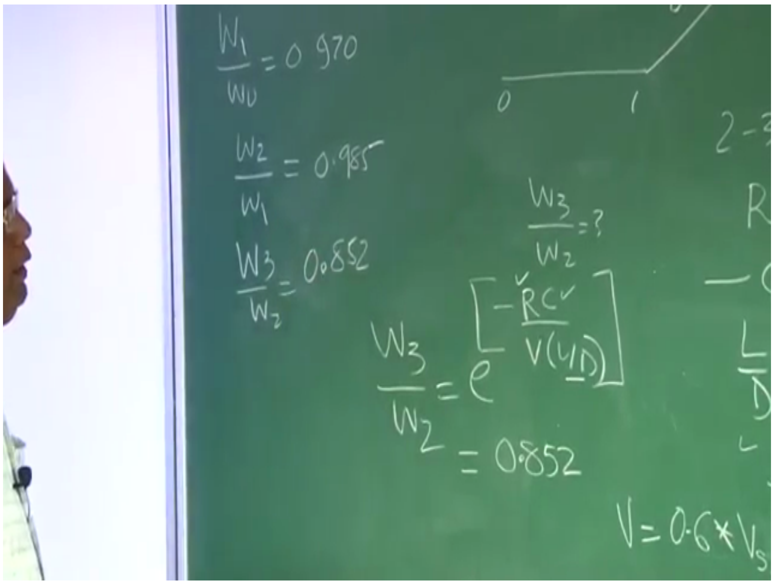

And from the range formula you will see W 3 by W 2 can be written as e to the power minus R C by V L by D check by the range formula from the from there, you can find out this and if I put this value this will come around 0.852, R you know C you know V you know, if I want to compute this I need to know the value of R, which is given there C we have corrected for dimension here L by D it is around 13.9 what is V, V is 0.6 Mach main 0.6 into velocity of sound at 30000 feet.

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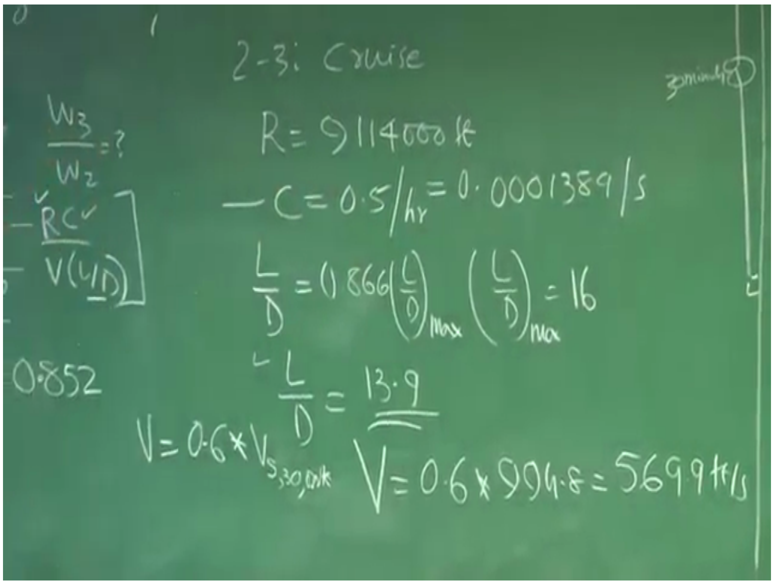

So, the V will be equal to 0.6, into 0.6 into velocity of sound and that is 994.8. So, this become roughly 569.9 feet per second. I repeat to get this ration W 3 by W 2, I need to know the value of R C V L by D all are given if you plug in you will get this ration W 3 by W 2 equal to 00.852 right. We are trying to calculate W 3 by W 2 we need to know the value of R C V and L by D. R we know C we know L by D we know V the airplane is flying at mac 0.6.

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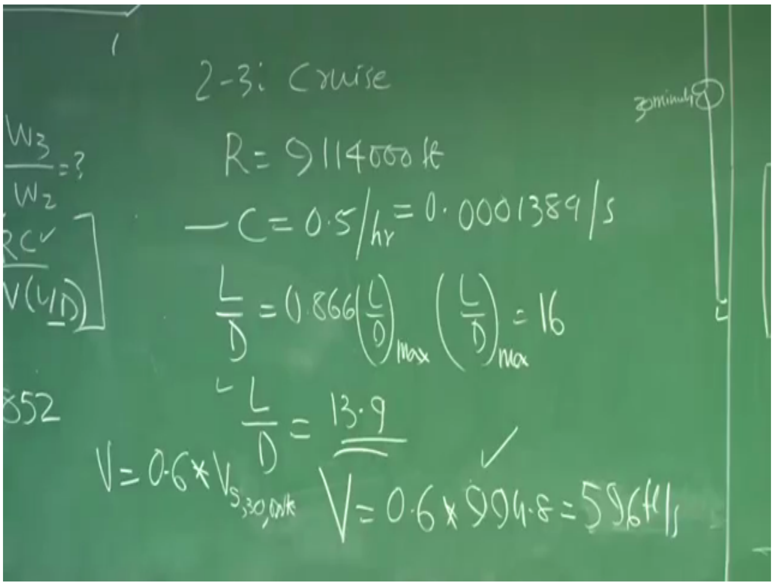

So, V will be 0.6 into velocity of sound at 30000 feet, 30000 feet the speed of sound is 994.8 feet per second. If I do this this become around roughly 596 right feet per second. These are rough number you are supposed to do it yourself and then W 3 by W 2 will be 0.852.

Once we have found out W 3 by W 2, now we are interested in W 4 by W 3. What is the machine during W 4 by W 3; it is support to loiter for 3 hours. So, now, loiter for 3 hours 3 hours means again we have to convert into FPS unit. So, this is 10800 seconds. And what value of C should I take since we are using high bypass turbofan we have seen for loiter the historical value is 0.4 right. So, we take C equal to 0.4 upto this is per hour to convert into per second and that will be 0.000111 per second. I repeat again everybody to compute yourself and discuss in the forum.

And this is for loiter repeat for best range the value of C s f C for a turbofan with high bypass ratio was 0.5 recommended, 0.5 what historical base number is a 0.5. Now if you go back to the formula for endurance for jet you see it is proportional to L by D. If you want e max you have to fly at L by D max. Unlike the range case where you have to fly at 86.66 percent of the L by D max. So, now, the L by D max since we have chosen around 16. So, the L by D required for loiter estimation will be also taken 16. And then W 4 by W 3, if you see that expression and take the inverse you will get this equal to e to the power minus e C by L by D. And put the value of e in seconds 10800 seconds C 0.000111 per second L by D as 16 this value will come out to be 0 0.9277.

Just to give here this value this whole in the brackets should come minus 0.075. So, W 4 by W 3 is known. Now we want W 5 by W 4. W 5 by W 4 is what is the operation from 4 to 5. It is again as per our requirement we said let it fly for this much of range assuming that it is again coming back to the base. This range could be different the airplane may land at other distance some other distance some other range, but do we have taken for example, is coming back. So, we have putting 91400 feet. So, W 5 by W 3 will be same as W 3 by W 2 right. And W 3 by W 2 was 0.852. So, I put 0.852 if the if this range was different then this then the ratio will change, but since they are same. So, I am not doing any repeat calculation. So, this value is here.

Now, comes this is W 5 by W 4 know. So, this will be W 5 by W 4. So, W 5 by W 4 is 0.852 which is same ratio as W 3 by W 2 because the rangers are same.

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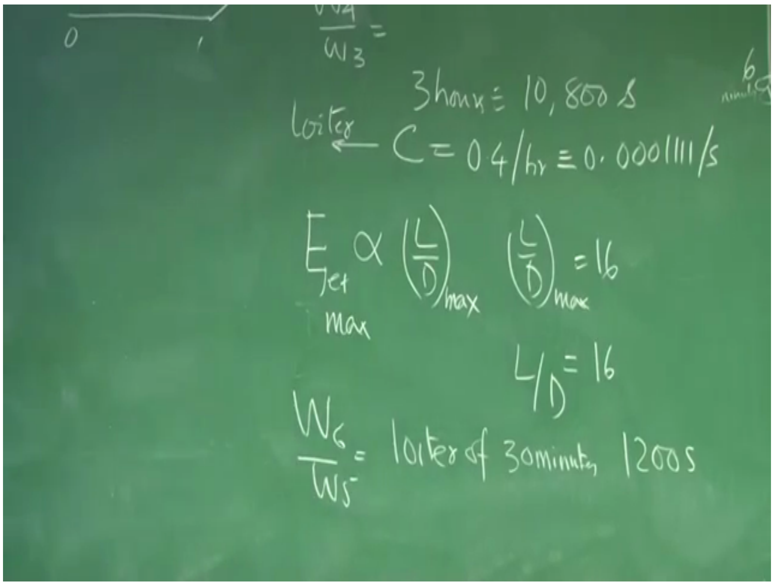

Now, you want to look for W 6 by W 5, W 6 by W 5 is what we looking for W 6 by W 5 that is that correspond to loiter of 30 minutes as per the machine requirement and 30 minutes means 1200 seconds.

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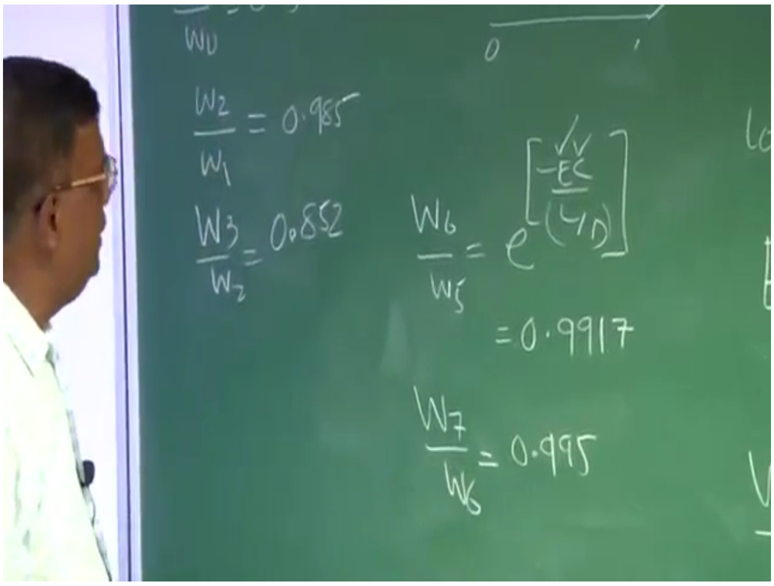

Again we have to use W 6 by W 5 equal to e to the power this say endurance equation e C by L by D. C we know e we know around 1200 second L by D we know it will be L by D max. So, this ratio will come out to be 0.9917. Then comes W 7 by W 6 which is as per historical data that is advisable to take around 0.995. As you will start learning more about design this numbers you can you will be refining right this is the starting numbers. So, what we have done we have seen W naught by W sorry W 1 by W naught W 2 by W 1 W 3 by W 2.

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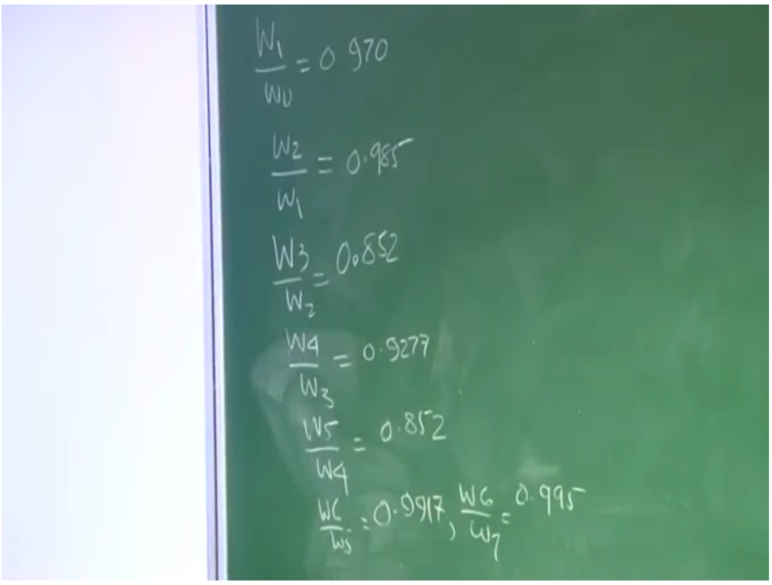

Then we have W 4 by W 3, then we have W 5 by W 4 then we have W 6 by W 5, and we have also W 7 by W 6.

So, let me write W not we will see here. So, W 7 by W 6. What to do with all these number by doing all these things mechanically with a clear (Refer Time: 21:37) understanding that at what unit is we are using and many times we are using historically based data and this exercise is to get just initial numbers design numbers for beginning a design starting a design. We have got W 1 by W naught W 2 by W 1, W 3 by W 2, W 4 by W 3, W 5 by W 4, W 6 by W 5, W 6 by W 7. Our aim is to see what is the fuel fraction.

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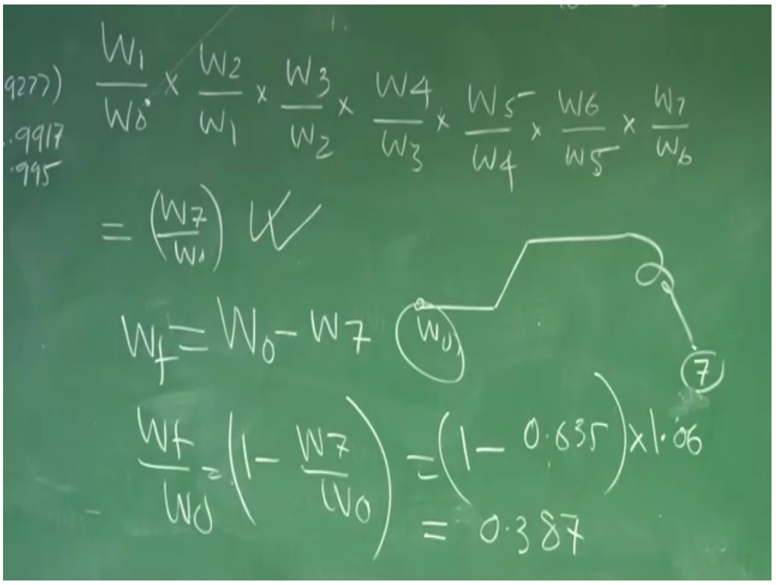

Now, we multiply this W 1 by W naught into W 2 by W 1 into W 3, by W 2 into W 4 by W 3 into W 5, by W 4 into W 6 by W 5 into W. If I write it like this, if I take product of all these fraction what final I get, I get W 7 by W naught right.

We are trying to find out what is the W f fuel consumption what will be fuel consumption you started with W naught, let us see from here you started with from here, when the weight was W naught. Did all the operations and landed here. That time the weight is W 7. And assuming that in in between you have not done anything, but flying. So, that difference between W naught and W 7 will be the amount of fuel consumed right.

So, I say W f is equal to W naught minus W 7. This is clear? I start from here W naught this this this whatever I want to do I land it here, say this is the 7th point right. I started with W naught and final is W 7 and the difference s the fuel consumed. Now where I W f by W naught that will be 1 minus W 7 by W naught. Now see here if I multiply the all the fraction fuels fractions at different stages I actually get W 7 by W naught. I have to simply subtract that value from 1; I will get fuel fraction W f by W naught. So, this is such a smart way of getting initial number.

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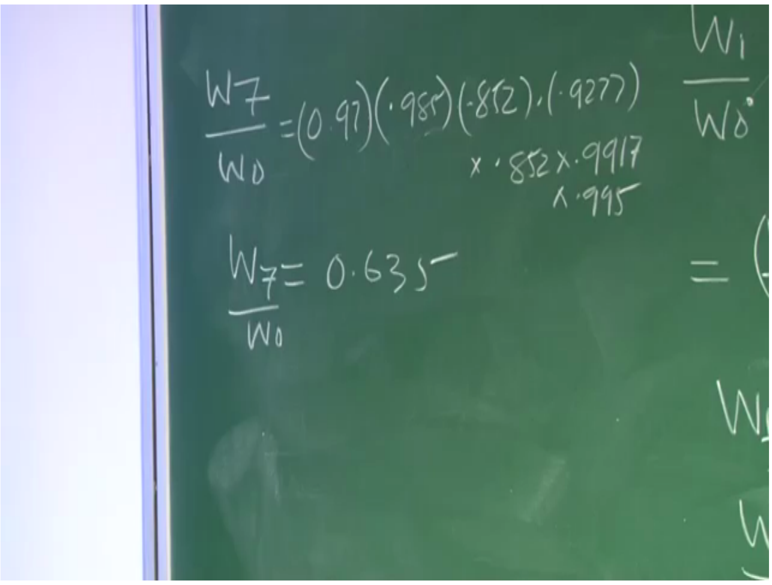

To complete this exercise let us do this. So, I will see W 7 by W naught will be let me just write it 0.97 into 0.985 into 0.852 all those fractions am writing into 0.9277 into 0.852 into 0.9917 into 0.995. These are all values of this fraction which we have already valuated. And if I do this I get a value 0.635. So, W 7 by W naught is 0.635, I want to find out W f by W naught then what will happen this will be 1 minus 0 0.635. W by W naught will be 1 minus W 7 by W naught and W 7 by W naught you know (Refer Time: 25:43) you yes put this, but you multiply with 1.06 usual practice. That is am increasing 6 percent because 6 percent is attribute towards trapped fuels in a fuel tank some percentage of fuel around 5 6 percent, will not be able to take it out. So, that is why we are keeping this margin. And if I do this this will become roughly if I am not mistaken this will be 0.387. Please do it yourself before you start asking sir right.

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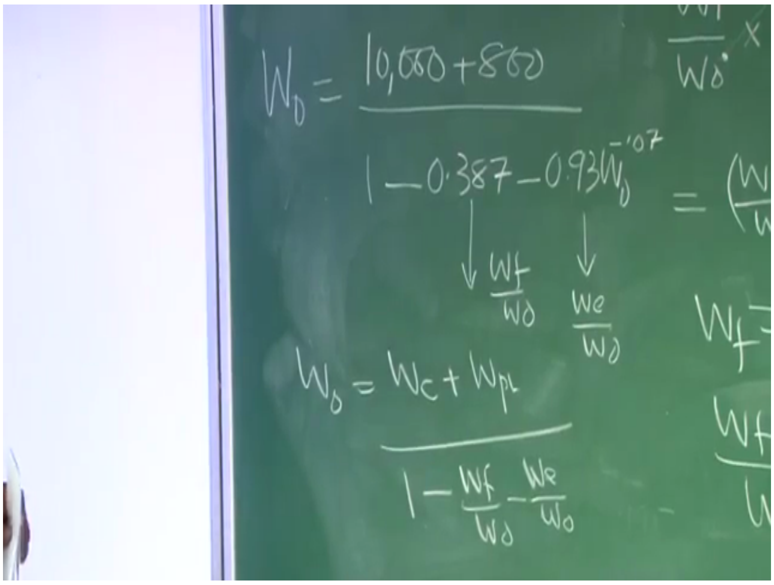

Why we are doing all these things? Again I go back you remember our aim was W naught W naught was as per the machine 10000 plus 800 pounds by 1 minus 0387 because this is W f by W naught, and for W e by W naught we have same (Refer Time: 25:53) W naught to the power minus 0.07. This correspond to W f by W naught and this corresponds to W e by W naught. Because our expression W naught we have seen this is W crew plus W payload by 1 minus W f by W naught minus W e by W naught. So, W f by W naught here and W e by W naught, I have already explained you in the initial stage and if you have forgotten, let me see how best I can help.

If you relook into my lecture on empty weight estimation, there we have given expression W e by W naught based on statical data again it is from raymers book, it was this Kvs.

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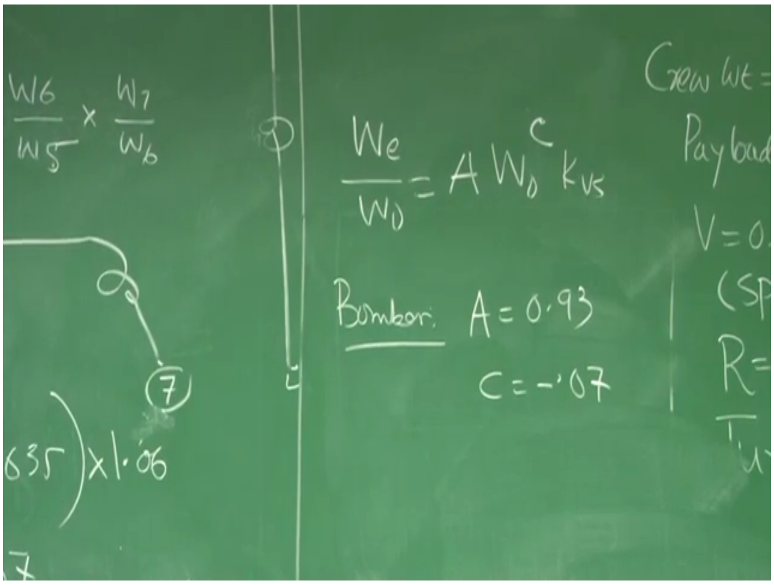

Of course this we have to get one Kvs value it may vary depending upon the material composites or any other material, but we are taking it one here. And for the type of airplane we are discussing, if you see that table to see how the value of A and C, this typically representing a number and for that case the value of A in the table is 0.93 and C is minus 0.07. If you are designing a civil aircraft, then accordingly you have to see the value which is historically generated generating using historic data.

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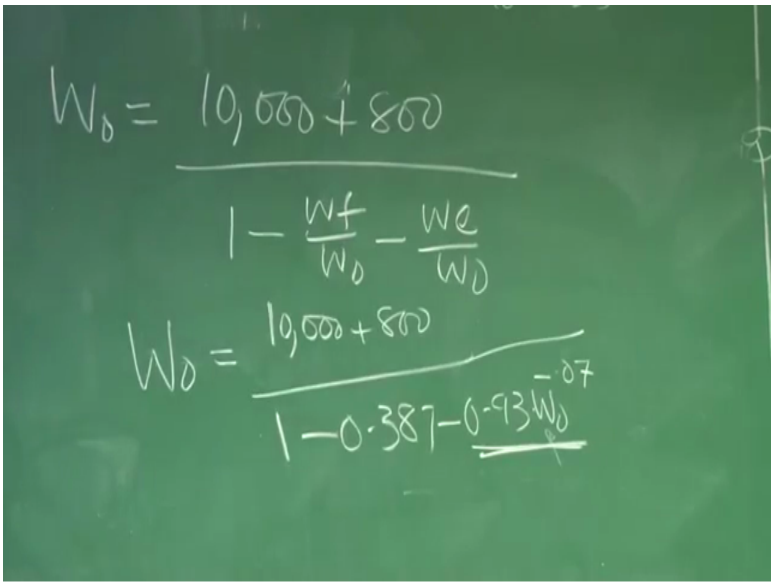

So, if I do this then I can simply write here W naught equal to 10000 plus 800 by 1 minus W f by W naught minus W e by W naught. So, W naught equal to 1, 10000 plus 800 by 1 minus 0.387 minus 0.93 into W naught minus 0.7 right. So, what is our aim? Now our aim is to get initial estimate of W naught. So, what the best way you can do you select some value of W naught put a and C whether it is they are converging or not or usually numerical method that is upto you.

The example says you take the simpler method, you go on putting the number here and see your left hand side that is equal or not and you will find these will converge when just to give is some numbers, but today's scenario you cannot do. So, many ways you can find out the solution.

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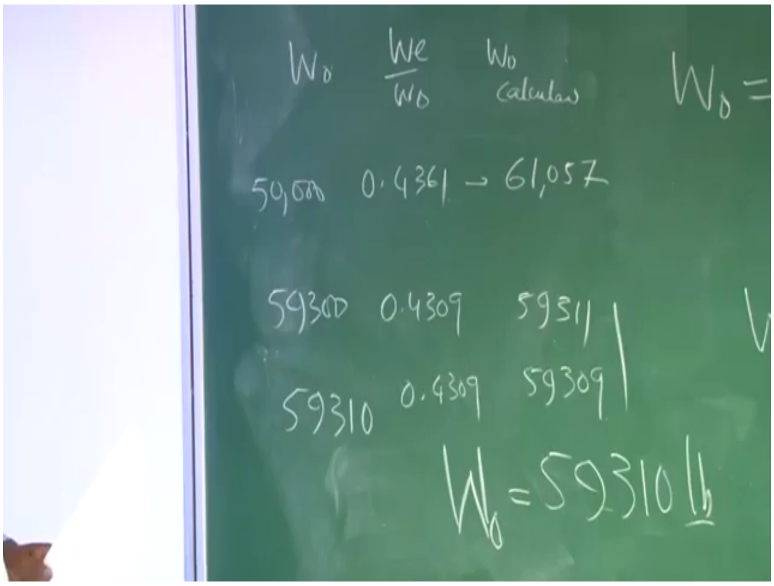

But what is being done here is W naught W e by W naught and W naught calculated right. Because see that W naught decides what is the value of W e by W naught, because this is given as a function of W naught right. That is why you have this sort of a issue we start with 50000 pound and this value is 0.4361, if you use that expression. So, that will give you around 61057 if you make it to 59300 then this value is 0.4309 and this is 59311 and if you have 59310 this value is 0.4309 and this is 59309. So, almost converging.

So, you say W naught initially I can start with as 59310 pound right. And then you see from the machine requirements what is the closest aircraft what baseline you have taken and you will find that these things are fine this number should be closer to the number which is being available through a base line. It cannot do widely different then there something is wrong right, will also talked about how to get feel for this number W f by W naught somebody says 0.4, 0.5 what should be the number. So, once you mechanically know how to do this will now put a designer perspective on how much W f by W naught you should expect for such an airplane, and that is what is the sense of a designer.

Thank you very much.