Engineering Thermodynamics
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Lecture 35
Reversible flow work, multistage compressor, efficiency of pump and compressors

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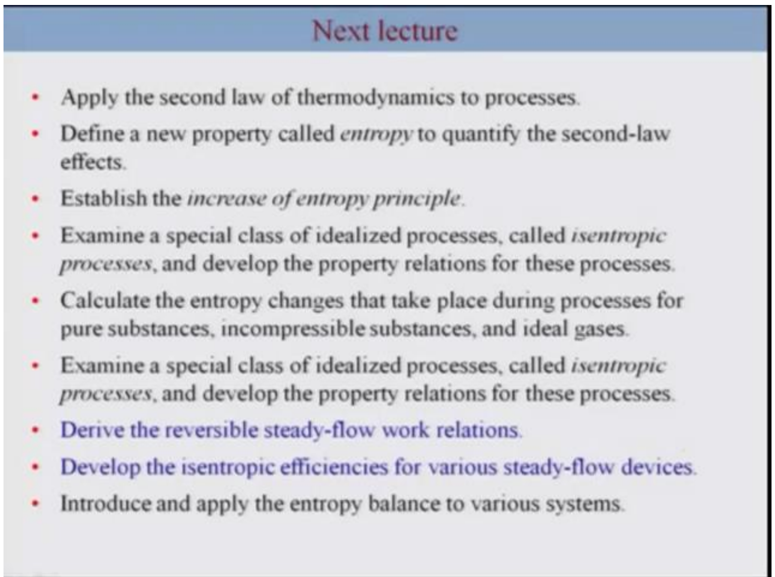

Welcome back, in the last lecture we were discussing the entropy change, particularly for pure substance, incompressible substances and ideal gases and as well as we looked at idealised processes called isentropic process. Ok and particularly we developed the prompt relation for this processes, so in this particular lecture we will we start with the derivation of a reversible steady flow work relation.

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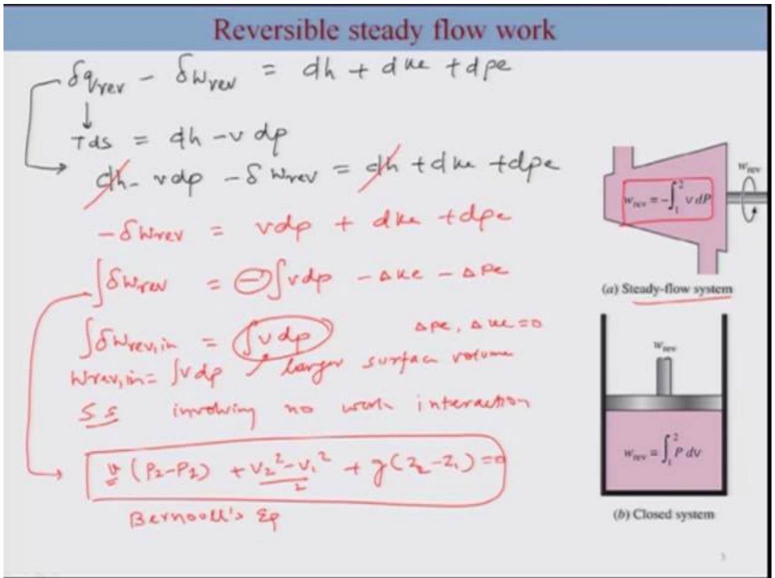

So let me start with a basic energy balance particularly for reversible process. Ok, so we are assuming this to be a positive value here, del Q reversible minus del W reversible and that will be for the steady flow conditions you have changed entropy or in terms of differential function or in terms of d h plus d ke plus d pe ok. Now for the case of a reversible process we can consider this to be your Tds and in the previous lecture we have derived that Tds relation in terms of entropy and then we can write this as dh minus Vdp ok.

So now we can express balance now in terms of dh minus V dp minus del W reversible that is going to be dh plus d ke plus d pe ok. So this can, it cancelled and what we have is del W reversible as V dp plus d ke plus d pe, ok. So if we assume the changes in the kinetic energy and changes in the potential energy to be zero then we can write this as minus V dp, ok. If you include the changes in the ke then you have to also include this terms ok.

Now this is the amount of work out from the system out work done on the surrounding, so if you consider this negative as this work as to be done on the system than this can be written as W reversible in that is input work, this can be written as simply V dp. Considering your delta P delta K to be zero ok. So it is very clear that if you have a larger surface volume, you will have this term that is the work consumed it is going to be greater.

Now in case of a steady state involving no work interaction, you can rear this expression in the following form if you are considering incompressible fluid then this is going to be p2 p1 plus v2 square v1 square by 2, the kinetic energy term plus gz2 minus z1 ok, so this is for incompressible fluid and this is W work is going to be zero or if you integrate here this should be also integral this, so this could be written as W reversible in as simply V dp ok.

So when this is zero (you) and the other terms are not zero let us say (de) del ke and del pe then you can write this expression in this form, this is equal to zero. this is nothing but Bernoulli's equation ok. Now you can clearly see there is a two different expression for the work related to the open system where we have (we) clearly written in terms of V dp, that is what we derived here and for the case of a close system is going to be P dv.

So you have summary where we have sent for a steady flow system, so you have W reversible in terms of minus integral of V dp ok. So here indicates it is out, negative indicates that you are doing the work on the system and this is the case for the close system ok.

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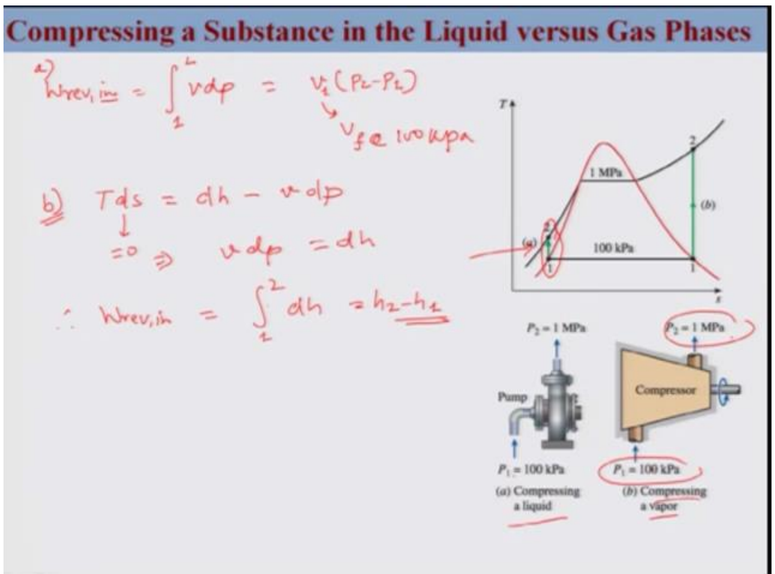

Now we can try to calculate this work reversible for two specific system, one is for the liquid, saturated liquid and other is for the saturated gas ok. So here is an example where you have a both this process which we are (con) going to consider is isentropic process that means the in drop is constant and then we are going to analyse how we should be considering the calculation of W reversible in.

Ok, considering in the negative are already absorbed by taking the direction, changing the direction of the out the in and this your, I can write this expression the work reversible in as simply V dp ok which is going to be for the compressible case that is case A ok, which is this. As simply V P2 minus P1 ok and V here is going to be V1 which is going to be Vf at that is fluid at 100 kilo per scale ok. What about B?

B is your gas now here of course we need to know the relation between V and Ap because V is not constant ok and in this particular case, we are going to make use of the Tds relation in terms of enthalpy, considering this is isentropic process, this is going to be zero, thus your V dp is nothing but dh and therefore we can write W reversible in simply as integral of dh for one to two for the case of B and that will be your H2 minus H1 ok.

This was the case where you had two cases, one for the compressing liquid and the other case was the gas where the compressing vapour inlet and outlet pressure were same as that for the compressive liquid but the expressions are quite different, the values are going to be extremely different as well ok.

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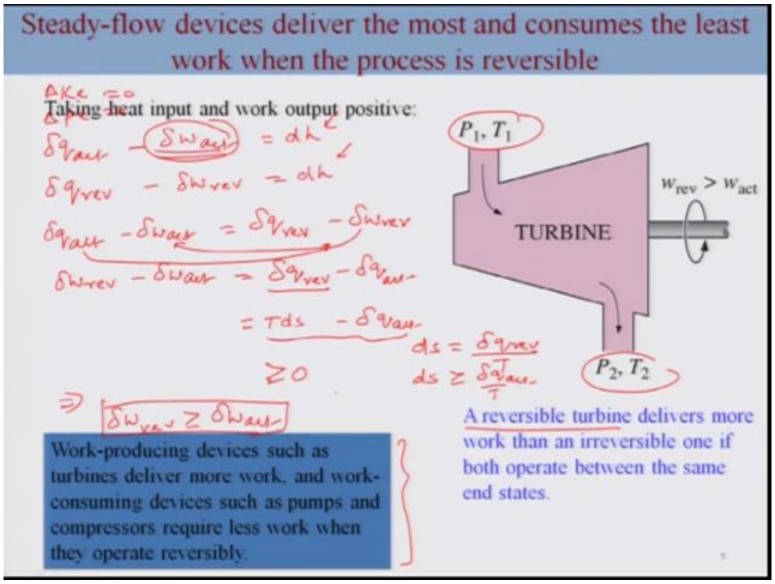

Now let us look at the case and we have discussed this aspect earlier also that the steady flow devices deliver the most and consumes the least when you consider the process reversible. So we have looked in this particular aspect when we are discussing about your boundary work ok and here we can now derive it very umm rigorously ok.

So you have turbine where the inlet pressure and temperatures were given and what we want to derive is that the reversible turbine work is more than the actual turbine, more than or equal to that of the actual turbine and inlet and outlet condition for the as well for that reversible as well as for the actual turbine would be (ca) considered to be sin.

So let us take simple analysis of the energy balance, we will consider again kepe to be zero ok, changes in the ke and pe start with basic energy balance here that means your, for the steady flow devices this is going to be del W actual is equal to dh ok and similarly you can write for the reversible process here ok. Now this as going to be the same because H is a state function is a that is, is a property and hence it depends only on the initial and the final condition the changes in the entropy, so you can write this expression that is this left had side should be same which essentially means that I can write as follows ok.

You can rearrange, bring this W together, so this is brought here and this is brought there minus del Q actual ok. Now del of Q for the reversible case is nothing but Tds ok and we know that ds is equal to del Q by reversible by T but for the actual case this Ts would be de Q by T which means your ds is greater than equal to the ratio of your Q by T for the actual system and thus this expression is going to be greater than equal to zero ok. So this (eh) clearly implies that your W reversible is greater than equal to for the case of actual ok. So this is how we can conclude that a reversible turbine delivers more work than that of irreversible one if both the devices operate between the same end points that means your P1 T1 and P2 T2 are same for the both the devices ok.

So we summarise here as follows that the work producing device such as your turbine deliver more work and work consuming devices such as pump and compressor requires less work when the operate reversibly. This is something which can now conclude here ok, based on this analysis now this here of course we have considered for the case of positive value of your work and essentially this is a work producing condition, that means this particular derivation was for the case of work producing device and you can do the same exercise for work consuming device as well ok.

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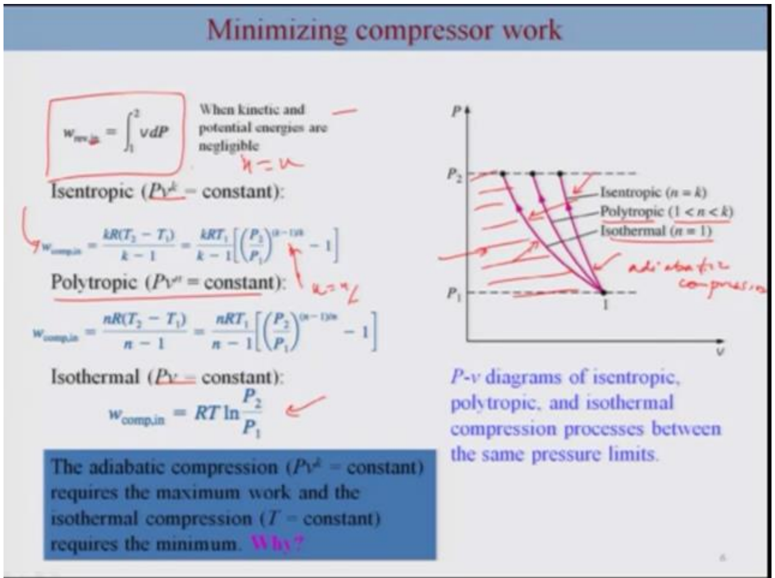

So let me just describe the compressor work and here the compressor again the can write this compressor work in this form ok, the in stands for the work needed by the compressor and this can be written as V dp assuming of course your kinetic energy and potential energy to be same. Now you can operate the compressor at a different conditions, you can operate at cost and temperature, you can operate adiabatic condition, you can operate at isentropic condition as well, so that means you can have different relation between P v and P and based on that your work required would be different and this is summary of the derivation, so this says the case when we have considered isentropic.

We have already looked into this that for adiabatic reversible case the polytrophic expression turns out to be K and this becomes your Pv to the power K is equal to constant this essentially means that the entropy is constant and for that this is the expression for the work compressor ok. And you can write in the general polytrophic PV to the n ok, so where basically nothing but K in this expression is turnout to be N here ok and for the case of isothermal Pv is equal to constant for the ideal gas here, this expression turns out to be this ok.

So of course this is the analysis based on the relation of the Pv here and one can draw this on a Pv diagram, so you have this is isentropic which essentially means the entropy is constant here and this is in general polytrophic for N less than K which essentially means here as we decrease your N, you slope decrease for the case of the path ok and this if you can remove the heat from the process in such a way that you can maintain the temperature than this turnout to be isothermal ok.

And what about the work here, work is nothing but the area under this Pv this would be the area for particularly isothermal, ok that means this particular area covered from here to here in this axis ok, would be the co spurning work because that’s what you said P is not a Pv integral it is a V dp integral. Thus it is very clear from this graph this plot that the area under isothermal curve is the lowest and area under isentropic curve is the most ok, so that means the adiabatic compression requires the maximum work, that means this is adiabatic compression ok, which is your isentropic process and the isothermal compression is going to be the one which will require the minimum work ok.

So this is directly based on this plot and this particular expression comes out to be there ok. Here of course we have considered that final limit for all this different conditions P2 and P1 and are same for the different processes which we have discussed here ok. So that means the all this diagram, the final pressure at 1 and P and 2 are same. Ok, so based on this exercise for the case of compressor, it is very clear that the work required is going to be minimum for the isothermal process ok . So it is a valuable to cool intermediately during the process in order to reduce the temperature and this is something which is clearly evident from out exercise just now.

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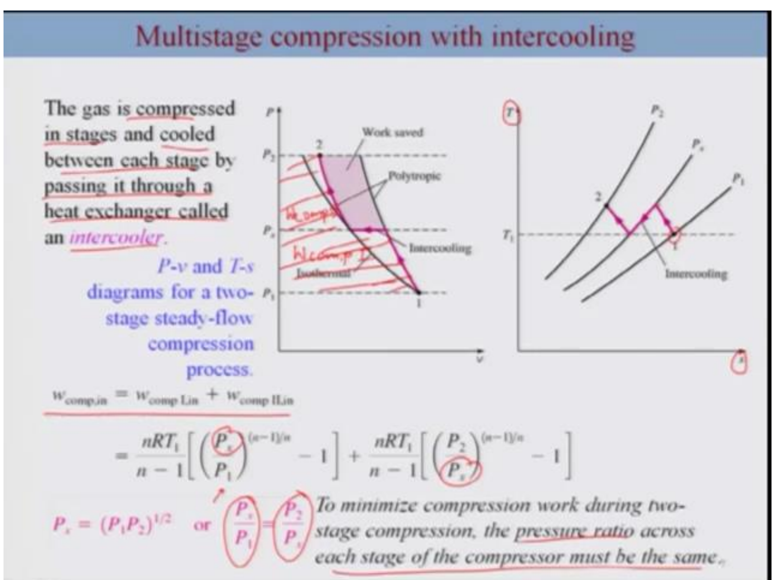

So this is the reason that typically the gas can be compressed in stages or rather is compressed and cooled between each stages by passing it through a heat exchanger which we call it intercooler and the reason is that is you reduce the temperature in order to reduce the work required for compressing the gas.

So this is an example here, so you have which we are drawing this process on a TS diagram, initially you are state one ok and you have this polytrophic process, you are compressing it, so you are increasing the pressure at this point you cool this system on the gas at a constant pressure and then again you pass through again a polytrophic process or you you compress it adiabatically again.

So this can be written in terms of this can be drawn on a Pv diagram and which would be written that you are on a polytrophic process as I said and then you cool at a constant pressure and then then you basically you you are bringing the system close to the isothermal condition and then you again operate on a polytrophic process and thus the overall area would be this that this is the amount of the work for the compressor. That means this particular shaded region is the work which you have saved by using a heat exchanger and by applying a intercooling mechanism.

Thus you the total work could be if it is two stages it would be simply the work done in this stage one and this is your W compressor two ok. And this can be written in this form, now the only difference is that the expressions are same as evaluated in the previous slide, the expressions are same the except for the final pressure for the intermediate would be replaced by Px ok and thus your Px is here and the here would be the Px the initial condition for the second stage would be Px as well.

So Px becomes the intermediate or the variable here and you can find out the pressure which will lead to the minimum work here by taking a derivative of this expression and finding out the corresponding pressure. It turns out that when you minimize this compressor work for this two stage (compress) compression the results indicate that the ratio of the pressure should be same for both the stages that means the pressure ratio across each stage of the compressor must be same. And this is the condition which is directly from minimizing this expression.

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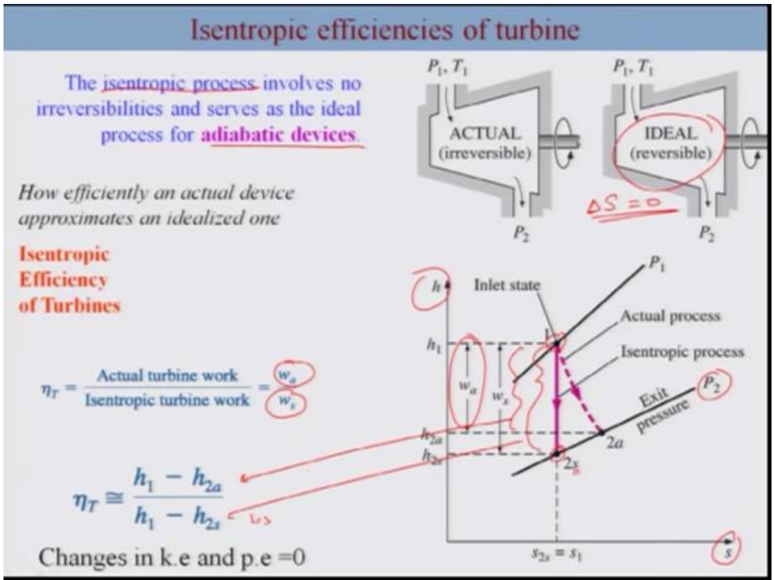

So we have already discussed the kind of work for the reversible case that the one we have looked at, we have looked at the compressor, we looked at the necessity of intercooling in order to reduce the work required for compressing gas for example. Now if you want to compare the efficiency of such a device we look at kind of idealised model ok and this is something which we have already looked into when we discussed the (ka) cycles where we have considered the convert cycle the the idealised fictitious kind of cycle as a theoretical limit.

Similarly in this case also for various flow devices which operates as adiabatic condition and if we consider such a flow devices then we will be considering another idealised device where there is no friction and other kind of irreversibility. Others we can consider a device which is essentially does not have any component of irreversibility or in another word we can consider an idealised reversible device where your entropy is constant.

So most of these devices will be operating in those condition and isentropic process will serve as a ideal process for the adiabatic device ok and this is what we are going to take as a kind of theoretical limit ok. So in order to find the efficiency of a normal turbine we are going to consider the following ok. We will be defining something called isentropic efficiency on a Hs plot the simple turbine would follow this process in idealised condition, there is going to be a drop in the pressure, so thus your one is initial state and it undergoes isentropic process for the case of a reversible adiabatic case where the entropy is going to be constant.

That becomes your idealised model ok and it goes up till to S indicates that it is isentropic process but for the case of actual process there will be deviation, so it will deviate from this isentropic path ok and it deviates until it reach this particular exit pressure ok. So naturally the work is going to be different because as we said the work is done and this difference in this change in the enthalpy would be indicating of the work, so to find out the isentropic efficiency of turbine, we will be making use of this expression which would be the actual turbine work divided by isentropic turbine work ok.

So actual would be Wa which would be simply this ok and clearly this work would be less compared to the isentropic one ok, this particular efficiency is going to be less than one ok. So we can write this in terms of entropy change, so for the case of actual it would be simply H1 minus H2s would be for your Ws and this is going to be for the actual ok. That will be H1 minus H2a ok, so that is how we are going to find out the isentropic efficiency of the turbine.

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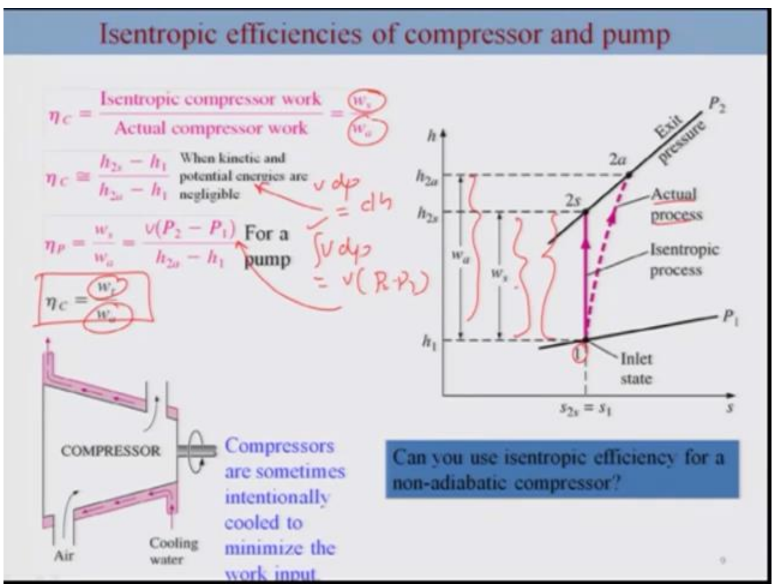

Similarly you can also find out the isentropic efficiency for compressor and pump, for the case of a compressor or pump the initial pressure is going to be low and along the process isentropic process for the idealised where there is no irreversibility ok, where is a constant entropy. So we are assuming that the actual and the idealised compressor the exit pressure is same so this is your total work for the case of idealised or isentropic process.

On the other hand there will be deviation from the isentropic process for the case of actual process and thus you need more work to achieve the same change from P1 to P2 state ok and thus your actual work is going to be more than Ws. So in order to have efficiency less than 1 we do not use the same definition which we have used for the case of isentropic efficiency of turbine, here we are going to use just opposite of that.

So the way we define the isentropic efficiency of compressor is isentropic compressor work ok which is going to be smaller than the actual work. So actual work is in the denominator, so this makes efficiency less than one. So we can wrote this based on the enthalpy difference here, so Ws is S2s minus H1 and Wa is S2a minus H1. In this analysis we are considering the change in the kinetic energy and potential energy are negligible ok.

Similarly for the pump you can come up with this, remember that we have already derived that for the gas V dp term is is approximated to dh and thus you can simply write dh. On the other hand for the case of a liquid which will be the case for the pump we are going to simply write V dp integral as simply your V P2 minus P1 and this the numerator is simply your V P2 minus P1 ok.

So now these are the case for your isentropic processes now many times as we already discussed that we take out the heat in order to maintain isothermal condition. In such case of course you cannot make use of this expression, these expressions were meant for adiabatic reversible case of isentropic case.

So you have to make use of a different definition of efficiency for the case of which deviates from adiabatic compressor for the case of isothermal and thus for the case of isothermal efficiency this will be simply Wt for the ideal isothermal work needed for the operation divided by the actual work ok. So this is how we are going to define for non-adiabatic compressor or in this case this is isothermal efficiency.

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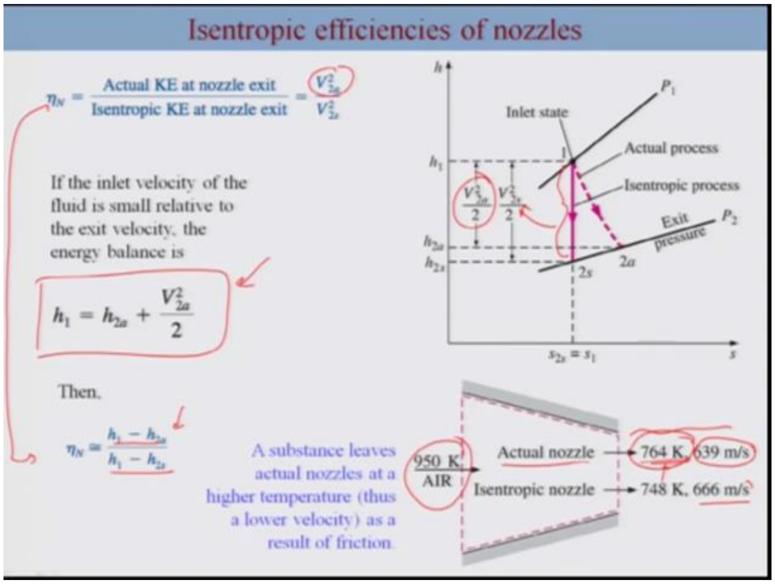

We can also extend this analysis for nozzle which are again adiabatic device and for the case nozzle which is of course an adiabatic device this are used to increase the accelerate the fluid that is increase the velocity.

If you assume that inlet velocity is small compared to the exit velocity ok then we can simply write the enthalpy balance in this form ok and the way we define the isentropic efficiency of nozzle is simply the actual ke at the nozzle exit divided by isentropic kinetic energy at the nozzle exit considering that for actual and as well as for isentropic, the initial conditions are same.

So this how we can depict on a Hs plot, so this is your pressure initial ok and the change the final P2 is same for both the actual and isentropic and they the delta H can be simply shown to be in terms of kinetic energy ok and thus your change here is simple the kinetic energy for the exit condition and isentropic conditions and this change for the actual one is going to be simply the kinetic energy and the exit condition for the actual case.

So this is more express in this form, so thus you can write this not necessarily in terms of kinetic energy but as well as simply the change in enthalpy for the actual case and as well as divided for the isentropic case ok. So that would be the definition of the isentropic efficiency of nozzle ok, not much different from what we have seen for the case of turbine.

Now you can take an example here for example this is nozzle, you have a inlet condition 950 k a at air and this is a nozzle for the case of a actual nozzle the temperature can be higher than compared to the isentropic nozzle due to the irreversibility of associated here and the velocity can be lower than the that of a isentropic nozzle ok. So which is of course related to irreversibility associated with actual nozzles. So you can do this analysis and find out isentropic efficiency of this flow devices under isotropic conditions.

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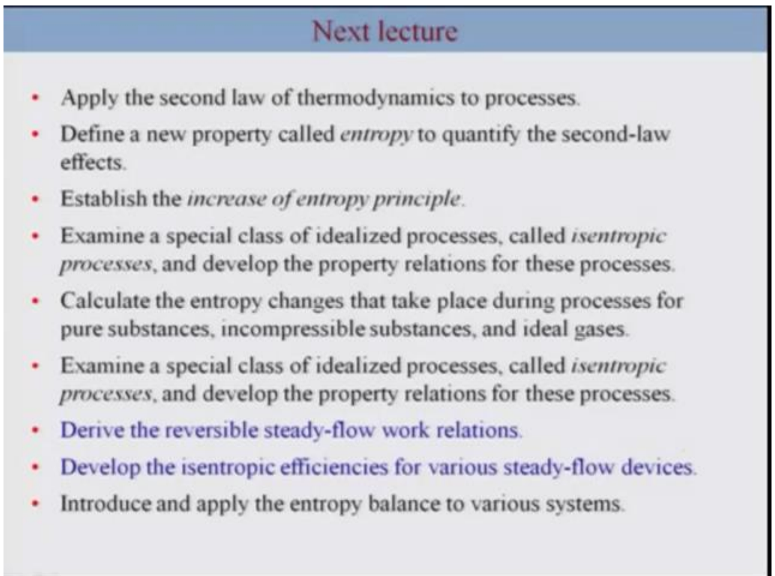

So with that we will end this particular lecture, in the next lecture we will be applying the entropy balance to various systems, so I will see you in the next lecture.