Aircraft Dynamic Stability & Design of Stability Augmentation System
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Module 2
Lecture No 09
Vector in Rotating Frame

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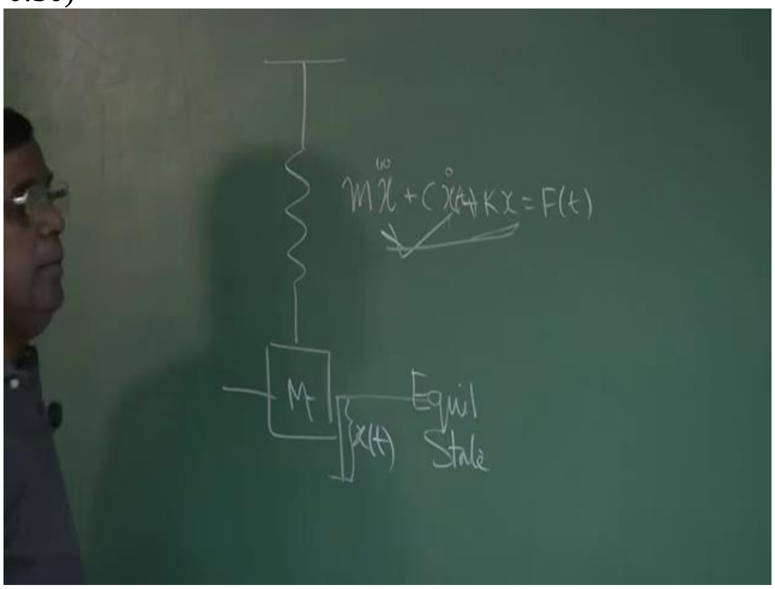

Good afternoon friends. We are continuing our discussion on Aircraft dynamic stability and we by now, we are very clear that our approach will be as I illustrate through mass spring damper system, our approach is, identify the equilibrium state, perturb it, small perturbation, very small perturbation and then write the equation of motion. So if you recall, we wrote the equation, MX double dot + CX dot + KX equal to F of T.

This X of T, these were perturbed variables. That is, distance or displacement from equilibrium. So in our sense, this is the perturbed quantity. We wrote the equation of motion in terms of perturbed quantity and then we tried to find out its roots and tried to study the behaviour of the perturbed quantity, X of T and if it is oscillatory, if it is coming just to the Trim without any oscillation, so we said it corresponds to either oscillatory return under damping case or overtime to case or a critical damped case.

But the catch point was, this X of T is basically measured with respect to equilibrium. Another way of understanding is we disturbed it and withdraw it and see how this motion variable which is a perturbed quantity is behaving. And based on that, we decide, we say that it is dynamically stable or not. Now we wanted to use this understanding, we wanted to extend this understanding for analysing and aircraft dynamic stability.

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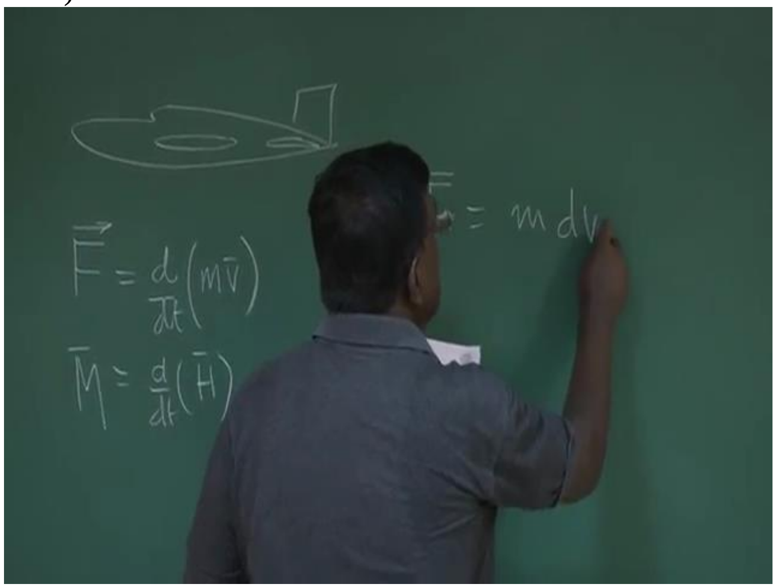

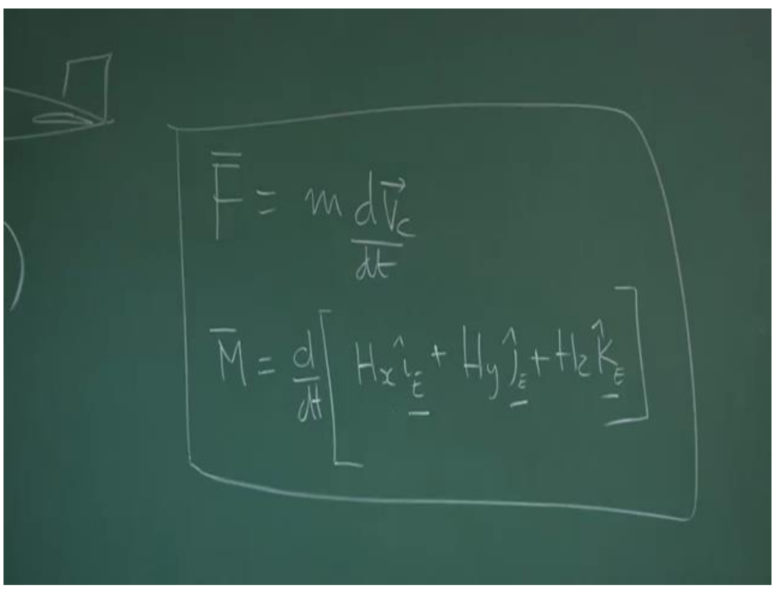

So what we did? We said we will also write equations of motion of an airplane. In fact to be more precise, we also know that we have to write perturbed equations of motion for airplane. Because these are perturbed equations of motion because these are measured from equilibrium when the perturbation was introduced. We wanted to develop such equation of motion which is a perturbed equation of motion for the airplane. But what was our approach? We said initially, we will write generic equation of motion of the airplane. Then we will introduce perturbation and try to get perturbed equation of motion.

And in that approach, we said that we will use Newton’s laws of motion and that is external force is equal to MV, rate of change of momentum. Similarly, externally applied moment is rate of change of angular momentum and we all know angular momentum is the moment of momentum or moment of linear momentum. To be more precise, R Cross MV. We all know this.

But what is another point? We also noticed here that if I want to implement these laws of motion, I need to ensure that we are applying with respect to inertial frame. And you know, inertial frame is the frame having no acceleration. So we started developing equations of motion but not perturbed equations of motion for airplane so far. We are all writing equations of motion.

And we found out 2 important relations that F equal to MDVC, the centre of mass by DT. Another we got was M, moment equal to if I write it correctly, it will be D by DT of HXI + HYJ + HZK. If you see, I am writing E, E here. Substitute E, F or unit vector because we are very much clear and cautious about the fact that if I want to apply this M equal to rate of change of angular momentum, I have to apply it in inertial frame.

So this IE, JE and KE, they are the unit vectors aligned in the inertial frame of reference and for our case, earth fixed axis system has been chosen as inertial frame although we know Earth is not inertial frame because it is rotating. But we know, since we are doing dynamic stability analysis which is for a very short period so we can make this assumption for all simplifications. Not much error will be introduced for dynamic stability analysis. Okay. This is one understanding we had and we also developed the expression for HX.

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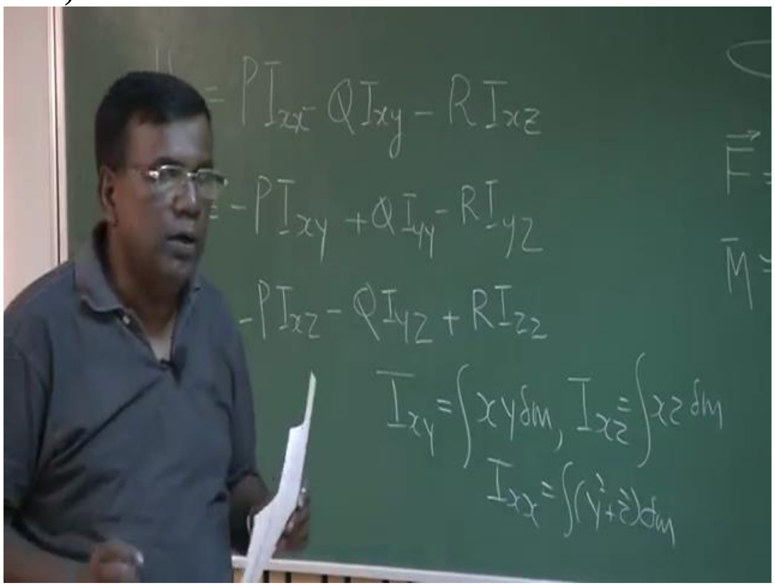

Let me write here. I was requested you to do these derivations yourselves because there may be some slip from my side. You young men should be able to derive this following my lecture. This is HX, this is HY. I say XX to be more consistent. HZ is equal to - PIXZ - QIYZ + RIZZ. Okay, let me see. HX is PIXX - QIXY - RIXZ. XY is PI XY + QIY - RIYZ and HZ is - PIXZ - QIYZ + RIZZ.

And I also wrote IXY is nothing but integral XY DM. Similarly IXZ is integral XZ DM and you all know, IXX will be integral Y square DM. These are by definition. Up to this point, we have no issues but we should not forget when we are deriving all those HX, HY, HZ, these are with respect to inertial frame but this P, Q, R over here are with respect to inertial frame. Now you imagine, even this IXX, IYY, all are with respect to inertial frame.

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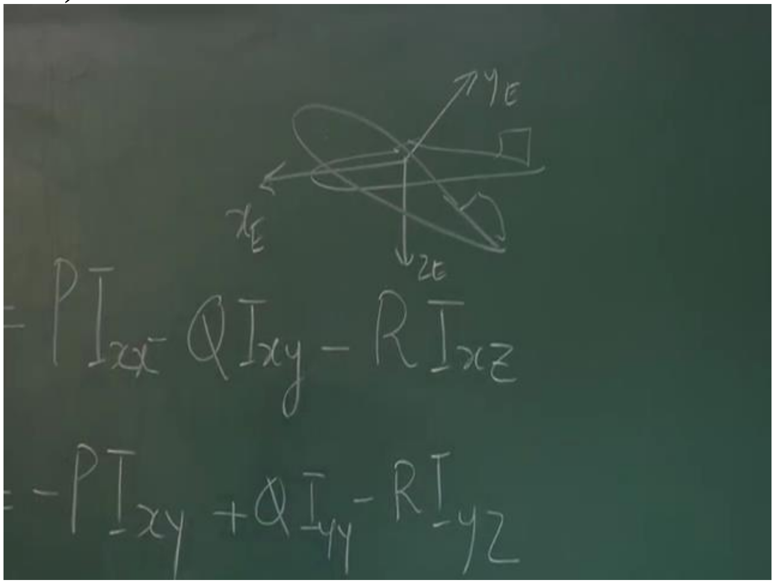

Now imagine a situation, this is the airplane. Let us say this is your inertial frame. I say X earth, Y and Z earth. Now the body will rotate because of the dynamic. So now what will happen? The moment of inertia of this body with respect to X, Y, ZE, initial frame of axis will go on changing as the body oscillates. So then it is very difficult to, very cumbersome to introduce those things. That means every time you to compute IXX for different different instants of time, finding the orientation, etc, etc.

This is one, the moment of inertia will go on changing if I am using X, Y, Z, earth frame reference system. That creates complications, it becomes very cumbersome. We are not very happy with dealing those. Also we understand that the aerodynamic forces acting on the airplane depend on the air relative speed.

I told you in the last class, if the airplane is stationary on the ground, it will experience no aerodynamic force. The airplane is stationary on the ground. However wind starts blowing, then it will experience a drag force or a lift force. So it is air relative speed that is important, not ground speed. So it is better to work in terms of body frame.

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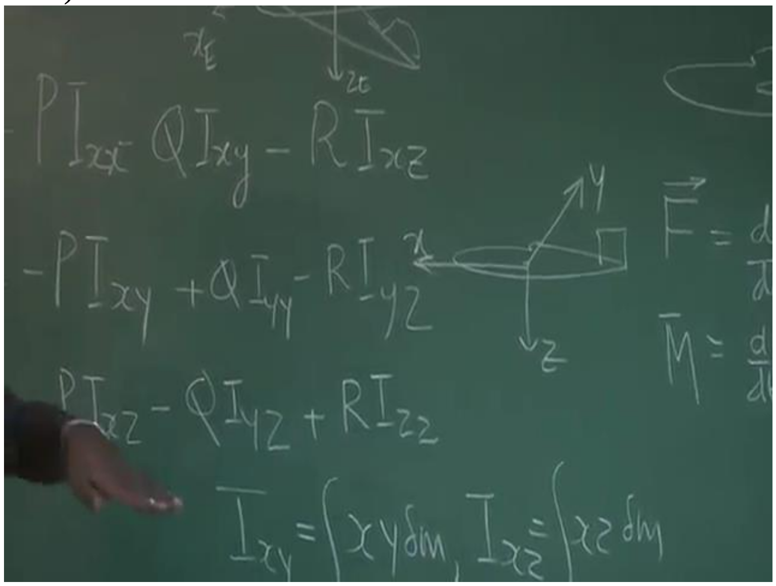

If I choose a body frame, if I am allowed to operate, knowing very well body frame is not an inertial frame, let us say X, Y, Z, body frame means, it is fixed with the body. And as the body rotates, this frame also rotates. So one thing is sure that moment of inertia now will not change about this axis because the axis is also moving. Then also, the aerodynamic forces will be proportional to the relative as speed. So along X, Y, Z, you can easily compute.

But what is the problem? The problem is, this is a non-inertial frame because it is a rotating frame. So how do I apply Newton's laws of motion?

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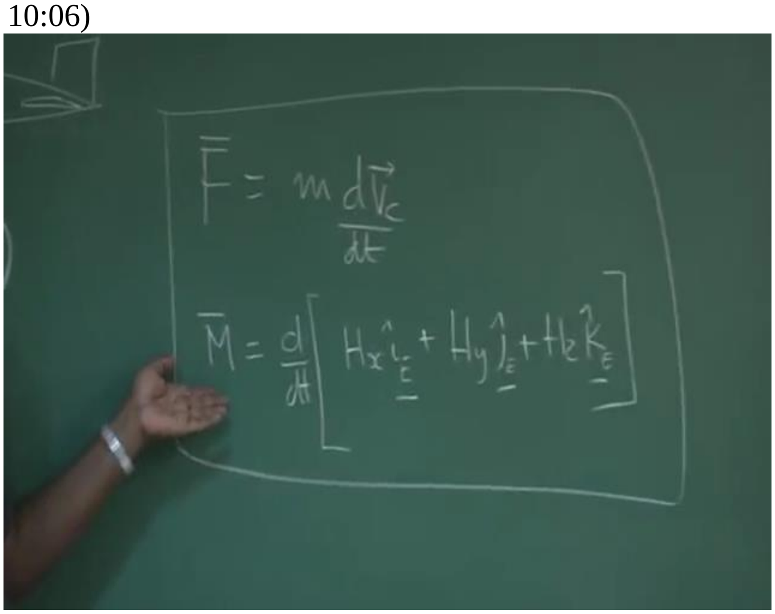

It says, it has to be inertial frame. So if I found out a way to apply this without violating this understanding that this has to be applied in inertial frame and still operate on a rotating frame, my job is done. That means, I am asking a question, what is the equivalent of handling this problem by operating at a rotating frame still this violation will not be there that it is not an inertial frame. That means, if we are operating in a rotating frame, if I know what sort of correction I have to add with a rotational frame derived expression, what additional correction should I make so that equivalently it will become as if I am operating in inertial frame as far as mathematics is concerned.

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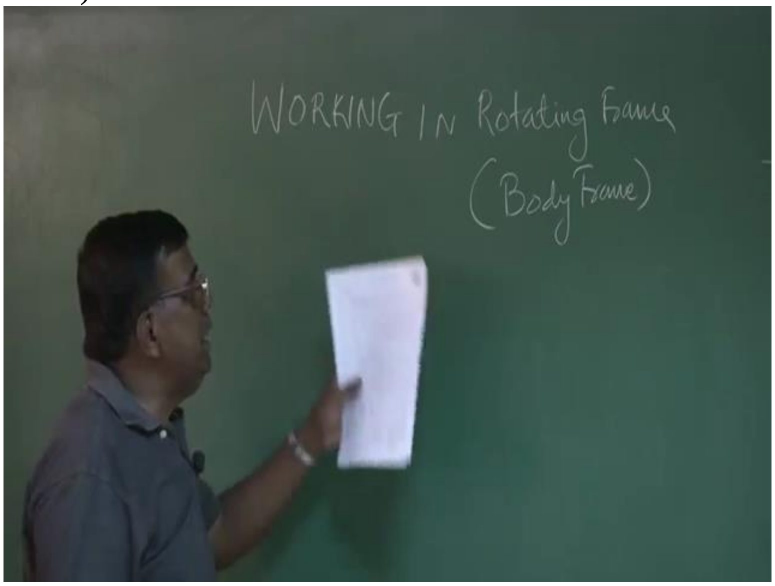

If that is true, let us have a closer look here. So I would like to make things clear working in rotating frame or body frame. So now our approach is very simple. I want to operate in interpreting frame. I know, if I want to apply Newton's law, it has to be non-rotating, non-accelerating frame. So I say, I will operate in a rotating frame because I have 2 distinct advantages and tell me what correction I have to make, add to this result mathematically so that it is equivalently as if I am working in a non-rotating frame or inertial frame.

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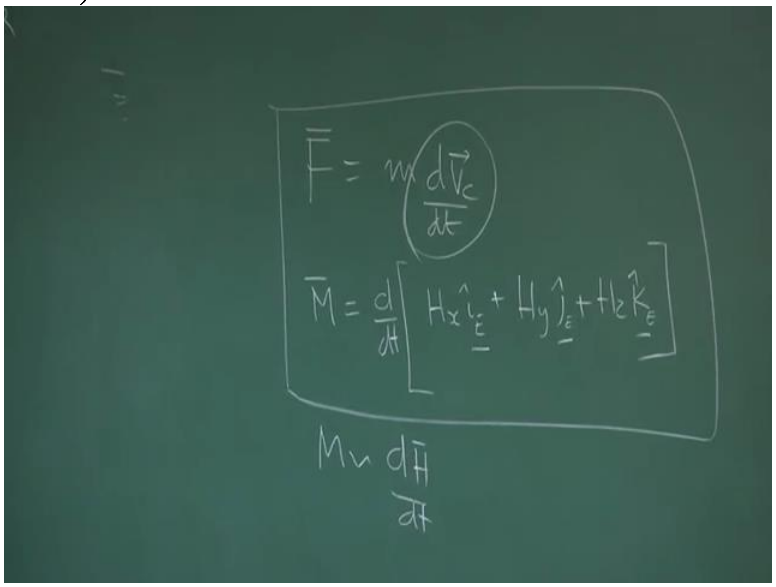

See here. Mathematically if I see, external force causing acceleration, mathematically I need a derivative, DV by DT. Similarly here also, M, you see it is a question of DH by DT. That is derivative of a vector.

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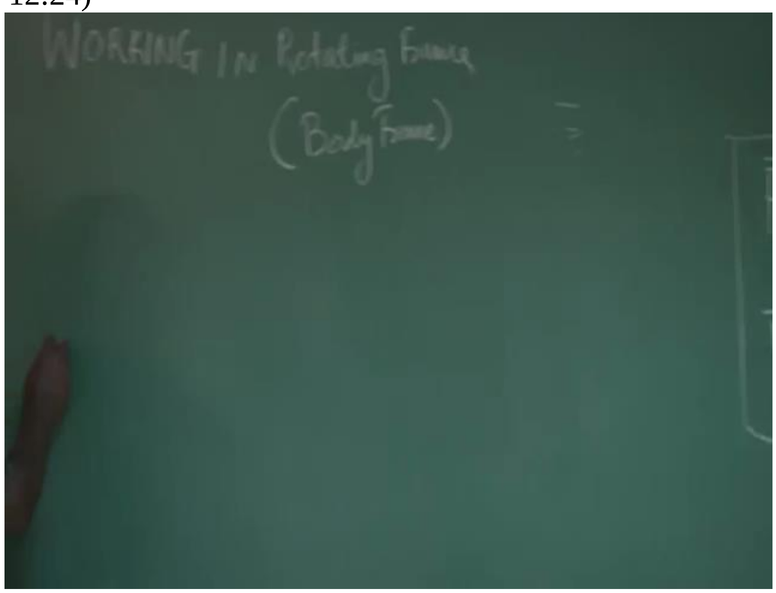

If I want to do a derivative of a vector in inertial frame, suppose this is our inertial frame, XE, YE, ZE. Let us say, V is equal to UI + VJ earth + WK earth. If I do DV star by DT, I see one is DU by DT into IE, second term is U into DIE by DT. Similarly, other term will come but if I am working in inertial frame as it is here, DIE by DT is 0 because it is fixed.

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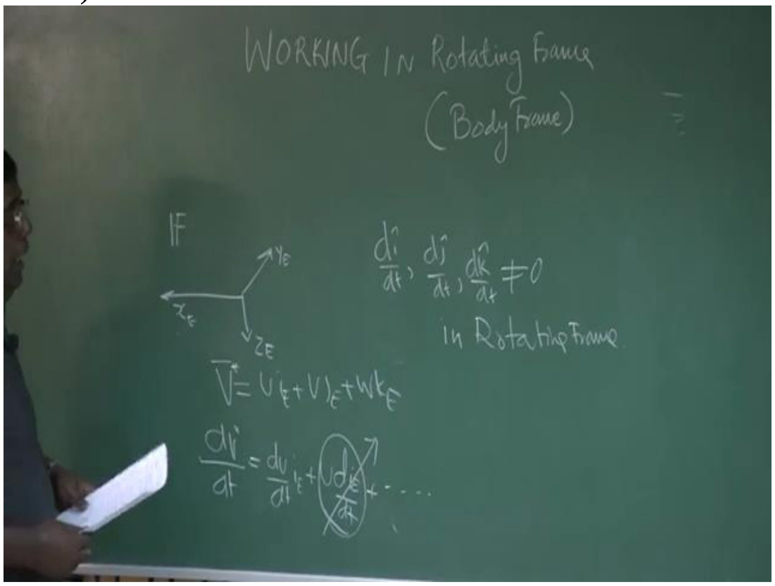

But if I want to do this derivative in a rotating frame, if we want to find in rotating frame, then DI by DT or DJ by DT or DK by DT unit vectors, they are not equal to 0 in rotating frame. That is the problem. So naturally, I have to put some extra terms so that there is an equivalence. And what is that?

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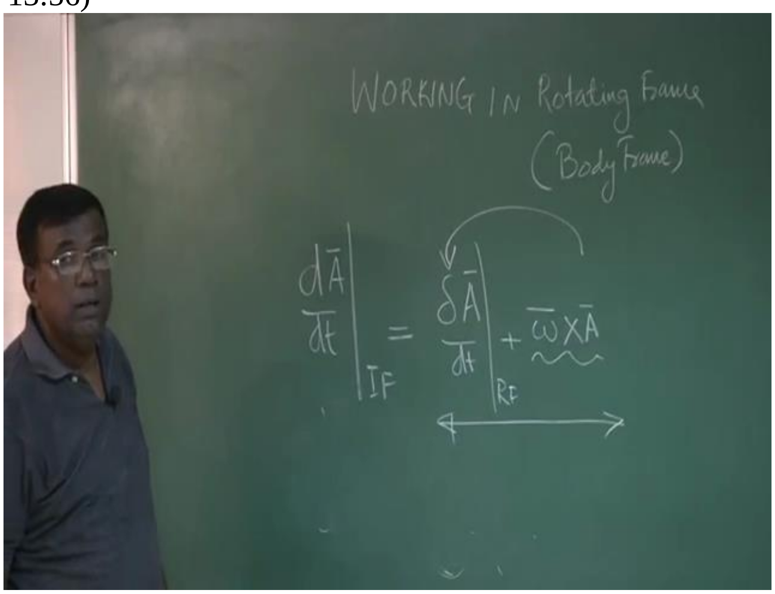

You know from my last lecture, that is if I want to derive derivative of a vector, let us say DA by DT, if I want it in an inertial frame, this is equivalently DA by DT in rotating frame + whatever the frame rotation is there cross A, this is the correction. If you add on DA by DT evaluated at rotating frame, if I add these 2, it is equivalently as we are working in inertial frame.

The derivation which I had in the last module, I have already shown, you can see your textbook also but as far as application point is concerned, you should now see that we have solved that riddle that we want to operate in DA by DT, inertial frame. The rate of change of momentum, rate of change of angular momentum, we want to evaluate in inertial frame, but now what we are doing? We will evaluate in rotating frame because we have some advantages and to make it equivalent to inertial frame, we will add this omega cross A.

We will add these 2. So it is equivalent to working in inertial frame. So I look now towards this, right-hand side. And that will fulfill my desire that I still work on rotating frame because I know there are some advantages for this specific case to work in rotating frame. It is not a generic statement. For this case, yes it is okay. So now we will apply this understanding. Let us see what happens.

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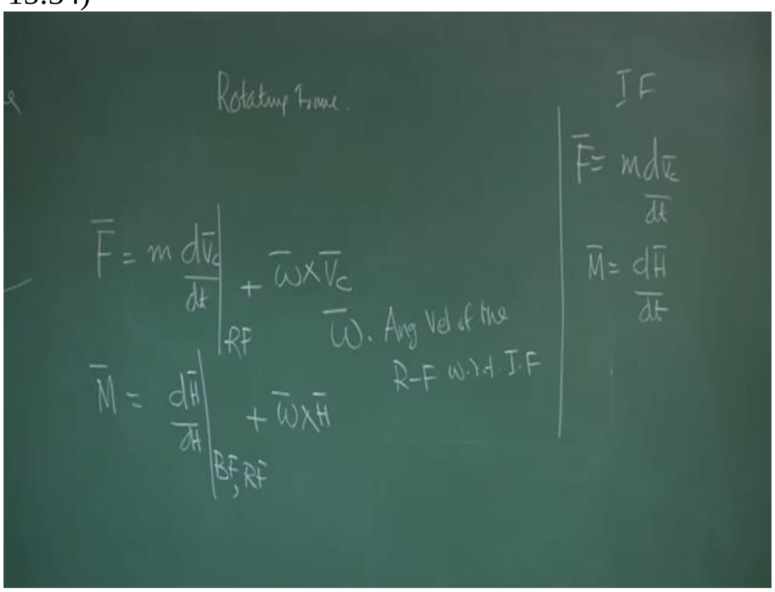

I will write here, inertial frame it was M D VC by DT. So this VC, everything was measured with respect to inertial frame. M was the rate of change of angular momentum. Again with respect to the inertial frame. But now we want to work in rotating frame equivalently. What we will do? We will say, F equal to M DVC by DT evaluated at rotating frame + omega cross VC by using this relationship.

Similarly, we will write M as DH by DT evaluated at body frame or rotating frame with same thing, + omega cross H. What is omega? Omega is the angular velocity of the frame with respect to inertial frame. Correct? What is omega? Omega is the angular velocity of the rotating frame with respect to inertial frame. Now. Please understand the next step. What is VC?

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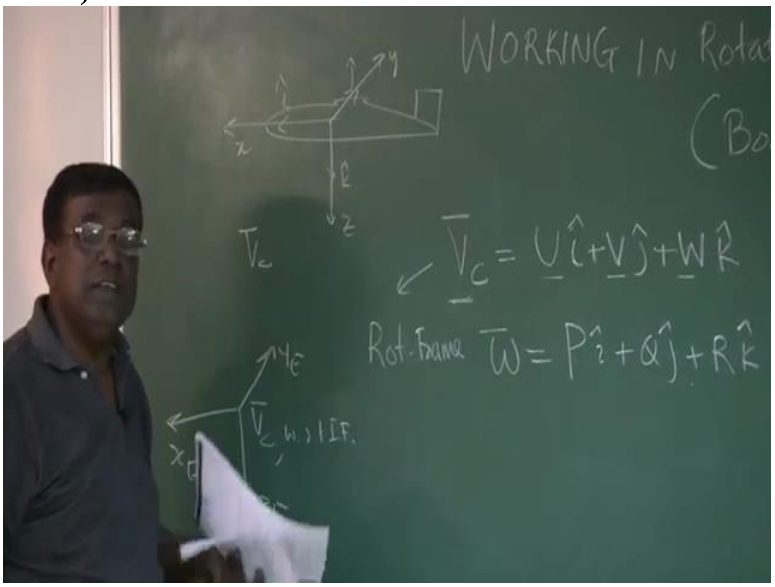

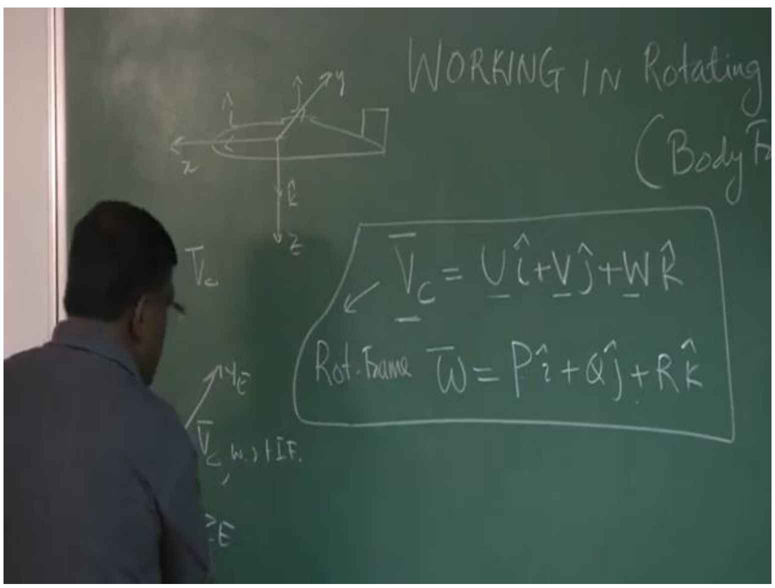

How I am going to expand VC in rotating frame? How I am going to express it? I wanted to work in a rotating frame. So I will write, this is equal to U into I + V into J + W into K. They are the unit vectors. What are these I, J, K? I, J, K are the unit vectors of the body frame. Remember, DI by DT, DJ by DT, DK by DT is nonzero. But since we are giving this correction, omega cross VC, we can directly take the derivative in a standard form and because of the rotation part, the vector gets corrected through omega cross VC.

So what are the U, V, W? U, V, W Waugh the components of the velocity of the frame or the body with respect to inertial frame but resolved along body axis, X, Y, Z. Let us understand this. If this is your airplane, if this is the body axis, X, Y, Z. And here it is, X earth, Y earth, Z earth, then what is this VC? VC is the velocity of centre of mass because we have seen from the first law that even if the forces are distributed, for translational motion, you can assume, whole mass is concentrated at the centre of mass as a point mass and take the resultant of all the forces acting at different components and transfer it to point mass and write F equal to MA.

So VC is the velocity of the centre of mass with respect to inertial frame. But then, when I write like this, what I am showing? VC is expressed as, which is the velocity of the body or frame with respect to inertial frame, resolved along local I, J, and K, unit vectors are resolved along local body axis. So we are now operating in body frame. Similarly omega when I write PI + QJ + RK, what is the interpretation?

Omega is the angular velocity of the body or the frame. They are same because axis is fixed, frame is fixed to the body to the angular velocity of the body or the frame with respect to inertial frame. However this P, Q, R are the components of that omega resolved along I, J, K body axis. Clear? These are 2 important understandings you must have. Once I have that and once I have this relationship, I can easily find out some expressions which will be very interesting.

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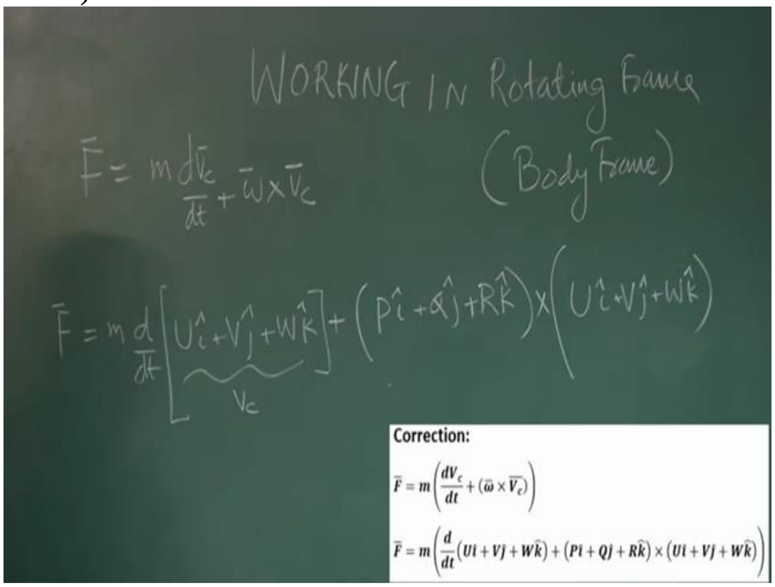

Let me start with F equal to MDVC by DT + omega cross VC. Let us to some vectors simple elementary operation. You know VC is nothing but UI + VJ + WK. So if I can write F equal to MD by DT of UI + VJ + WK + omega + PI + QJ + RK Cross UI VJ + WK. Right? And D VC, this is nothing but VC. We have seen. This is nothing but omega and this is again V. These are the stands when I am operating in a body frame now.

So this when I, when I am doing this derivative in body frame, I am not bothered about D I by DT. No more. Because those things have been corrected here. Simply you take derivative it respect to the scalar term. We will get F equal to MU dot + QW - RV. This is one equation you will get because this is FX. Because you understand F is nothing but FXI + FYJ + FZK.

So what are FX, FY, FZ? They are the components of force, F resolved along body, I, J, K axis. So if I do this vector operation, I simply have to write here, FXI + FYJ + FZK. When I do this derivative, this will be U dot and this will be V dot but J will be there. W dot K will be there. When we find the cross product, we will get this equation, FX equal to that. FY equal to MV dot + UR - WP. And FZ equal to MW dot - UQ + VP.

You could see here. If I was operating in inertial frame, everything was inertial frame. Derivative was on inertial frame. Then it will be FX equal to MU dot. This thing will not come because at that time, there is no such angular velocity correction, omega cross VC. Now it is very clear. We are operating in the body frame. But at the same time we are maintaining the understanding that, Newton' law application is valid only in inertial frame by using this correction.

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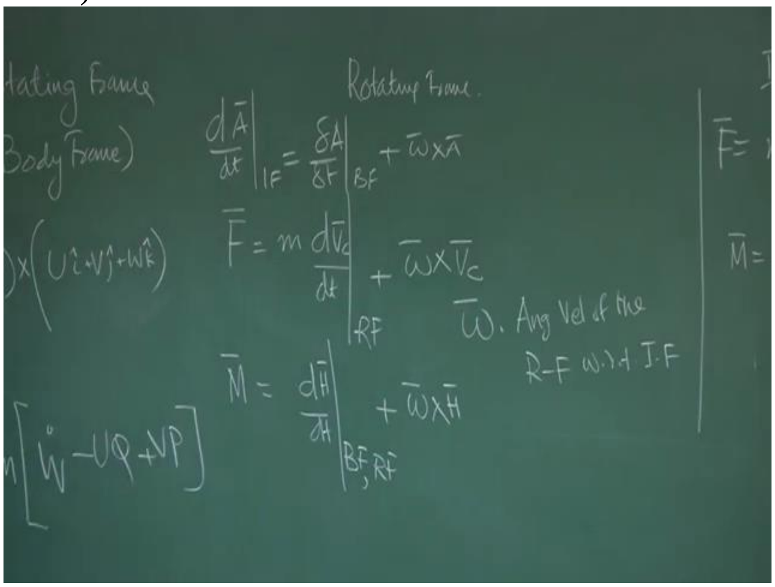

What is that? Correction is DA by DT, inertial frame and equivalently operated at body frame + omega cross A. This understanding was applied and you have got FX, FY, FZ expression. Why we are getting all these things? Because we want to write equations of motion. But these are not perturbed equations of motion unlike the mass sprint system. This is a generally equation of motion. We will be extracting perturbed equation of motion by doing another stage of mathematical treatment.