Indian Institute of Technology Kanpur

National Programme on Technology Enhanced Learning (NPTEL)

Course Title
Introduction to Experiments in Flight

Lecture – 19
Aerodynamic Parameter Estimation Using Least Squares Method

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Hello and welcome to the tutorial. So in the previous class we have seen this experiment about parameter estimation using least square methods. So there we have seen what is parameter estimation? And how to estimate the parameters and why do we lead the parameter estimation and everything. So we understood and I also discussed about various methods are available to do this parameter estimation and I have discussed about least square method.

So that we have learnt what is the basic principle of least squares, how least square works and we also have seen the mathematical formulation and I have also talked about some examples like how can you incorporate the least square estimation philosophy in the problems and other things right. So in this tutorial what we will be learning is this like earlier lecture was about generic parameter estimation like any process can be modeled BE mechanical, electrical or from any other process.

And if your models are parametric then you can use the technique of parameter estimation and maybe least square estimation technique to estimate with parameters right. So I will be discussing today about aerodynamic parameter estimation, so I will be talking what is aerodynamic parameter estimation and how aerodynamic parameter estimation is very much important. So let us start for today's session.

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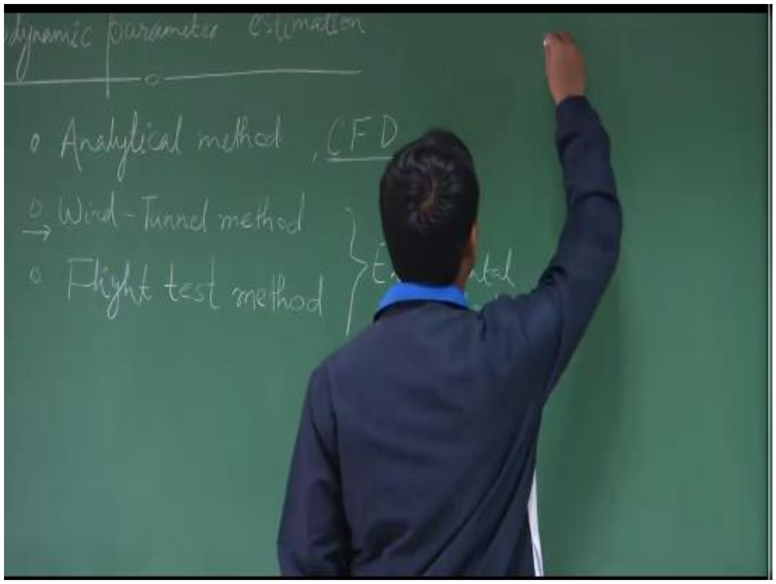

So again the experiment is about parameter estimation, so this parameter estimation is nothing but aerodynamic parameters. So we discussed about definition of parameter estimation and few methods I have listed which can be, you know like from there you can estimate parameters using the flight data, using the states of input data and output data. But we also should know there are other ways also to estimate aerodynamic parameter estimation right.

So these methods are like this, first one is analytical method, next is wind-tunnel method, and then third one is flight test method to which we will be learning in detail today flight test method. So earlier part of the lecture was about this method right. So this is a theoretical method you do the parameter estimation using analytical method from the geometrical detail of the aircraft, but what happens there like you assume some complex phenomena, you simplify some complex phenomena.

So the estimated parameters may not be so accurate, but for that you have other methods also which is picking up so fast this state and that is based on the CFD computational flow dynamics also all of you know about CFD. So CFD based methods also can give you the computed parameters right. But again to corporate those parameters whatever you got from the analytical methods or from the CFD based methods you need some experimental methods right.

So these two are experimental method when you treat these parameters from the experiment doing some experiment is experimental method. So as I said you need to corporate to which rigels or the parameters then we should go for experimental based method. So let us see how wind-tunnel method work.

So what it does like you have a tunnel and then you do not, usually you do not put a full scale model you put the scaled model inside the tunnel and you will try to simulate the accurate mass that you try to give the same speed or you maintain the desired flow which is usually in the flying condition and then wind-tunnel methods gives you the measurement of the force and coefficients there you get those parameters. But again since it deals with the scaled model and then there will be issue of the Reynolds number you cannot exactly similar the Reynolds number in the wind-tunnel.

And apart from those things you will have issue with this like dynamic derivatives and other stuff. So again we should look for some prominent method where you will get better estimate of the parameters and with that we will start this flight test method where you will collect the data during the flying condition and from the method variables or flight variables you will try to estimate with parameters right.

And already I have discussed which parameter estimation in aerospace area is very important because once you talk about designing of control algorithm or designing of card algorithm in aerospace application, then you will lead to have a very accurate model even if you want to do something for the fault, detection and diagnosis you should have a better model. Aerodynamic model is becoming so important because the accuracy of aerodynamic model will dictate the accuracy of your equation to motion or whole aircraft model.

So I will not talk about equation of motion here, so we will start with this aerodynamic model okay, aerodynamic model of forces and moment.

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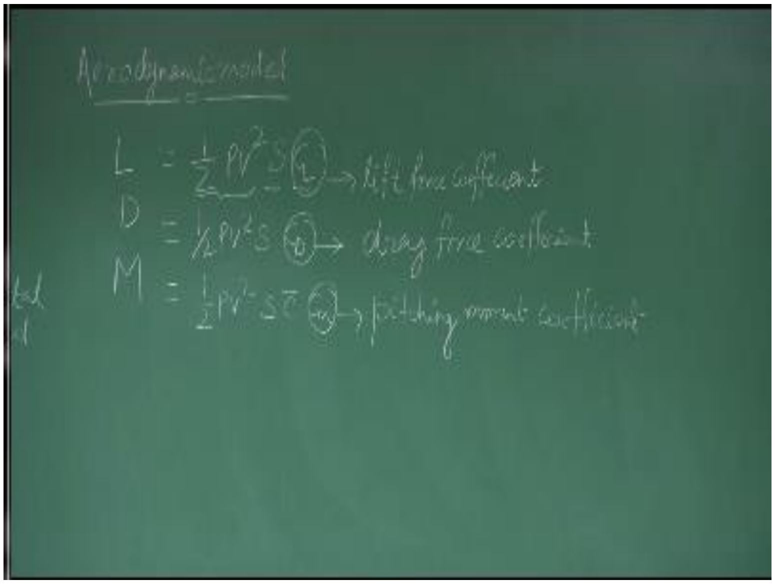

Aerodynamic model is so as you know in this aerodynamic model like if you see we have force lift force drag force and movement I'm taking up pitching mode so I am giving you the example of the longitudinal derivatives or longitudinal forces in moment so lift drag and pitching movement right so you can write lift as ½ ρ v^2 S into CL right this part is your dynamic pressure and plane form area and this called lift force coefficient so you can write this as general lift force coefficient.

Right and drag also you can define similar ½ ρ v^2 S CD where this coefficient CD is your drag force coefficient correct and preaching movement is again ½ ρ v^2 dynamic pressure times S times C bar here this is cord length and this Cm which is your preaching movement coefficient okay so you know force and movement can be simplified in this term so these are basically your non dimensionalize term right they are very important to study in this equation of motion now with the introduction of lift force lift drag and pitching movement coefficients.

So I will talk about like this model like how do you model them right so now you know what are those coefficient so.

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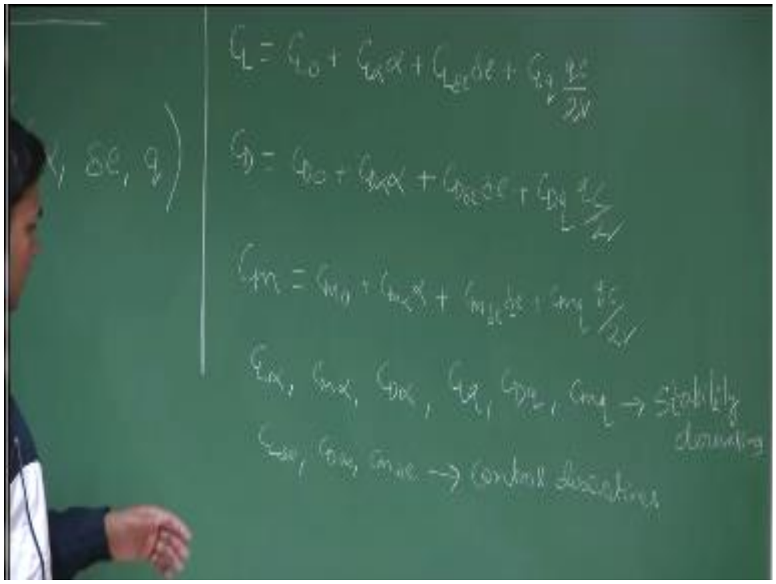

CL CD and Cm so you know they all are function of α angle of attack δ A change in elevator deflection and q pitch rate it can also be a function of other variables like Reynolds number V and all so just for simplification I will take this three variables so CL CD Cm they are function of C α δ E and q right so if you write the aerodynamic model which already you know just for revision I am doing it again so CL can be written like CL0 + CL α into α + CL δ e times δe + CL q here we write CL q time qC/ 2v.

To make it non dimensionalize and everything will be in same term and CD you know it will be written CD0 + CD α and α + CD δ e x δ e + CD q x qc/2 v right similar Cm can be written as Cm 0 + Cm α x α+ Cm δ e x δ e+ Cm q c/ 2v so they are called aerodynamic model yeah so already I have written here so this these things are your aerodynamic model force and movement coefficients right.

So here you see that all the variables like CL CD and Cm they are well structured right so this is the structured model where CL 0 CL α CL δ AND CL q they have their own physical significant and they are called aerodynamic parameters so if fewer diameters like CL and CL α Cm α which is related to your α or may be CD α that talk about the stability basically these things about talk about the stabilities so they can be called as stability derivatives also your CL q CD q CD q will be very week parameter and Cm q so that talk about the stability so they can also called as stability parameter or stability derivative okay.

And the parameters which are related t control here control is δe elevator control here so CL δ e CD δ e and Cm δ e they are called control derivative now why they are derivatives so it is quite clear from the nature of the parameter because if you see.

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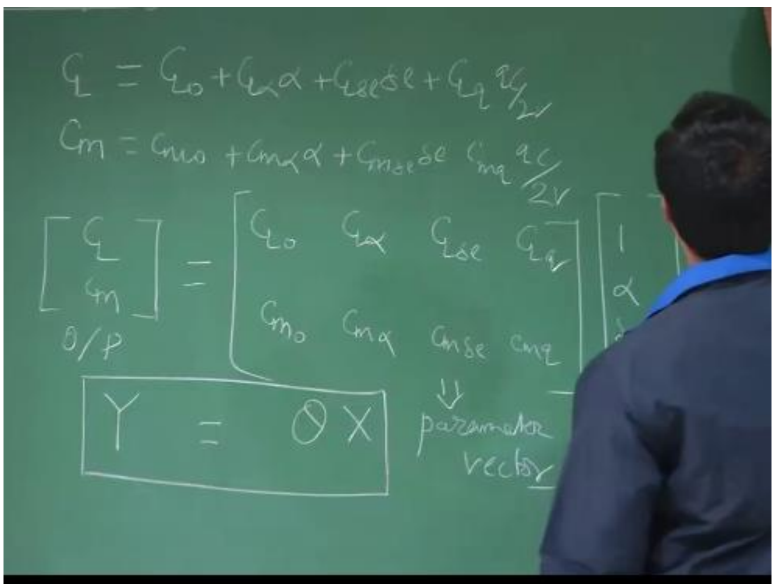

CL α ∂ CL ∂ α v it says the change in α what about change you observe in CL like the change in CL because of change α this is defined by CL α right so accordingly everything will be like that CL δ e change in CL with the change in δ e so this is called your control derivative right so once you have the structured model right so they again become a parametric models.

So today I will talking about the A is said earlier about the aerodynamic parameter estimation and I will show you with examples and this is also the demonstration of the technique what we have learn in previous class right so I will take two parameters two variables sorry force CL and movement Cm pitching movement coefficient.

So you now you know the model CL is CL0 + CL α into α + CL δ E into δ E + CLQ X QC/2B yes and CL0+ CM α times α CM δ E times δ E CLQ QC/2V okay so it can also be written like this yeah, CL and same I will write in a vector form like this and if you write all the parameters here like CL0 CL α next is your CL δ E and CL q and this you have CM 0 CM alpha, CM δ E, CM q.

1 α δ E and QC/2Vyes so this is the same equation actually represent as in terms of matrix so now if you see it will look like this y about the output force and moment co-efficient and if I can call this as a θ matrix which is a parameter matrix and input if I represent by S so this is again it is again in terms of this very well known equation right, y = θ Xa linear equation right, so now this had become a candidate for less square we have seen in previous class.

How to estimate the parameters from the linear model which what like there yeah so I will demonstrate the principle of less square on this example right, so now let us talk about the flight data right because now as I said this is output this and here this is input right, output this is your input and this is parameter vector matrix, and you want to find those aero dynamic parameters so this is the problem for this in this example.

Now how to estimate those thing before that I will be talking about ho9w do you get these outputs CL and CN, and how do you collect all the inputs right so as we know there is no sensor which can directly give you the force and moment co-efficient during the flying condition so CL and CN we cannot directly measure right, so now equation comes how will you get CL and Cm.

Because now we are talking about the system identification or parameter identification it is about the input and output so how will you generate the output or how will you get that so it means can we do something do we have other side variables which is related to force and moment and from that we can get this force and movement co-efficient so can we do something like that, so let us see again CL and CM.

And as you know most of the sensor which is in build in aircraft which is mounted in aircraft they give you the information in terms of body frame and for in your information you know CL specially this lift force they are in wind frame not in body frame so you need a conversion from body frame to wind frame right so CL how do you establish the relation like how you will get Cl.

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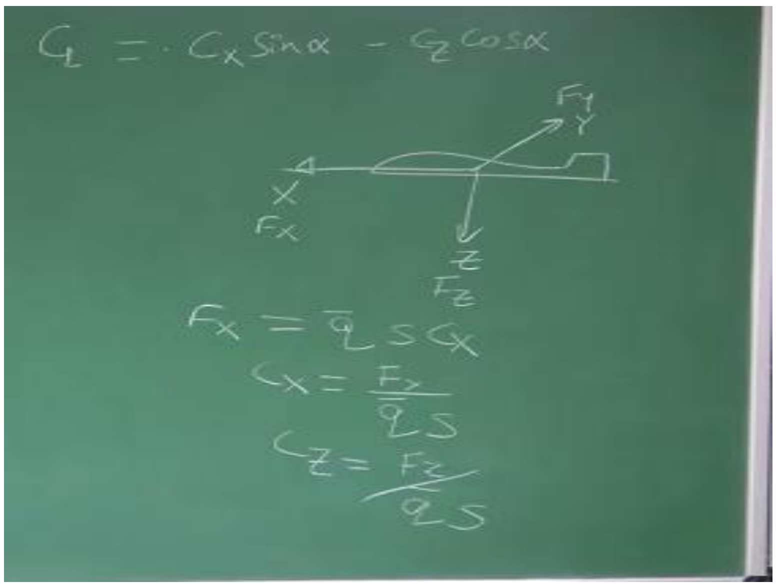

So CL as we know CL can be also written as C_x Sin α right minus Cj Cos α right so now they are also force co-efficient in body frame and you know that actually you see that the aircraft if your x is related like this if you define XX is true also know y here and Z so the force in this direction will be FX, FY and FZ, right and again you know force can be written as Q bar then represent a time S x Cx.

So you basically get Cx from this or Cz from this dynamic pressure times surface area right so you know the meaning of Cx and Cz right so just I have told you so that you can get the standing.

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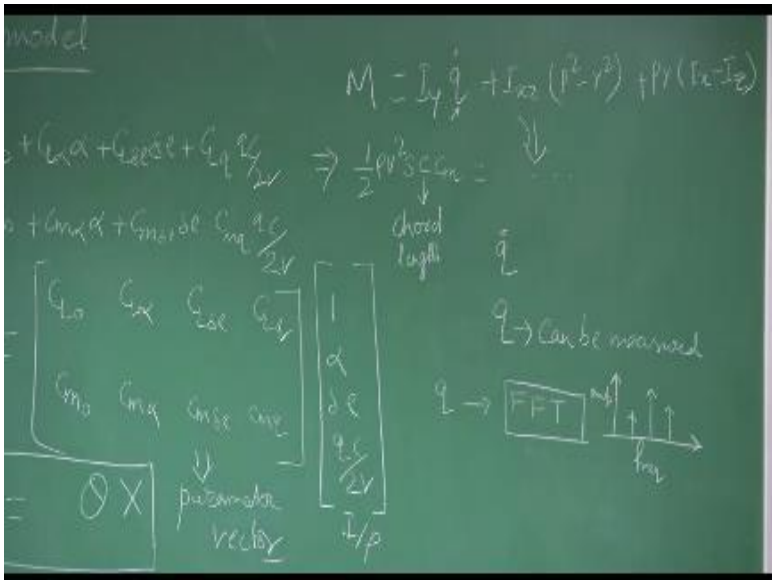

You can align your understanding whether right but again we do not have sensors directly which directly measures CX and Cz then Cx as you know it can be written as mass times acceleration actually expression in X direction minus trust hydrogen with FE so please remember this is your trust force right, okay. Divided by dynamic pressure time area but if there is inclination angle with the incidence so this is called engine insertion angle so that component also will come here.

So that will be Cos σ T right and this is your engine inclination angle, so that I though which we will be talking where the value will be 0 so this is not a very dominant term but to therefore the sake of completion of this equation I have written this so that you should remember. And see that you will get maz, m time az minus not will become plus Fesin component on this angle, okay.

Now we know from this equations we know that data for ax trust force dynamic pressures, surface area this every things are know rigjt, because IMU will give you the details of acceleration and from the geometrical information of the aircraft you will get S and sensors are there to measure dynamic pressures you will get the dynamic pressure, right.

And you will also get az so in this equations everything can be measured through the flight right, so once you have measured details about the variables you will cx and cz and by knowing cx and cz you get CL right, so that is how you will get the CL lift course coefficient. Now coming to the PC moment coefficient this one how will you do that let how will you measure that or how will you derive from the flight variables just write the moment equation, so all of you know moment can be written like this $M=I_y\dot{q}$ right, $+I_{xz}(p^2-r^2)+pr$ times I_x-I_z so this is the moment equation right.

And further you can write M as $1/2\rho v^2 sccm$ basically this is your chord length c we can write c I will just use this notation as a c and $\bar{c}$ interchangeably so here I have written c so let us write c, but here please understand this is your chord length, okay right and then the same equation okay, so here you can get same by knowing all those variables. So you know the I'm contains the accelerometer, gyroscope, magnetometer so from the gyroscope you will get pr and q everything and from the geometrical details you will get Ixz, Iy and Ix znd Iz to you see parameters you will get.

Now this is the only thing where we should look for it is $\dot{q}$ because most of the aircraft will not have the sensor which can directly measure $\dot{q}$ or in our aircraft we do not have the sensor which can directly measure $\dot{q}$, so how will you get this $\dot{q}$ you know $\dot{q}$ variable right, so but we know like we can measure q, q can be measured or the data is evaluable, okay. So from any numerical difference in technique if you have those q you can get that $\dot{q}$ this is not a problem.

But problem will come when your q is not accurately measured it has some amount of data noise some amount of noise in this signal and if you take the derivative of that signal it will further amplifier a noise so the signal will become even worst, and once you do not have the accurate $\dot{q}$ you will not be able to get accurate output Cn and once you do not have the accurate output then the estimated parameters will not accurate, so that is why we have to be very careful the kind of data you are selecting or we are feeding in this algorithm those data should be very accurate originally accurate.

So now questions will come like how can we region ably estimate or obtain this $\dot{q}$ from q right, so now suppose there is small amount of error or some amount of error in q right, then we need to identify the error or we need to select if there is any presence of error so how will you detect that so you apply FFT from the FFT it is called fast Fourier transformation right, so what it does like it will give you the frequency components present in the signal so if you pass through at and then if you see this amplitude verse frequency draw so it will generate some spics right like this.

Wherever frequency terms are there you will get those spics, so I think let me tell you about the little bit about the FFT, so that you will be able to appreciate okay, you might not have learned it right. So before that so suppose you have a very clean signal or sinusoidal signal like everyone have seen this signal right, sinusoidal signal, okay.

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So sinusoidal signal if you see mathematically it will look like this right, $2\pi ft \sin\theta t$ so here this signal have only one frequency component which is f right, and if you do this fast Fourier transformation or Fourier spectrum analysis of this signal through fast furrier transformation the smart this would I was trying to discuss here then it since it has only one frequency component it will generate only one spike right.

And it was add frequency of f so you will get the spike at f with the same magnitude or amplitude you can also see the amplitude of the signal if it has two frequency components then you will get two spike generate for corresponding frequency right.

So this is the idea of your fast furrier transformation okay so as I said if q has noise then usually noise enters through a very high frequency atoms so you will get to know once you do the spectrum analysis using FFT and if you have seen instead few glitches or few spike at higher frequency it may not be so big in the amplitude it will be a small maybe in multiple spikes at higher frequencies then this will be dominant one so it may look like slightly bigger right.

So from here you can easily identify that noise high frequency noise entered in the signal here and it will look like this these are the high frequency noise usually noise is higher of high frequency you will be able to identify the noise in the signal by doing this first data formation.

Now you can easily identify these are the frequency which we do not need ion the signal so you can eliminate those higher frequency noise terms but designing a suitable low pass filter right so low pass filter what it does like low pass filter LPF it rejects all the higher frequency component terms right all the higher frequency terms and it will give you the signal which is related to low pass like a flow frequency.

So you can reject all those higher frequency noises ort higher frequency term by employing the low pass filter and then you can give the information of cut of frequency in the low pass filter from the FFT analysis of this Q signal right and once you got the q signal clean then you employ any numerical difference in technique to get this q dot right so now you got the q dot so in this equation you have the information about q dot now you already know about inertia parameters you already got information about p and r from the magnet like from the IMU through a gruel scope.

So all the flight variables which are use in this equation or known and from that we can get this CM so that is how you will get Cm right, okay so ion the output you got CL and CM and you can directly measure α from the α sensor from flight test you have the information about the elevated deflection from the sensor so this also it measure and q definitely you know and C is your code line and velocity is also measured.

So now ion the set of input and output everything is known so what is not known or what is unknown of this, this parameters we do not know about Cl_0 CL_α CL_δ and CL_q and Cm or not Cm α CM δ Cm q and it is now it has taken the structure of y = θ x right so you can think of using a least square in this problem and then you will get the θ so how will you do that like from the simple technique you can y = θ x now right so how will be the how will your estimated parameter will look like this so look like in this thing again least square technique says like it minimize the summation of the error if you remove the error part what I discuss in earlier class.

So what you do like you multiply first multiply with x^T here or both the side right so now your θ will become y x^T times x x^T so that will be your estimated parameter okay, so what I will do like we have gather the data on unsure graph so I will show you the how data has been collected so you will be able to see all the information about the flight variables α velocity and qrp and everything and from there we have derived CL and CM so what I am do like.

I will show you through the slide so that will be able to appreciate more once you see it visually and then from there employing this least square technique we have estimated this parameter vectors θ there you will get to know about all the CLL α CM α CM δ here whatever actually parameters we have talked so all the parameters can be obtain using this re square technique so from the Mattel of simulation we have done ity I will shoe you in this slide okay.

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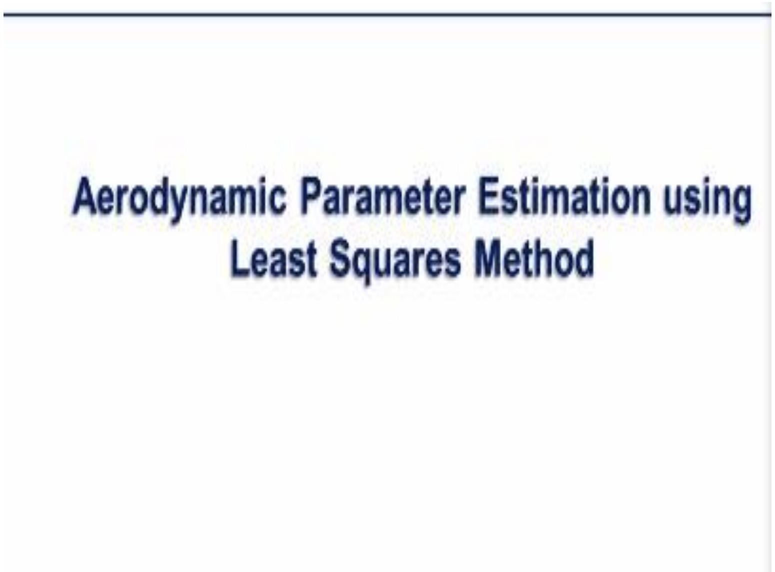

So I will show you the results which we got for this parameter estimation right so yes.

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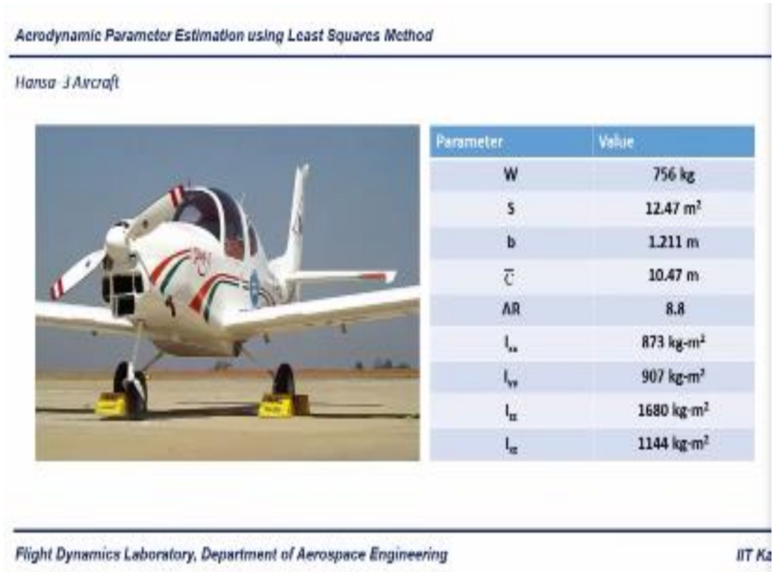

Parameter	Value
W	756 kg
S	12.47 m^2
b	1.211 m
$\overline{c}$	10.47 m
AR	8.8
I_{xx}	873 kg-m^2
I_{yy}	907 kg-m^2
I_{zz}	1680 kg-m^2
I_{xz}	1144 kg-m^2

So this is air craft Hansa 3 air craft we have use to gather the data and if you see that it is a twin seated air craft research air craft so we use this for our research work. And then where it is 760kg as 12.74 meter square in span is 1.221 meter cord length is 10.47 meter as the procedure 8.8. And inertia parameters are like I X is 73kg meter square IYY is your 907 kg meter square IZZ is 1680 meter square IXZ is 1144 kg meter square.

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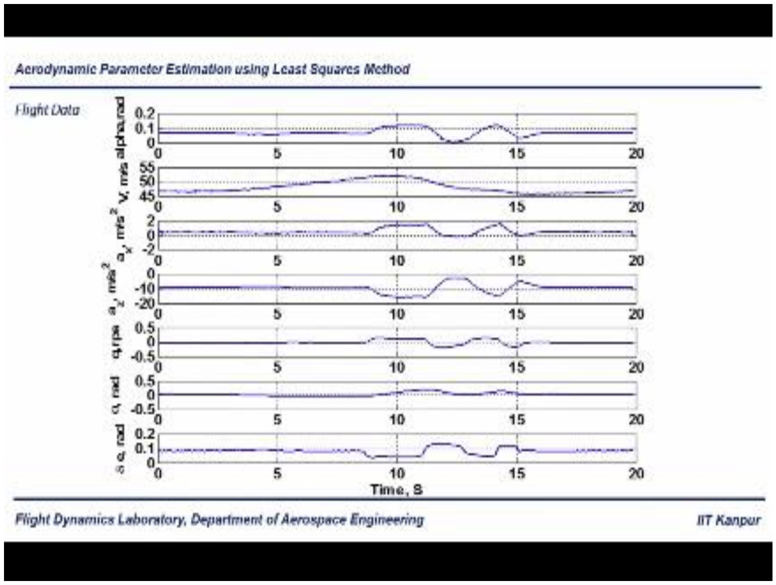

This is your geometrical parameters of this hansa 3 aircraft yes. So these are the longitudinal data we have gathered during the flying condition you see here elevated has been deflected so this is a standard signal which we say 3211 yes. So if can see the elevator which is right at the bottom here and we tried to make it like a 3211. 3211 says like it should be 3t time's upper down 2 t times if it is down then it should go up then three t times down and up.

So this is over flee 3 2 t t and t. so we tried our best to get those 3211 signal through this elevator to side the short period more right. And then this is the deflection in θ 2 ax a1 ay az ax will ascertain alpha with the change in the with the change in the elevator deflection so you have seen the changes in the alpha so it is quite following. Right like we have a deflection here and corresponding to here started seeing the deflection in the table alpha velocity.

We tried to velocity for short period should be constant but it is stably constant in the accuracy of may be 5 or 7 meter per second it has gone off and you see the deflection in the acceleration data the x data it is of 2 meters 2 meter per second square you can see roughly and sz also changes may be -15 to it has gone slightly lower to 0 and there is a change in q right.

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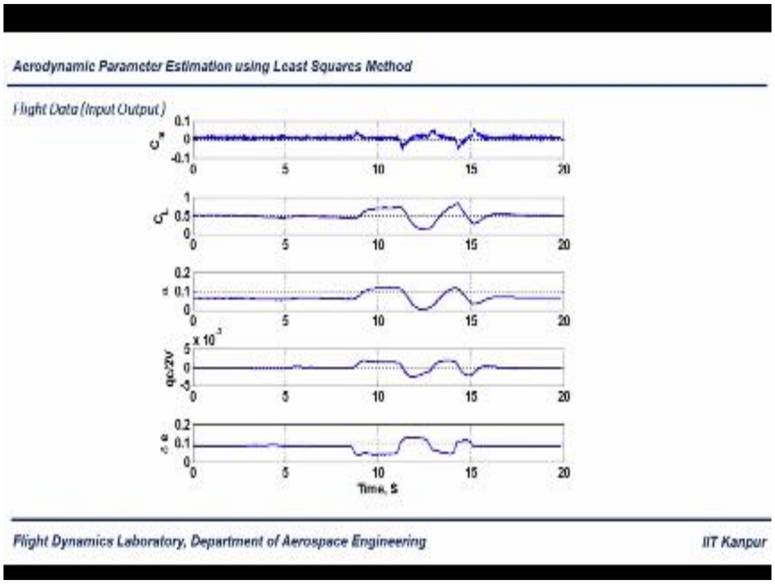

The digital in radian per second units are already written here so you can see so you saw the changes in the q and θ and other variables are not listed in this but we collect all those data fight data which has been discussed during this lecture right. so I have shown you those important variables through this graph right next thing was which I was talking about the longitudinal input and then output I have talked about two derivates so in this graph. You see like these are the four set moment co efficient.

So Cl is your left for this co efficient cm is your pitching moment co efficient and which we call as a set of output data cl and cm. So the data has been collected from the 20 second you can see here in x axis it represents time in second. And these are the state of input data right so these are the eqce/2v alpha for the same order time so. Now we have a set of input data and output data once you use this technique or least square method then you will get parameter like this.

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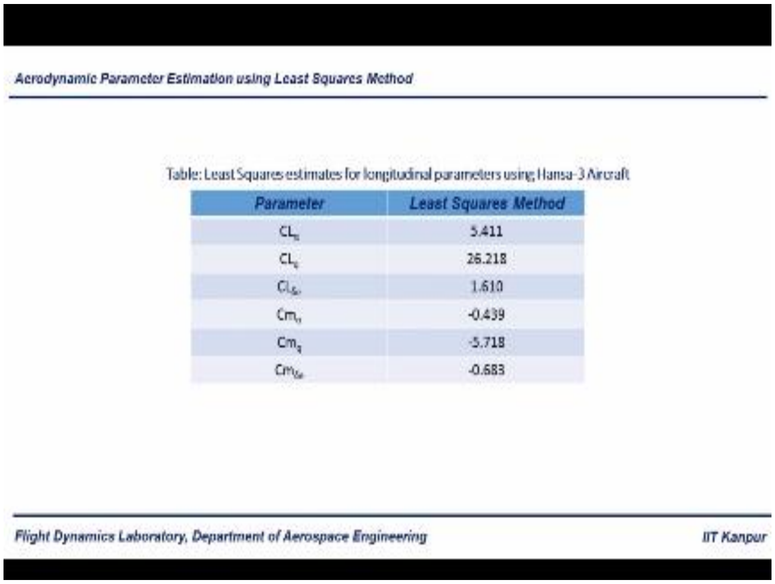

Table: Least Squares estimates for longitudinal parameters using Hansa-3 Aircraft

Parameter	Least Squares Method
CL_{α}	5.411
CL_{q}	26.218
CL_{δ_e}	1.610
Cm_{α}	-0.439
Cm_{q}	-5.718
Cm_{δ_e}	-0.683

So these are the few parameters which I have shown you here they are usually strong parameter cl alpha clq cl is cm alpha cmq cm e yeah. And so this least square method gives those parameters in one short computation so least square method is about the single shot computation and it is very efficient here the value of clq might not have come correctly basically q derivatives we have not exacted the complete dynamics like so maybe there is a slight change in the in cq derivatives but other variables are quite okay in comparison to internal values.

Yeah so now in the next lecture the next tutorial we will be learning about the same parameterized estimation technique or aerodynamic parameterized technical from the different methods so here we talk about least square method right and then in the next class we will learn about method which is based on the black box model right and then you will be able to appropriate the method of method which is based on artificial method. And then we will have the comparison of the results from the least square method.

And then you see which one is better or they are comparable or what is the confidence interval of those methods so thank you so much.

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