Aircraft Dynamic Stability & Design of Stability Augmentation System
Professor A.K. Ghosh
Department of Aerospace Engineering
Indian Institute of Technology Kanpur
Module 1
Lecture No 08
Six Degree of Freedom Equations of Motion

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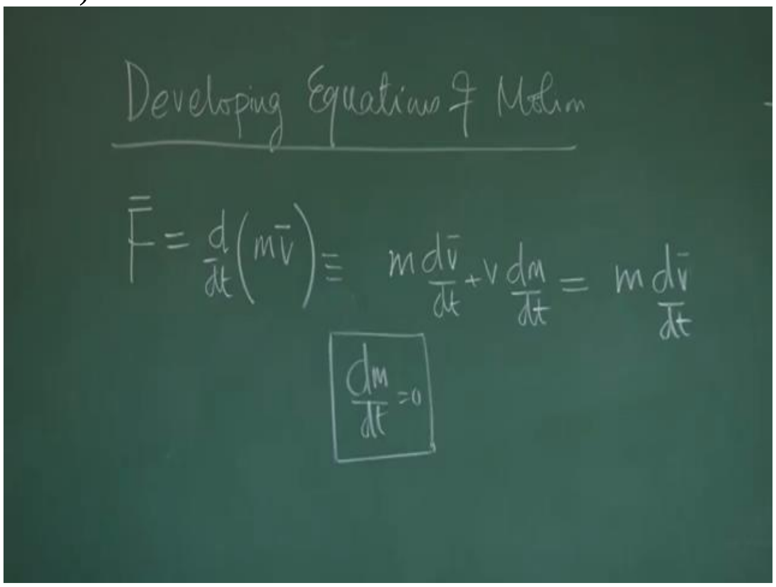

We are continuing our effort to develop equations of motion and we all agreed that we will follow Newton's laws of motion where it says, this D by DT, external force is equated to rate of change of linear momentum and if I expand it, it is MDV by DT + VDM by DT. For our stability analysis, if the duration is small, even if there will be very small (())(0:55) assumption during the study, we will neglect, we will say DM by DT is 0.

Please understand, we are putting it 0 because we are building our equations of motion for stability analysis, dynamic stability analysis which is a transient analysis. Very small time, we will be using. In practice, DM by DT is nonzero for an air craft because a lot of fuel consumption will be there. Almost 30% of the weight will be fuel. But for stability analysis, dynamic stability analysis, since it is for a small duration, a transient analysis, we will assume mass is remaining constant.

So DM by DT is 0. So I write this as MDV by DT. Now I ask you a question, what is this V? This velocity vector is what? This is the velocity vector with respect to inertial frame.

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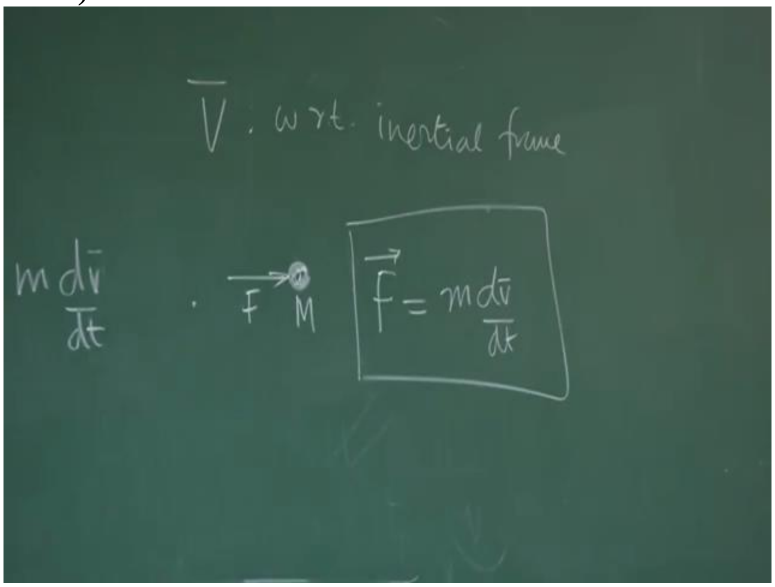

This is V with respect to inertial frame. The acceleration is with respect to inertial frame. DV by DT is also with respect to inertial frame because we are writing F equal to MA, Newton's law of motion which is correct and valid only when you implement it in inertial frame.

When I write such equation or anybody writes such an equation, this is in your mind, it is something like this, there is a mass, concentrated mass and there is an F, external F being applied on this concentrated mass and the effect of this mass or the effect of this force is to cause acceleration. This is typically the understanding if mass remaining constant. What is in your mind is, like a point mass. The whole mass is concentrated at a point. But now what is happening you see. For an airplane, what is happening for an airplane?

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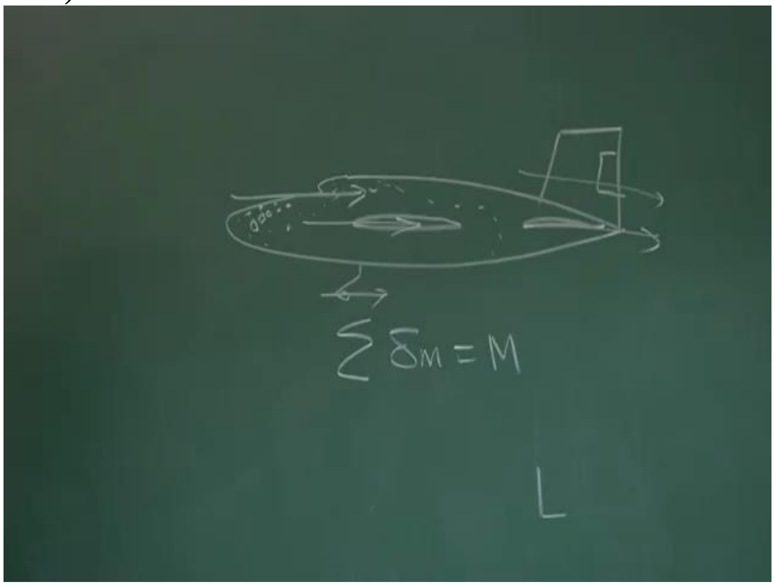

Please understand this concept very clearly. If you understand this clearly, tomorrow, if the vehicle is changed, instead of aircraft, it is a missile or a rocket, you will not find any difficulty in understanding. The moment I write F equal to MA, I am biased with an understanding that the mass is a concentrated mass. But when I see an aircraft, I see that it is composed of so many delta M. So, summation of Delta M is actually the mass of the whole airplane. That is number 1. Number 2. Forces on the wing, forces on the fuse launch, forces on the rodical tail, here. If there is a landing here, then force here.

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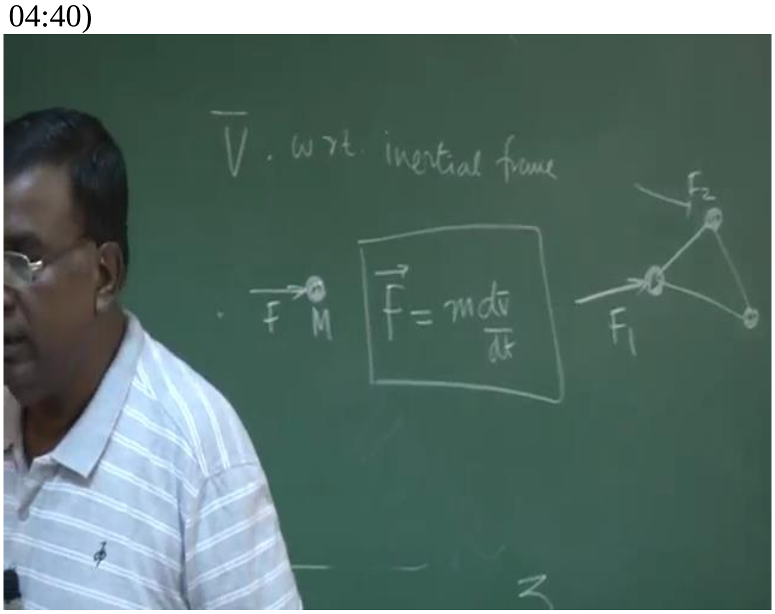

So it is not that one mass, point mass is there and resultant force is acting here. This force is distributed over the whole airplane, mass is distributed over the whole airplane. Now, how do I apply this concept, F equal to MA? That is the question. If you go back to your 11th or 12th standard, if there are 3 masses like this, they are connected like this. Here, F1 is acting, here F2 is acting, here F3 is acting.

If we want to find the acceleration of this, assuming there is no rotation, then how do I model it? How do I write F equal to MA. Remember, there we used the concept of centre of mass. We will be using that concept also here. Just to prepare yourself, you will see that whatever you have learnt in 11th and 12th, we will be using that here as well. That is so important. So let me draw a diagram.

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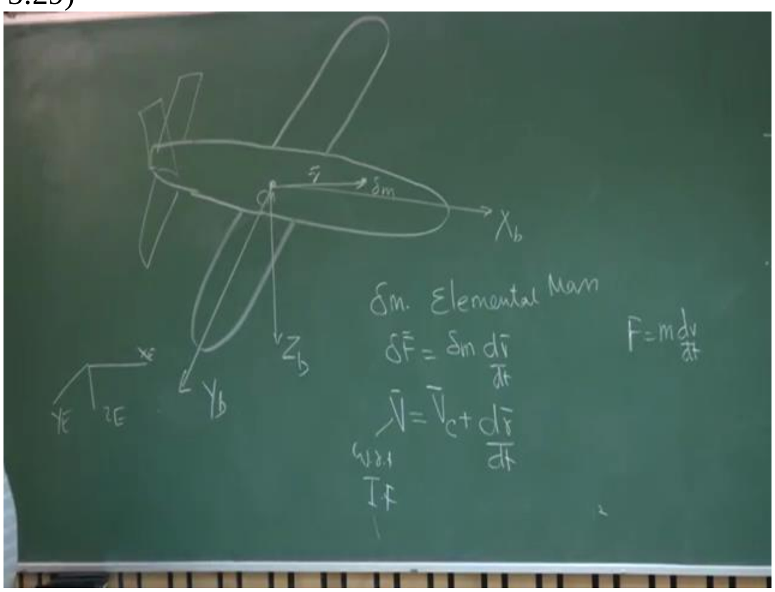

So this is YB, this is XB, body axis YB and ZB. And this is the location of Delta M denoted by R bar measured from centre of mass. What we are doing? We are trying to develop equation of motion or we are trying to apply F equal to MA using inertial frame of reference. That means, whatever measurement will be made in terms of acceleration or velocity will be with respect to inertial frame.

Now we could write easily, Delta M’s as elemental mass and delta F is equal to if I can write Delta M DV by DT. Right? We know that F equal to M DV by DT. If M dot is 0, so I have for an elemental mass which is acted upon by force, delta F, I can write this expression. More importantly, B which is measured with respect to inertial frame. The sum of this inertial frame is here, XE, ZE, YE.

This V is measured with respect to inertial frame because writing Newton's laws of motion, this I can represent it like VC + DR by DT. No objection. R is this. What is the rate of change of R with respect to centre of mass and what is the velocity of centre of mass with respect to inertial frame, these are vectorly added and I can represent it like this. Okay. No issues.

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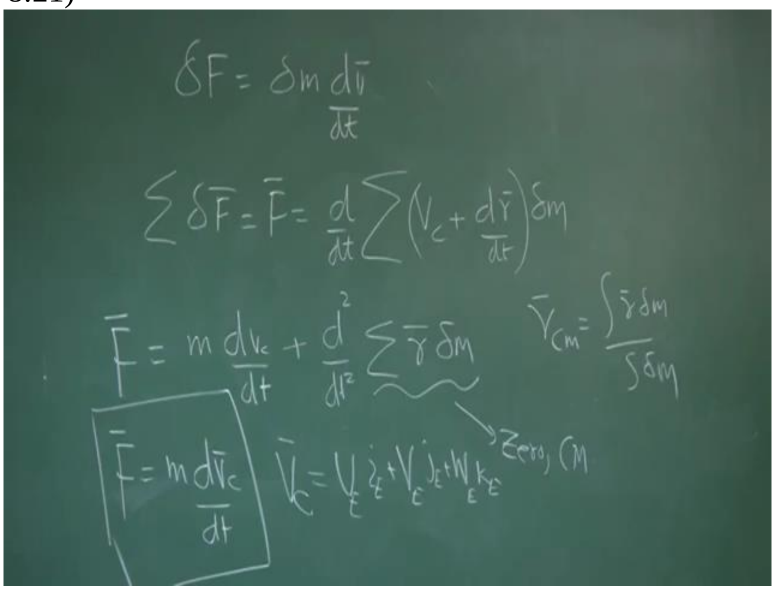

Now what do I do? I start with delta F equal to Delta MDV by DT. So summation of Delta F let me write so that I can explain it, will be D by DT of summation of VC + DR by DT. Right? I have done summation means resultant force vectoral summation and V is VC + DR by DT. So, D by DT of V is V C + DR by DT into Delta. Summation is done.

So if I do this then I can write easily, F is equal to MDVC by DT + D Square by DT Square summation of R DM. No objection. This I have done in last lecture also. But could you recall what is this value RDM? We have selected this location of the body fixed axis at centre of mass and by definition of centre of mass, RDM will become 0. Because if you have forgotten, let me write this, R centre of mass is RDM by DM. The moment values.

So if RCM is 0 because we have located the axis where the RCM is 0. Plus we have put this here. So naturally, integral R DM becomes 0. So this term goes. So what you get is, F equal to MDVC by DT. What it says, let the mass be distributed, let the force be distributed. Now if you want to apply Newton's laws of motion, you need to find out what is the centre of mass of the distributed mass and assume whole mass is focused, concentrated at the centre of mass and the hold resultant force is being applied at the centre of mass, then solve it like a point mass, you will get the acceleration because of external force F.

So we have come back to that understanding of point mass. That is the beauty of this analysis. So when I write DVC by DT, and if I ask you, what is VC? VC should be for your understanding, so that you do not forget, when I write like this means what? IE, JE, KE are the unit vectors along the inertial frame and UE, VE and WE are the components of V measured with respect to the inertial frame resolved along IE JE and KE of inertial frame. So all we are talking about inertial frame here. We are not talking about body frame at all here is now till this point.

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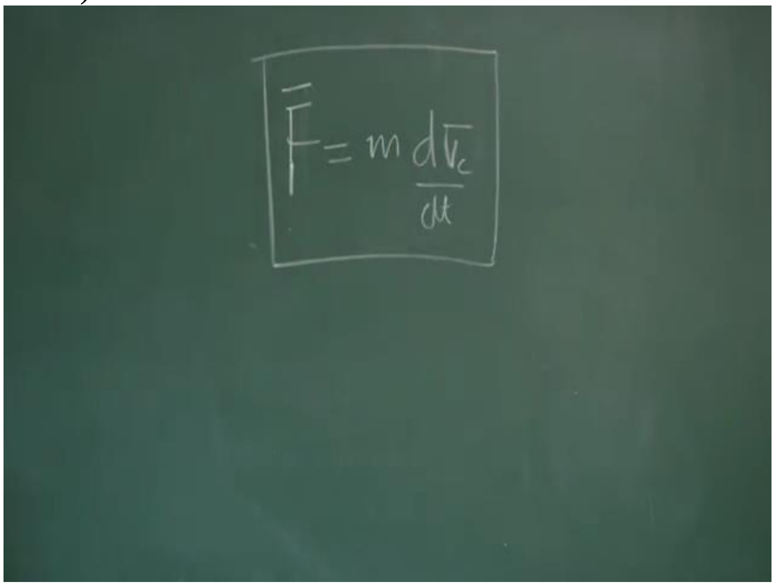

Once we completed the linear motion, we write F equal to MDVC by DT. We know that this airplane has not only translational motion, it also has angular motion. When it comes to angular motion, we know, it is the angular momentum that will decide. That is, in linear case, we say the external force will cause change in the linear momentum and in a similar way, we say, external moment will cause change in the angular momentum.

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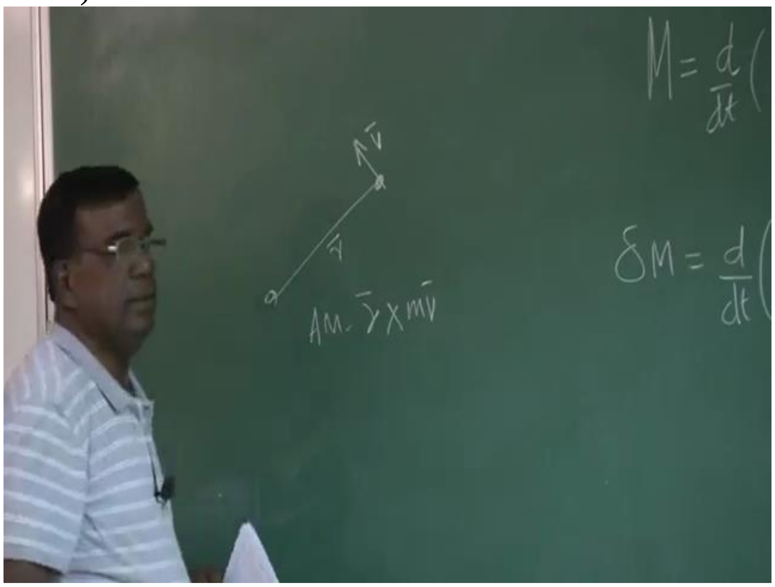

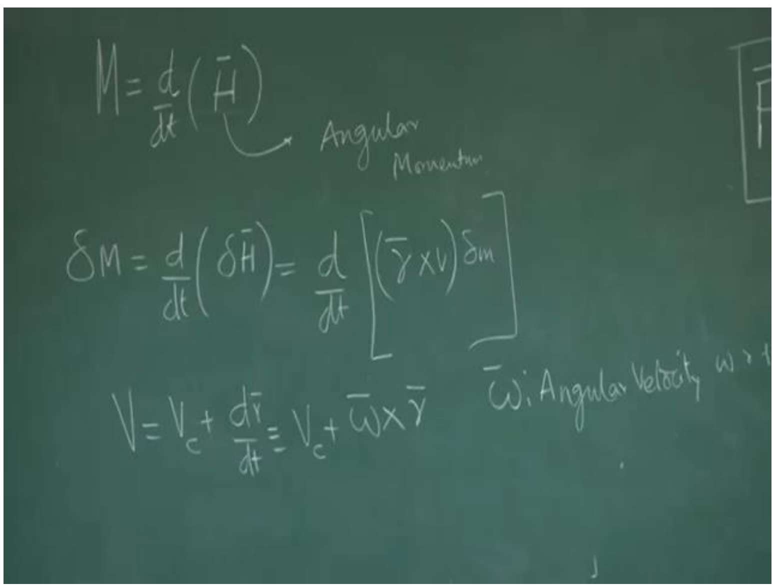

So we have to use this equation, external moment, M will cause rate of change of angular momentum H. H is the angular momentum. Please understand, we are all operating with respect to inertial frame. Now what is the understanding of angular momentum? We say angular momentum is the moment of momentum. R Cross MV. Go back to 11^{th}. It is the moment of momentum. So we will use the definition.

We will write Delta M as D by DT of Delta H which I can write as D by DT of R Cross V. This is R Cross MV, angular momentum, remember? If this is body moving with the velocity V, this is R, then angular momentum is defined as R Cross MV which we say moment of the linear momentum about a point or an axis. Same definition has been used, this is R Cross MV only.

Also we now, V equal to VC + DR by DT which I can write now, VC + omega Cross R. Is this clear? DR by DT is nothing but omega Cross R. Say for example, this body is rotating with omega, then what is the velocity? Omega across R. Omega is a generic, 3 dimension also, omega Cross R is there. So I can replace DR by DT by omega across R.

What is important to be known is that omega is about the inertial frame. This omega is the angular velocity with respect to inertial frame. This is important. R bar was measured with respect to centre of mass. Which is again very simple, again I repeat, if I am rotating about the CG like this, if the point of reference is at CG, omega across R will be 0 because R is 0.

At any other point, the velocity will be if I am moving like this, rotating like this, it will be VC + velocity introduced because of omega Cross R. So vectoraily, that has to be added.

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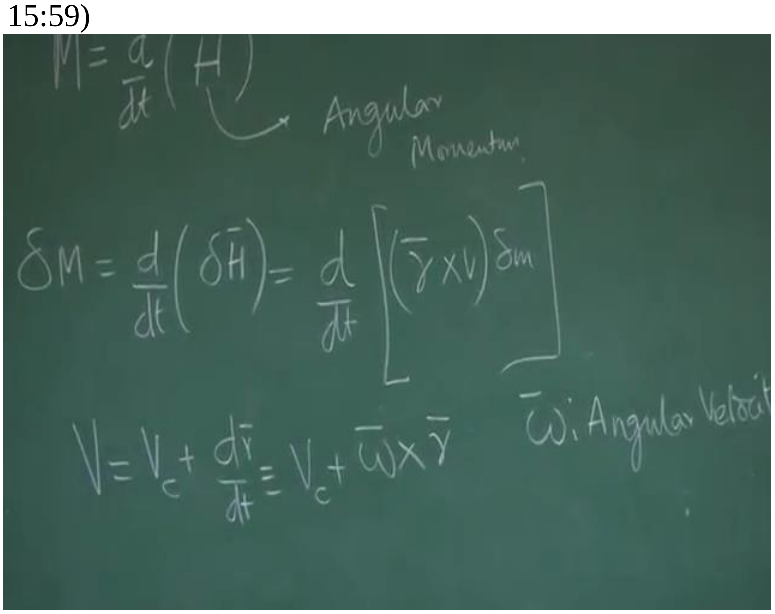

Now let us do a little bit of algebra. So H, momentum, I can write as summation of R Cross V DM which again I can write H is equal to summation of R Cross, for V, I will write VC + omega cross R. Then DM. Nothing great I have done. V I have replaced V by VC + omega cross R.

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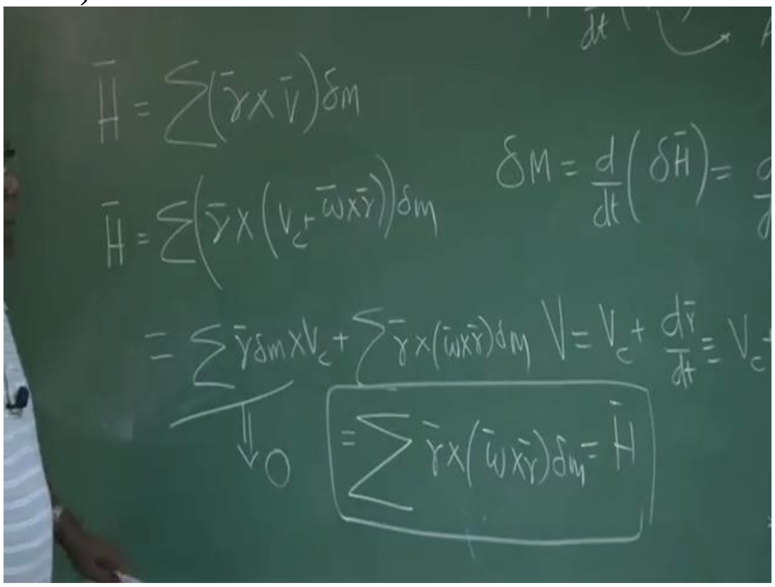

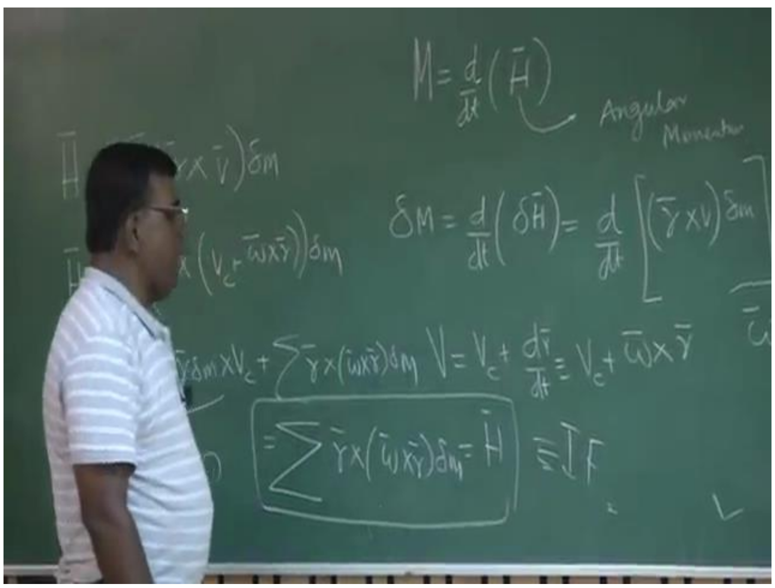

And this I can even expand, I get summation of R DM Cross VC + summation R Cross R omega cross R DM. By now you are smart. Summation R DM is by definition of centre of mass, this is 0 because we have located the axis at the centre of mass. So this term goes. We have only this term, summation R Cross omega Cross Cross R DM. That is equal to angular momentum H. And these are all done with respect to inertial frame. This I will keep on repeating, all with respect to inertial frame.

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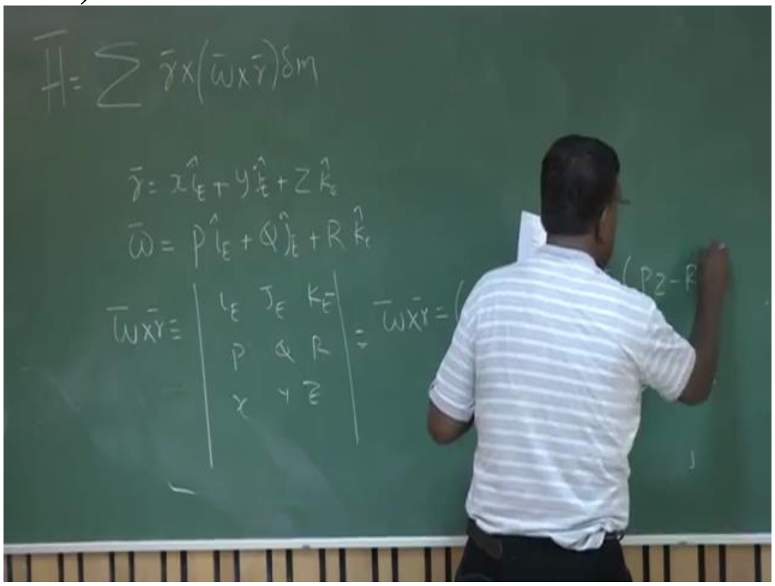

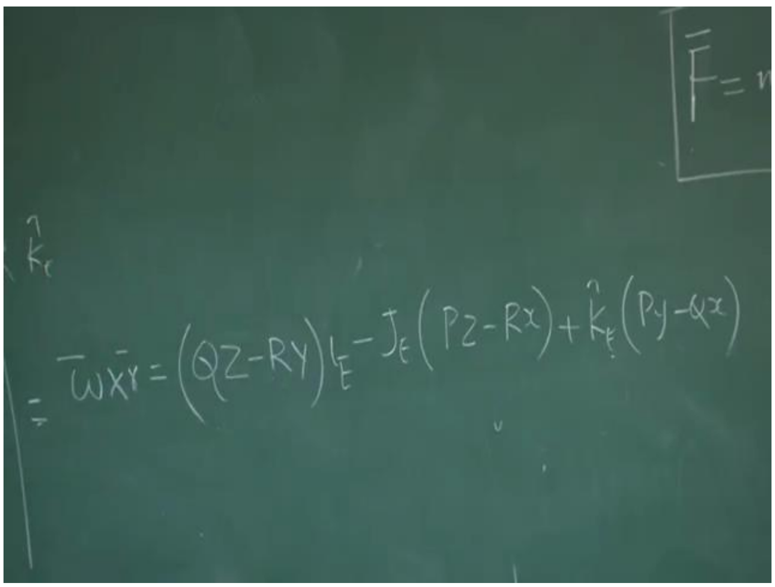

We like to understand by expanding this. Before you do this, please understand, R is the distance of any elemental mass with respect to centre of mass. Omega is the angular velocity of the rotating frame, that is the body which is rotating with respect to inertial frame and Delta M is the elemental mass. What I do? I write R as the X component resolved along IE + YJE + ZKE. IE, JE, KE are the unit vectors along inertial frame, XE, YE, ZE.

So resolving R, expanding R with respect to inertial frame. Similarly I write, omega, the rotational velocity as PIE + QJE + RKE. No problem. It is the angular velocity of the airplane resolved along inertial, IE, JE, and KE axis. So now if I do omega cross R and I will simply tell you, you can do yourself. This is IE, JE, KE, P, Q, R and X, Y, Z. If I do this, if I am not mistaken, I will get omega Cross R. Please, you should do it yourself also. Do not blame me if there is some mistake. IE - JE into PZ – RX + KE into PY - QX.

Right? You know that how to do it. Use it. RY - J, RX, PZ. You could see that. Edit. If you see here, IE into QZ - RY and - JE into PZ - RX + KE. What is our aim? We want to find R Cross omega Cross R. So what will happen? I erase this now. Omega Cross R expression is there.

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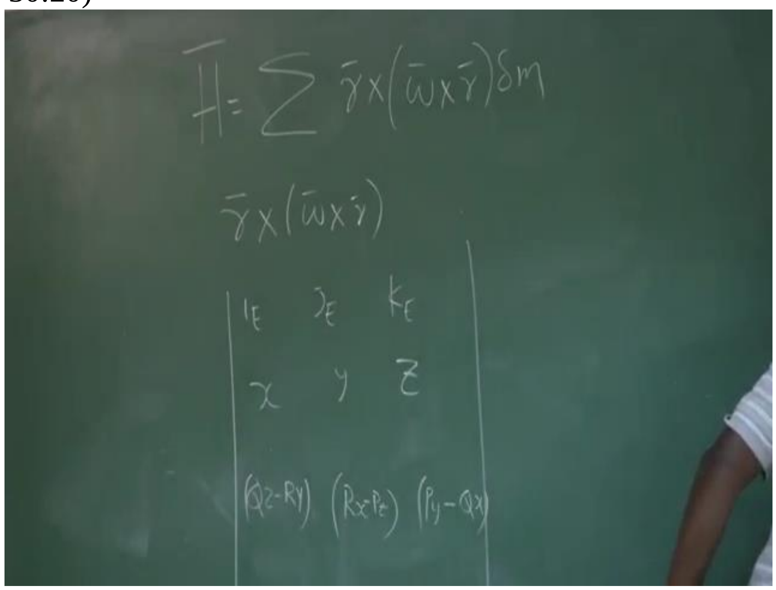

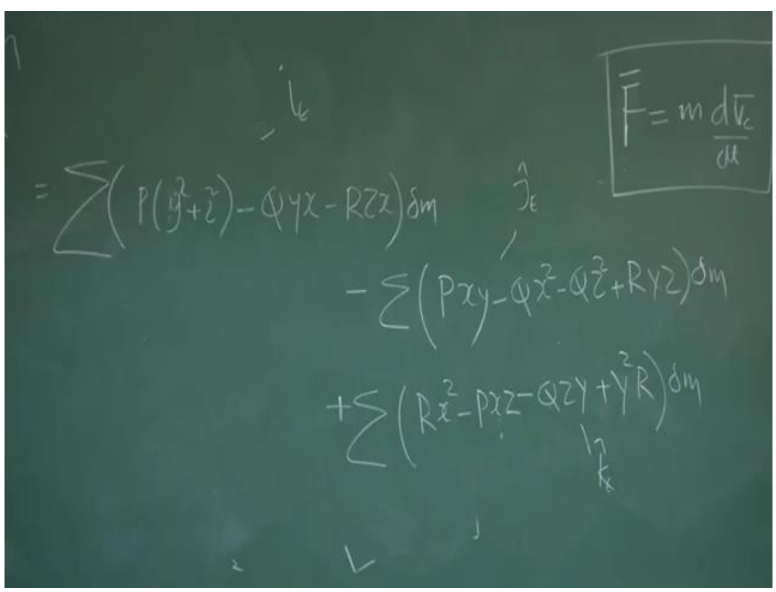

And now we write R Cross omega across R. You can do mechanically. It will be again IE, JE, KE. And then, it will be X, Y, Z and then it will be QZ - RY. Then RX - PZ. Then PY - QX.

Again mechanically we do. IE into this, this - this - this. So you will get simply this vector multiplication.

Once we again do this operation and put it into this equation here, you will get, this whole term will be equal to summation P Y Square + Z square - QYX - RZX. Then this is DM - summation PXY - QX square - QZ square + RYZ DM. And then + summation RX square - PXZ - QZY + Y square R DM. You could see here. First one is IE component. Second one is JE and third one is KE component.

I will strongly advise, you must do this simple operation. And do not take me to court if there are some sign mistakes here. I assume that you will do it and you will crosscheck.

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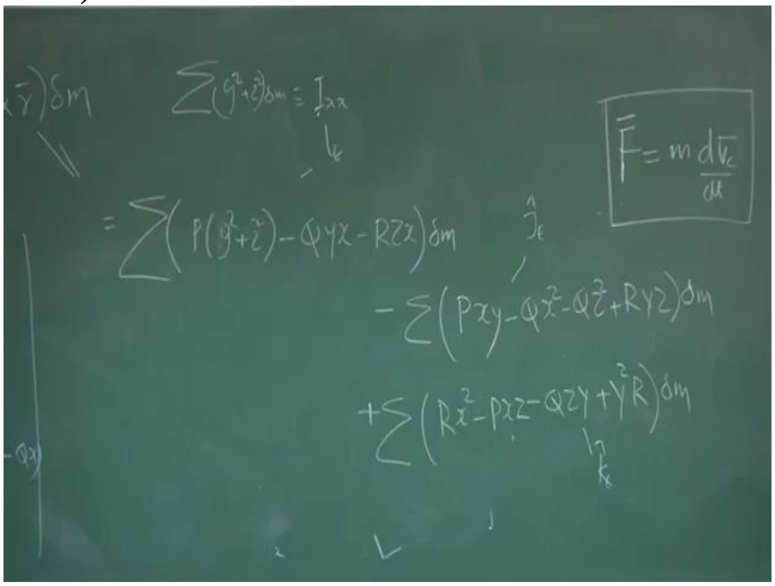

If I do this now, further operation, I can write, you could see what is summation of Y square + Z square DM. Y square + Z Square DM is nothing but IXX, moment of inertia about X axis. Similarly, the Cross moment of inertia, all those things you can do and finally you could see that if I do this operation, what I get is important.

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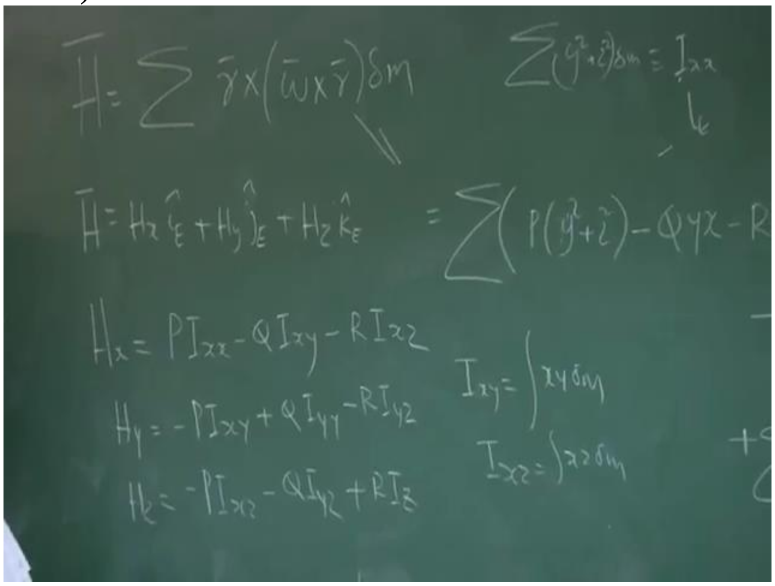

I get H is equal to HXIE + HYJE and HZKE where HX is nothing but the first term with IE. So we get PIXX - QIXY - RIXZ. And HY we get as - PIXY + QIYY - RIYZ and HZ is equal to - PIXZ - QIYZ + RIZ where we will appreciate IXY is XY DM, IXZ is XZ DM. Like that. What is PQR? PQR are the components of angular velocity of the airplane with respect to the inertial frame resolved along IE, JE, and KE. That is along the unit vectors of inertial frames.

We are all talking about the inertial frame .This should be very very clear The only thing is that you have initially realised that if I operate with respect to inertial frame, then as the airplane rotates, this moment of inertia, it will go on changing because with rotation, the distance of Delta M from the X, Y, Z axis will also vary. So then, this IXX, IXY, IXZ, all this will go on changing as the airplane rotates in motion.

That makes things complicated. So we will try to use this and see how best, what could you do so that we can use it in body frame. So that if I am operating it in body frame, then even if the body rotates, the moment of inertia will not change because the axis also will change. So the trick is, we operate in the body frame but do not violate Newton’s laws conditions. Implement the condition, it should be with respect to inertial frame.

The way out is yes, you can do that as long as you apply a collection then life will be simpler. In next lecture, we will talk about how to operate in a body frame and still do not violate the condition that if I want to apply F equal to MA, I need to operate in inertial frame. Thank you very much.