Indian Institute of Technology Kanpur
National Programme on Technology Enhanced Learning (NPTEL)

Course Title
Introduction to Experiments in Flight

Lecture-18
Parameter Estimation Using Least Square Method

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Hello I am Ajith and once again I am here to assist you in this course of introduction to experiment in flight okay. So in this tutorial we will be learning about the Parameter Estimation using aircraft data. So the experiment which we will be learning today in this Aerodynamic Parameter Estimation using list Square Methods.

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Here you see these two terms got introduced first time maybe parameter estimation and list square method. So I will be talking about; what each parameter estimation, why it is required and how to do the very impressed emission and some of the techniques? We will discuss so here I

will discuss about list square method, so we will learn what is the square methods and how it is applied for parameter estimation with some examples.

So I will show you some example, so let us start with parameters. Parameter Estimation ok, so what is parameter Estimation? How can we understand it, how can we define it? So parameter estimation is nothing but a subset of system identification, so you can write is a subset is some identification right. So here you saw this term system identification so what is system identification. So system identification each about the identifying the model with the help of input and output data right.

So let us understand this so suppose here you have set of input data so I will write input and output data sets and you see the models right.

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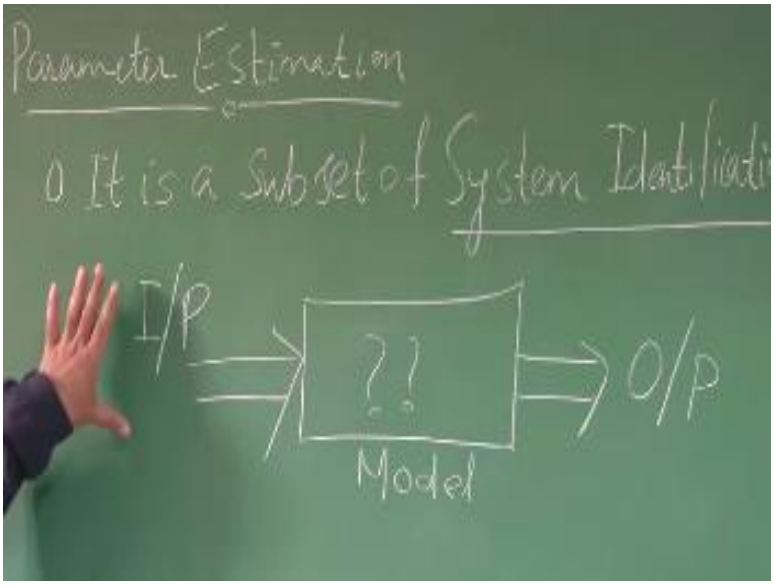

So once you have sets of inputs and output and you are trying to identify the model, so this is basically a problem of system identification now. So once you got this model and if model is structured in terms of unknown parameters then identifying parameters can be a problem of yours parameter estimation. So how will you define the parameter estimation, it is like this if

model is structure is model is structured with unknown parameters then identifying parameters is Parameter Estimation okay so this is how you can define the parameter estimation.

Now you can take some simple example also like a very basic model I will just show you it is called the spring mass damper system. So all of you know, it is expressed like this so MX.. + BX. + KX equal to first of all force right here. So each rotation will have the standard representation this is mass, this derivative is called damping coefficient side, and this is spring constant. Here your X is lets write in terms of this displacement and FT force okay.

So what we observe in this kind of structure this is you see like this model is structured with the in the form of parameters, so here M B and K can be the parameter for this model. So how will you argument this problem in the parameter estimation, here you can say suppose now ft, ft is input and the displacement which observes output. So with the help of input and output means for data for the force, applied force and displacement if you can identify mass b & k then it can be a problem of parameter identification or parameter estimation.

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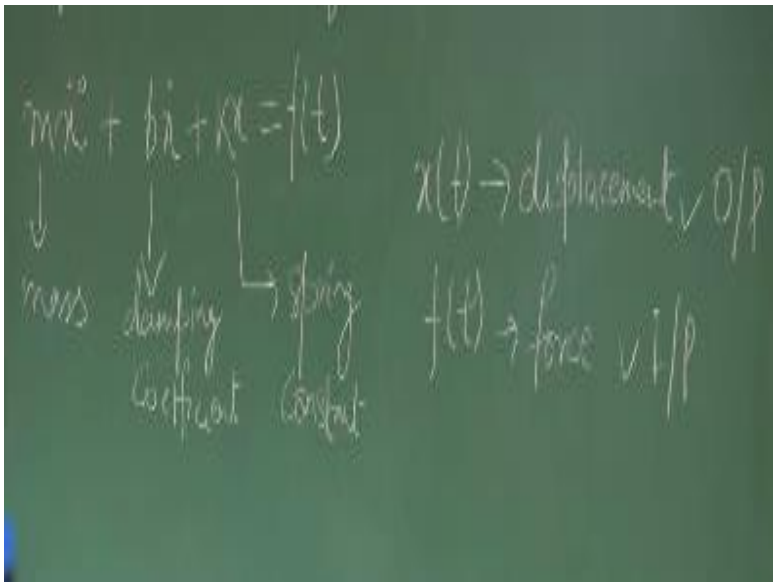

So this was a simple example and now next thing come why do you need parameter estimation so this was about what is parameter estimation, so next thing comes why parameter estimation? Let us read PG which required okay. What is the motivation behind parameter estimation? So as we know most of the design algorithm and if you talk about control design, guidance designs, for the elusive application and design of aerodynamic database.

If you want to detect the fault in the system or you want to identify the false then you need an accurate model and accuracy of the model depends on the parameters like if model is parameterized, then accurate estimation of parameter is equally important in that way we should know what are the accurate parameters and then with that we can have the accurate model.

So next thing now will come how to do the parameter estimation? How to do, parameter estimation? So we talked about what is parameter estimation and why it is required, now next thing comes how can we do this parameter estimation right how to do the parameter estimation? So various models are available if you see the literature, then model can be classified into categories we talked about the system identification, then observed model can be of two types.

So first type is called phenomenological model, which can also be called like physics based modern, physics based model right and then next stage is black box model. I will tell you the difference what is difference between these two methods of these two class model, black box model right. So under physics based model there are several model, methods available for parameter estimation actually.

So let us try to understand what is physics based model and what is black box model, so in the class of physics based model where we have the basic understanding of the physics we can derive the system with the physical understanding of system, so this will fall in the category of physics based model. Next class of model black box model where we do not know anything about the system like we have no knowledge about the system we just have inputs and then object output but system is like a black box so that is why it is called the black box model.

Now if you want to do the parent estimation of these two categories to several methods are available. So I list few of the methods here so under this category they are method which is called equation error method, next is output error method and other method can be filter error method okay.

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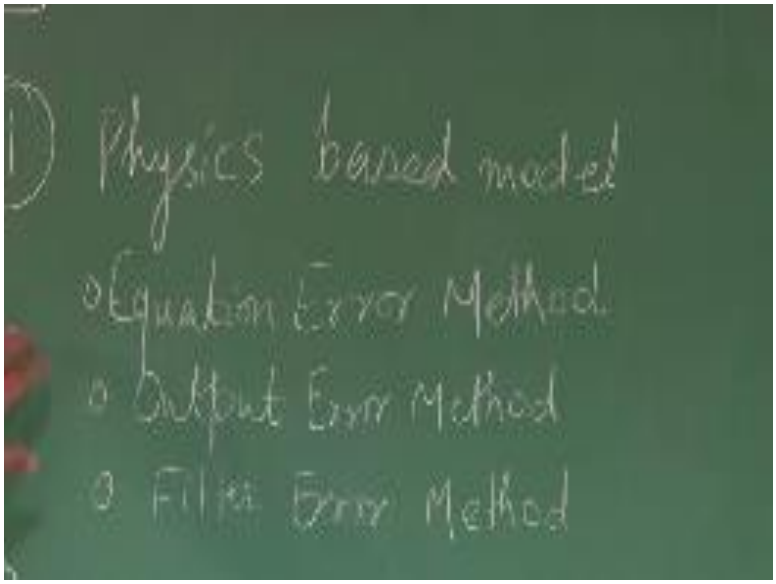

So in this category this are the methods, it depends like what kind of system you are dealing with so application of this method depends on the system also complexity of the problem. Suppose the data which you acquired input, sets of input and output data, they are kind of very accurate in most of the aerospace application we see instruments are of very high quality so you don't see much noise in the data.

So there you can apply this equation error method assuming that there is no noise in the instrument, instruments and in the process, so when there is no measurement noise and process noise so this method is preferred. Suppose there is just some error in the instrument like your instruments are not functioning well it means you have measurement noise available.

Then output error method is preferred their output error method can be a best option this next class of error filtering method where you have noise in both the things like process, you have process noise as well as measurement noise, then you can go for filter error method right and next class of model is of black box model where as I discussed as I said like we do not have the physics understanding for this model.

So in under that several method can also be listed so, 1 I will just write few of the method. So first method can be of your artificial neural network, second can we have fuzzy logic okay and third can be combination of these two methods. So it is said neuron-fuzzy and other method can be your support vector machine SVM and many more methods are there we can also write about machine learning based method.

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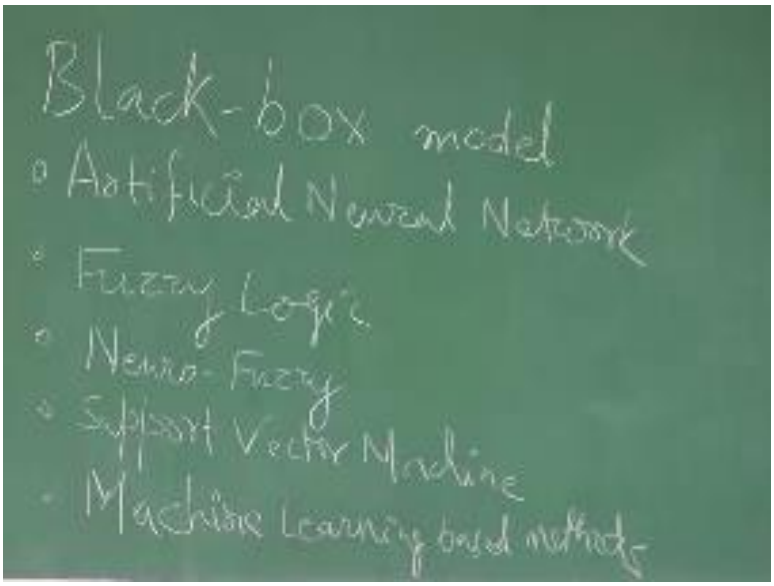

So these all are your black box model. So I will not discussed everything in the detail, so what I will do like I will take one method from this class which I will say least square this is also called regression analysis or least square method will fall in this category equation error method. So this and from this I will take very popular method which is proposed by Professor Ghosh and it is called delta method.

So we will learn what Delta method is and how it is used to estimate parameters right. So start with this least square method, so let us see what is the square. So this is least square based methods, so I will write least square estimation. The idea of least square estimation had been proposed way back by a very renowned scientist Karl Gouss in 1809. So what is the basic principle of least square estimation?

Basic principles, what actually do we mean by least squares? It has been a very popular method and is not limited to the application of parameter estimation of system identification it has been widely used for various kind of problems right. So least square estimation see is it minimizes the sum of the square of the error between measured output and model response here. See if the very catchy term.

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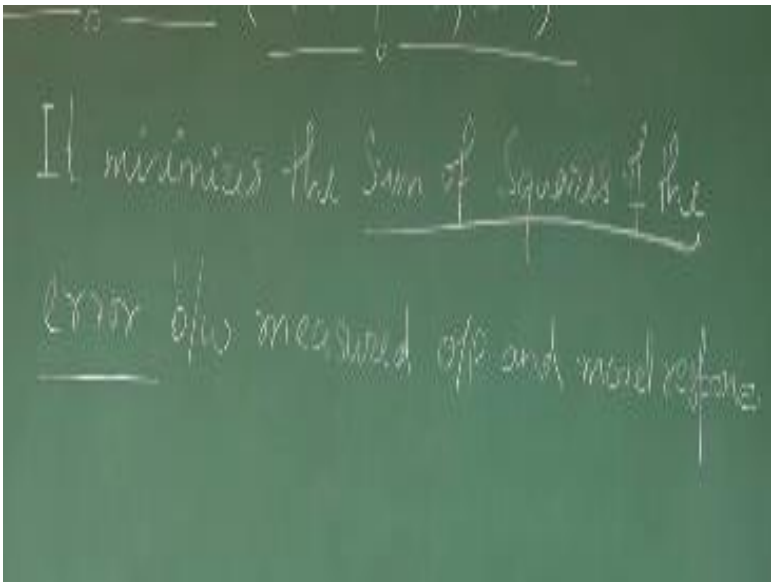

It is like about minimizing the sum of squares of the error. Now what understands behind it you like, what is the inference of this statement? It minimizes the sum of squares of there and while sum of square of the error why not absolute error? So let us try to understand that, so if you have some data I will start with a very basic example ok like, we can give the name like Y or X ok. So here what we are trying to do like we have some amount of some data like this.

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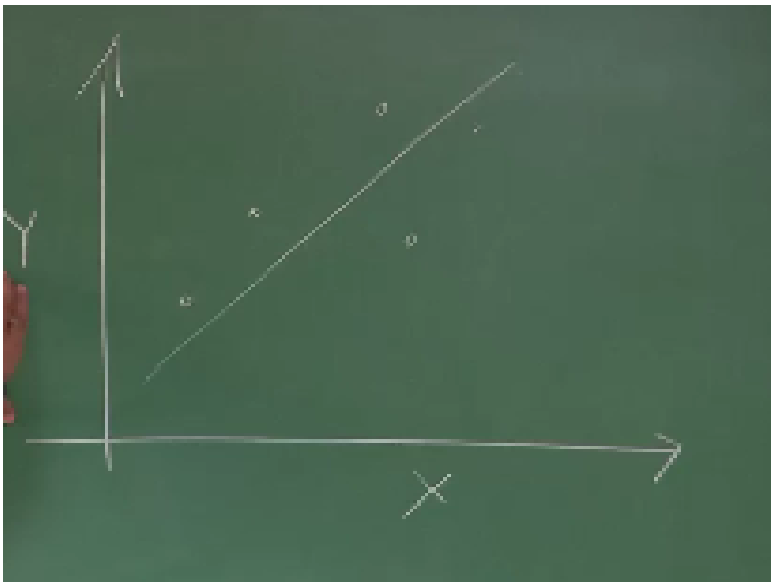

We are trying to fit a curve, a best fit which minimizes the error actually so for these values what could be the best fit? We are trying to understand that with the concept of least square, so care least square estimation help us to find a best fit for this kind of problem. If you see let us try to make the inference of this statement or with this example.

So these are the error, with respect to each data samples of data point. So once I say squares of the error then it is something like this so you make a square here hope this is clear. So now you see that how many squares one, two, three, four and five. You five different squares now one, two, three, four and five okay.

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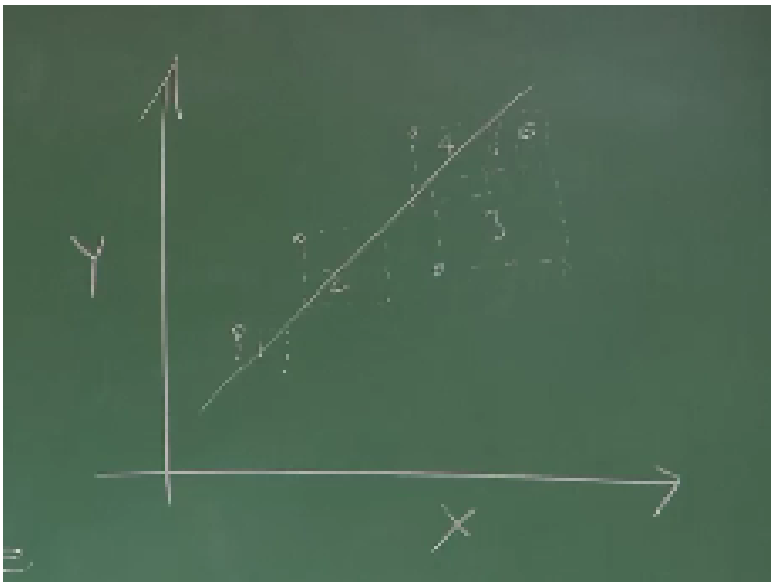

So summation of squares means like you are adding all the square; square 1 plus square two, third square whole square and five square sum of all the square okay. It is sum of all the squares yeah.

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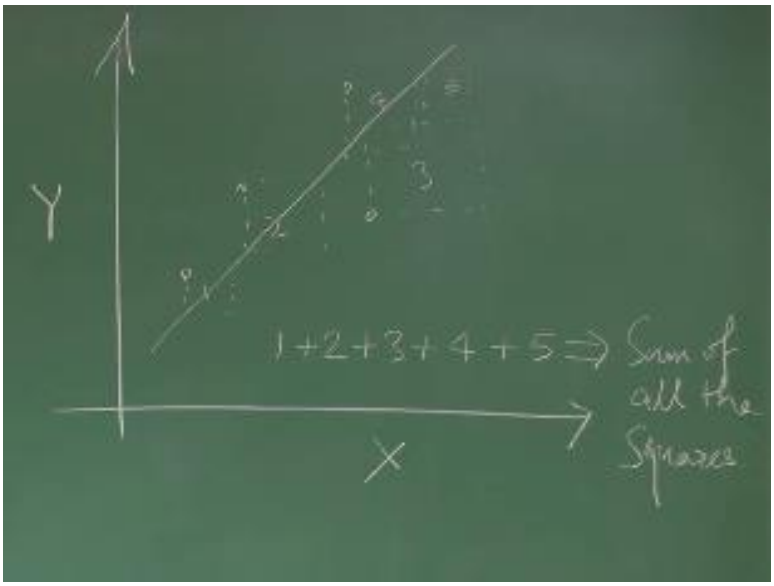

Now you want a line, a best linear fit for which summation of squares will be minimum, so that is a philosophy of this least square estimation. So least curve works like that it gives you the line corresponding line of best fit line for which summation of all the squares will be minimum right. Now what, what I was talking about like why it works on summation of squares of the error not absolute errors and before that if you see the error here.

So once you do the square, what is have it does like it gives you always the positive value. If we do not do that, if we do not do the square what happens like it can give you some of the negative errors some of the positive error. So once you accumulate all those errors, then what happens like it will cancel each other, some big error in positive side can cancel the bigger in negative side.

So it is very much important to have the positive error. Now next thing comes why square is needed? If you take the absolute value of the error, that also can give you the positive value right. It is not necessary that we have to do the square, so what the theory behind it is like if you ask any calculus maybe professor or the students has taken the course.

If you want to minimize something how will you minimize it like so they will say like what you do you just take the first derivative equal to the 0 and you get the solution you get the minimum value corresponding to that. So it becomes very much easy for us or to deal with the calculus once something is in square, if you do not make it square if it is absolute value then it may be a discontinuous derivative.

So it will very difficult to do the analysis on this discrete derivative, so for that it is very much useful and easy to work with it square term. So the objective function will be in terms of square which I will be talking now. So next we will see there are mathematical formulations, so now I think this should be enough for us to understand about this principle yeah.

So for least estimations we will talk about the mathematical formulation. So now how will you understand it mathematically or how it works, so we will see the derivation of this least square estimation. So now we will be learning this mathematical formulation okay. So suppose you have n samples of data for the output, so I will write yi1, yi2, like this you have n samples. So these are basically your output or dependent variables of n samples.

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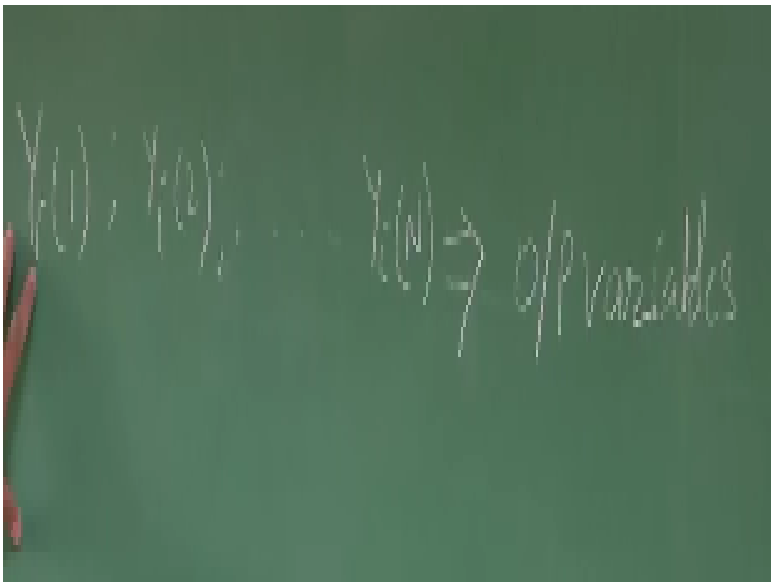

I write here y because I am assuming this is high output, output can be of like multiple or two can also be possible, so I am taking the highest outputs right corresponding input variables you can write like X of 1, X of 2 and X of n this will be your input variables are independent variables.

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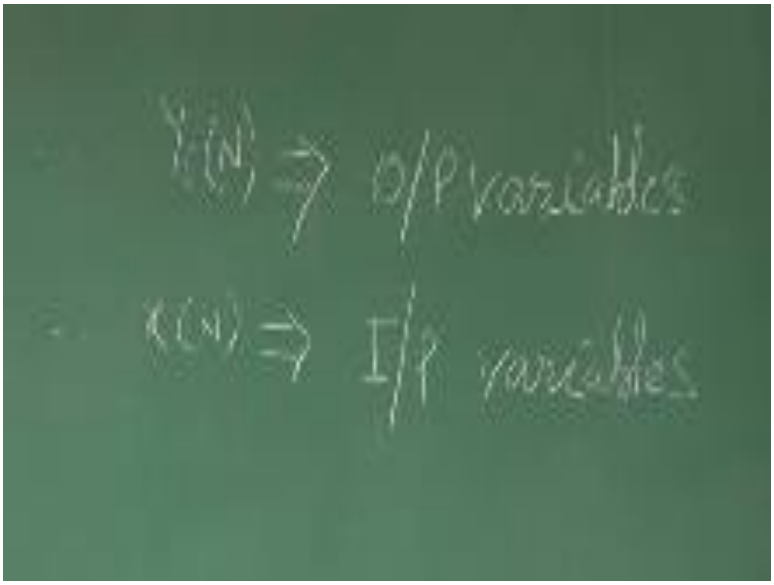

So see now slowly we are progressing through this idea like you have input variables or input samples and output samples and here X is a vector which have X 1, X 2, X and Q. So it has is these many inputs you can write like that, now you make our simplified assumption here assume output are linearly related to input yes. So how will you write you can write YIK= , so you have to introduce now the constant team Θ1X1K + Θ 2X2K like that Θ n cube X and Θ a with some error epsilon k this is called equation error all right okay.

So in the matrix form, you can write like this YK = x Θ (K) + epsilon K ok. Now if you generalize it like if you write sample the sample so Y1 corresponding Y1 will be your extra charge one Θ of 1plus ok and the sample will be in X transpose n to the samples and this okay. So this is how you will right.

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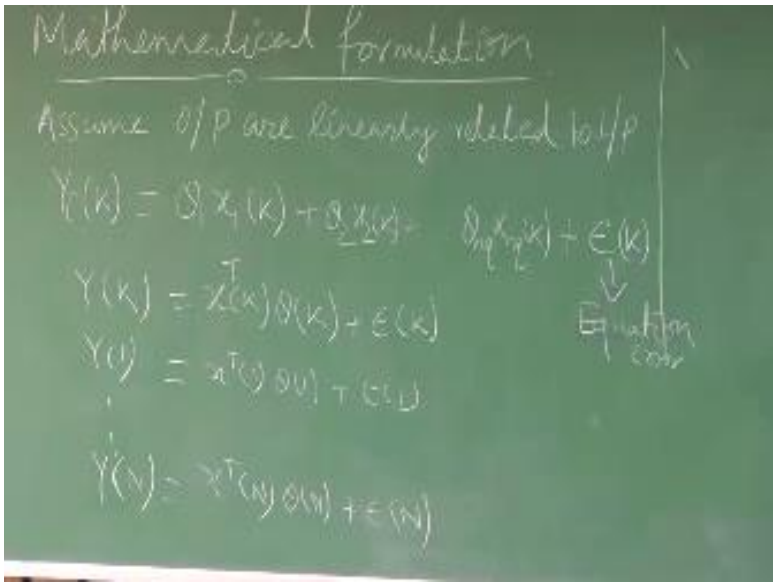

Now further in a general form or in a matrix form again you can write Y= X Θ + the error yes. So this is our platform rotation and X matrix will have x1(k) and I will not use k so x1 of first one, X2 of first answer x and q / x 1, then X1 second sample like this is XNQ.

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Now you know Y set up all the outputs X is this and equal parts all the associated errors for each samples, so this is the thing now we have to estimate this Θ which will be in terms of vector Θ corresponds all the Θ1, Θ2, Θ till NQ. So how can we estimate this Θ with the philosophy of least square. So what least square is a state with minimizing the error summation of squares of the error, you can find the estimates.

So if you write the cost function corresponding to that so J Θ can be written like half of K equals to 1 2 n e transfer ok, so what is e epsilon here error. Error is your Y- X Θ. This is no so now we are trying to minimize the summation of square of these errors right here k sorry it should not be like this first square of error right. So it should be at this is one fit square of the error now you can further simplify you can write epsilon transposes epsilon right and once you expand this with this relation.

So it will become Y-X Θ transpose right, so you can also write Y transpose minus theta transpose X transpose times Y minus X beta. Now if you take the derivative with respect to theta. So J(Θ) / Θ right. So here I have half, half, so what happens like what will happen like once you take the derivative of this because you want to minimize you have to take the first

derivative so it will give you minus of y transpose X plus theta transpose X transpose X and how do you do the minimization you equate this is 0 right. Equating with zero what you got this one

So now if you take that transport both the side then it will become your X transpose X transpose and theta equals to here if you take then it will become your X transpose Y some taken transport both the sides like this yes. So with that you will get theta equals to X transpose X inverse times X transpose Y okay, so this is the estimated parameter okay.

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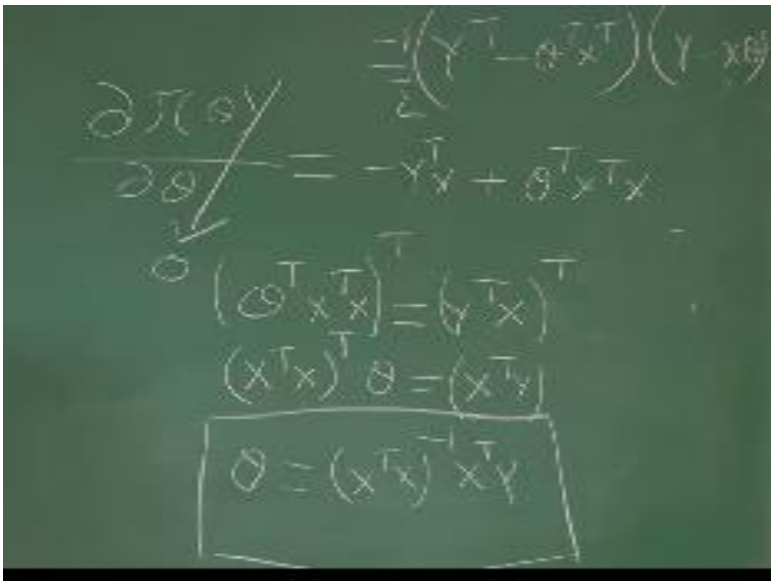

So what we got this parameter vector theta, theta we got X transpose X inverse times X transpose Y, so this is your estimated parameter vector or matrix with the philosophy of least square estimation okay. So if you want to verify it actually then as you know this equation y equals to x theta plus error, if you make this error 0 then it will become your y equals to X theta okay.

So indeed equation what you do you multiply pre multiply with x, transpose x + 4 y and to become X transpose times X theta right. Now theta data will become X transpose X inverse times X transpose Y right. So this is a theta you got with this algebraic manipulation, so which should be like this so you see both are similar.

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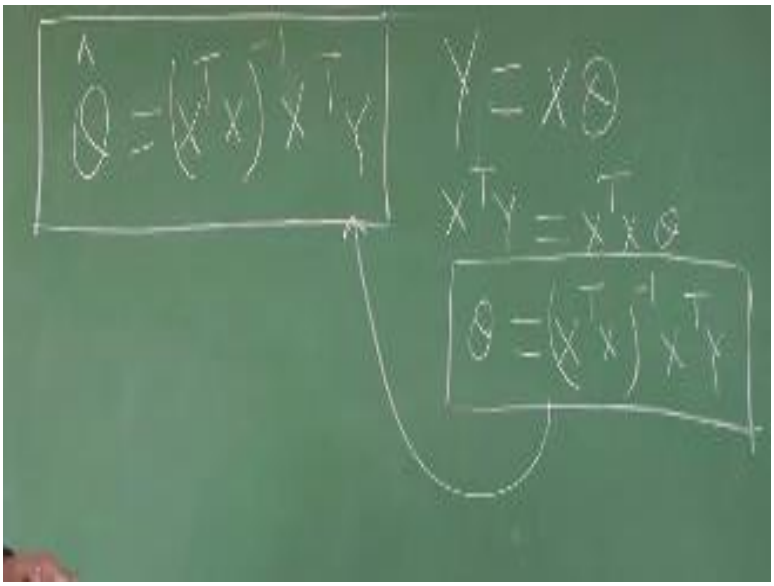

So now you got the expression for theta with the least square okay, so let us try to understand the application of this method again it can be a exercise problem for you. So for the same example I am taking let us say Y=5x +4 so, so this is the mathematical equation linear equation right. So the structure is what Y= mx+ c right.

This M represent flow and see each intercept now to exercise this method what how can you generate how can you make this a takes part of your exercise, so what you do you first generate thousand samples may be 1000 samples of data for this because here we know slow pitch five and intercept which force, so with Y=5x +4 you take different value of x, so you will get corresponding Y.

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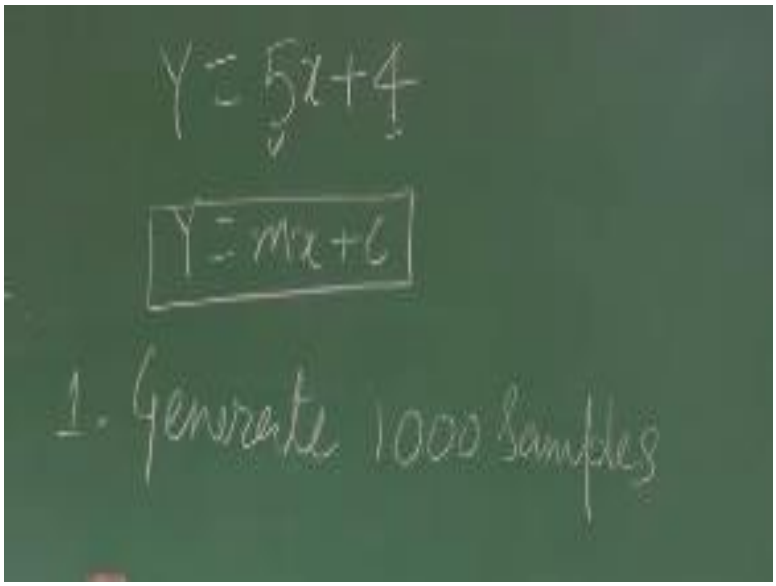

So gendered thousand samples of input output data right, so it means now you have the data input data and output data and then the structure is like this, with that now you will try to identify those parameters if I am assuming these are the parameters m and c are the parameters can you find m and c correctly and second to check the efficiency of this algorithm you can add the noise in the data.

So you add noise to the data all right and then estimate slope m and intercept c right, so how will you proceed with that like now we will try to develop the understanding how this kind of exercise or this kind of problem can be broken into this format right. So if you see Y= MX +C. So you can also write y equal to x1 and M & C, so for the problem we will have this 1000 samples of the data so you will definitely will be knowing about X and you have this Y data corresponding output data.

So now you see this is again becoming the format of y goes to theta X make it bigger and this is what you want to know right, so this is a theta so this is again in the form of Y=XΘ same thing.

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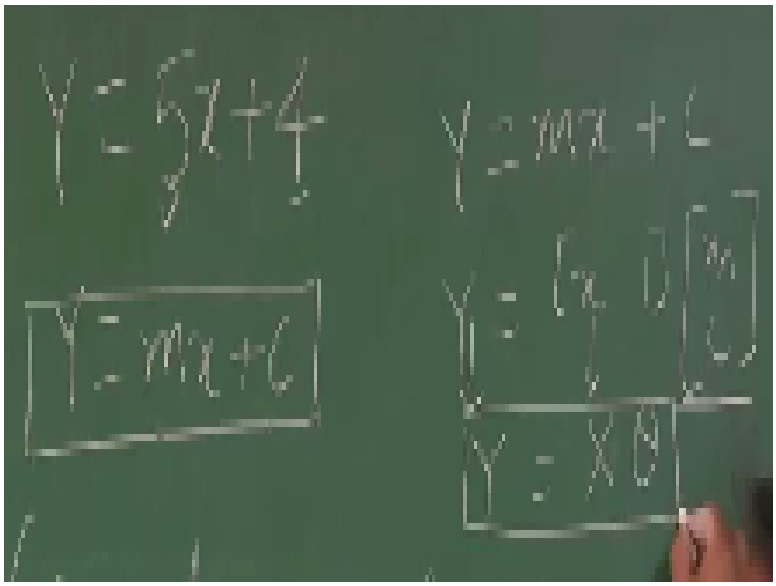

So if you want to estimate theta parameters from this equation or for this data then use this least square estimate value you will get slope and y-intercept and you add the noise of different degree five percent, ten percent, twenty percent and you see how this square is performing for different sets of data. Once you raise the level of noise if you make it higher, so how your estimated values are going to be, is it going to the same or going to be different so you can do it by yourself. So thank you so much.

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