Aircraft Dynamic Stability & Design of Stability Augmentation System
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Module 1
Lecture No 06
Numericals: Week-1

Welcome friends. I am Vijay Shankar Dwivedi, TA in this course. In this week, in previous lectures, you have studied about spring mass and damper system and you have visualised that you can understand on modes of the aircraft with the help of this Spring, Mass and Damper System. In today's lecture, we will be solving a few numericals. Before that, let us take a review of what we have studied in the previous classes. In the spring mass damper system, we have 3 basic elements.

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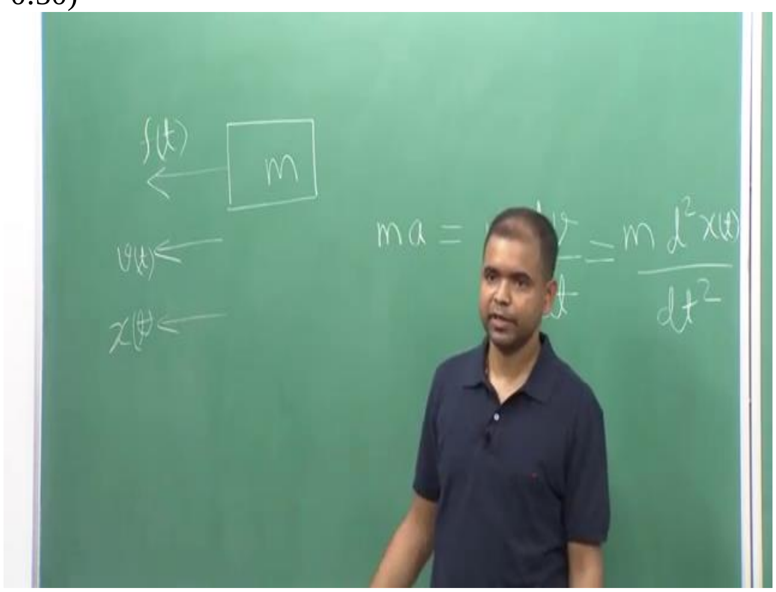

The first one is the mass. Whenever we apply a force on a body, the opposing force and the applied force, the net summation of them is 0. And due to the mass, there is an opposing force and this is the inertial force. Suppose we are applying a force here, FT. There will be an inertial force realised on this mass and this will be, from Newton's second law we can write mask into acceleration or we can write it MDV by DT if this is the V of T or if we talk in terms of displacement, this is X of T.

And this is equal to MD square X of T by D T square. And this is conservative in nature. Basically, it stores the kinetic energy of the system and they can retrieve all the energy without any loss.

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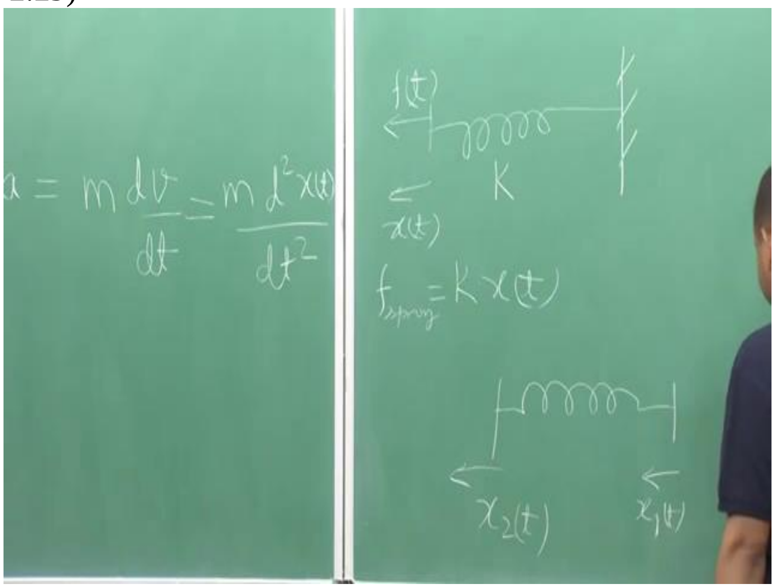

And the second element is the spring. This also applies a restoring force. Suppose the spring constant is K. The restoring force due to the spring is KX of T if the displacement is X of T. The F of spring. The X is the stretching the spring, is spring is connected in such a fashion that its both terminals are getting some displacement, so this part is displaced with X1 of T and this part is getting displaced with displacement X2 of T. So the force applied due to this spring will be, K is the spring constant of the spring and this will be KX2 of T - X1 of T.

The relative change in the length of this spring. And the third element, the spring is also conservative in its nature and it stores the potential energy of the system. So the spring and mass, both are transferring the energy of each other with a phase difference of 90 degree. But you can see here, the energy is conserved throughout the forces. So there is a spring mass system and if you deflect it, it will continue oscillating because there is no any energy dissipating element.

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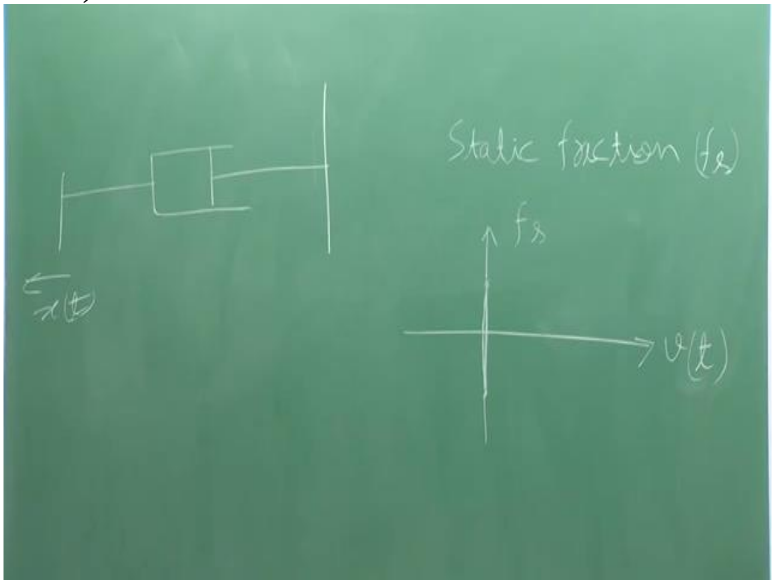

And the third element is the damper. Damper is basically non-conservative in nature. It dissipates the energy from the system to the environment and it works basically on friction. In the nature, there are 3 types of friction. One is static friction, F of capital S. Static friction is when we apply a force on a body, just before the movement we realise this friction. Once the body starts moving, this fiction disappears.

And if I plot it, it will be something like this. Suppose X axis is the velocity axis and this is my fiction. F of S. So it is something like this. Whenever there is velocity, it will be 0.

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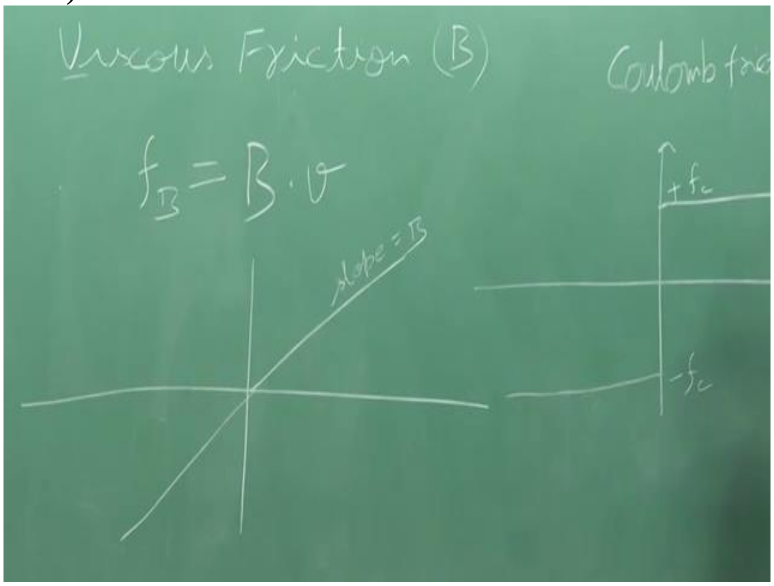

And the second type of friction is viscous friction and this friction is, whenever a body moves in a flute, we realise this friction and in our discussion, most of the time we realise this friction. So F of B is equal to B into V and if I plot it, it will be like this. And the slope is B.

And the third type of friction is the coulomb friction. This is constant in nature. Once the object starts moving, a constant force is applied on the body. And whatever the value of the velocity is, it will remain constant. In a study most of the time, the friction we observe is the viscous friction. So we will be ignoring the static friction and the coulomb friction.

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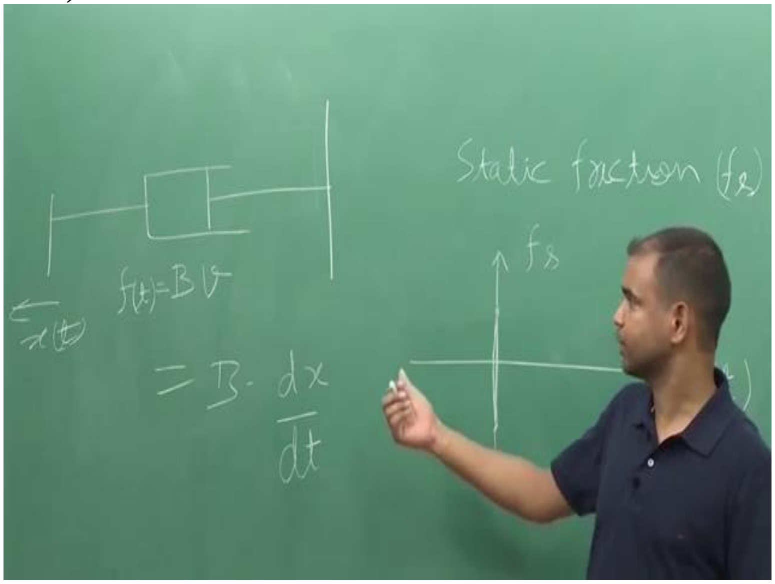

And the friction that we will observe is the viscous friction only and the force due to this friction is B into V or we can write B into DX by DT. And whenever the force is proportional to the rate of change of displacement, it will have an inherent tendency to dissipate the energy from the system to the environment. This will cause damping. That is why you call it damper also.

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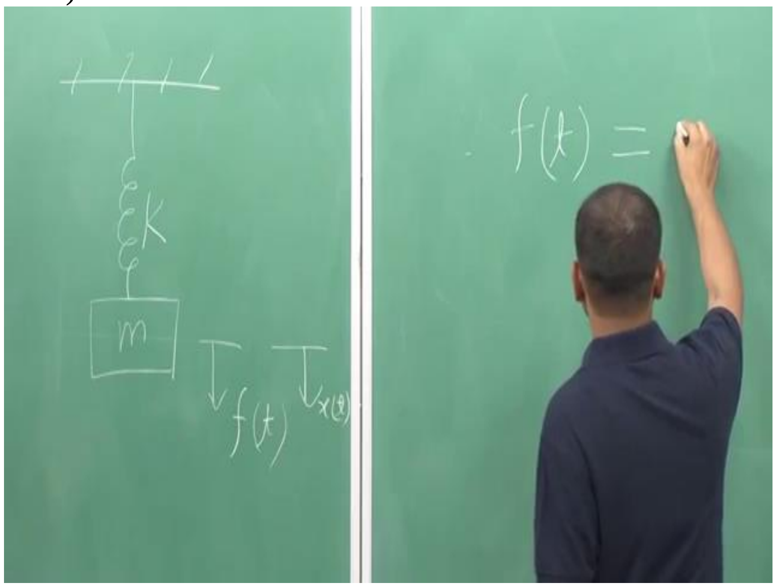

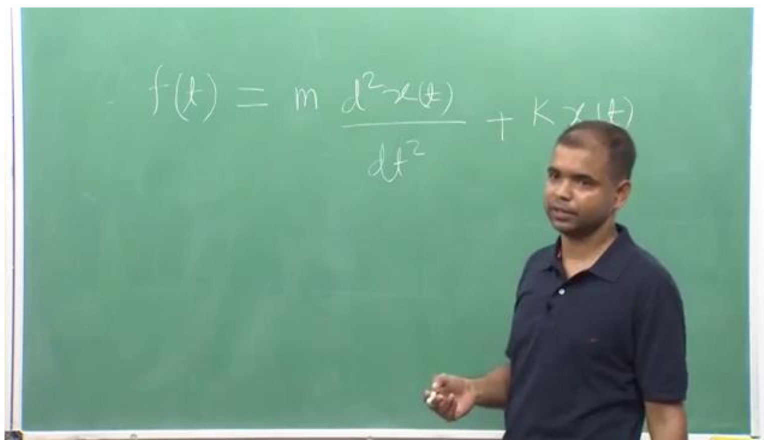

Now let us have a look at a spring mass and damper system. Suppose we have a spring and this is attached to a mass M. We apply a force here, F of T. The displacement is X of T and the spring constant of this spring is K. So when we apply the force, F of T, the net force, the opposing force and the applied force, they will be equal. So we can write, F of T equal to M D square X of T by DT square. This is the inertial force and the force due to the spring.

So both forces, this inertial force and this spring force, both are conservative in nature. That is why, this will keep oscillating and if I fall this differential equation, I will get the solution in the form of sine and cosine.

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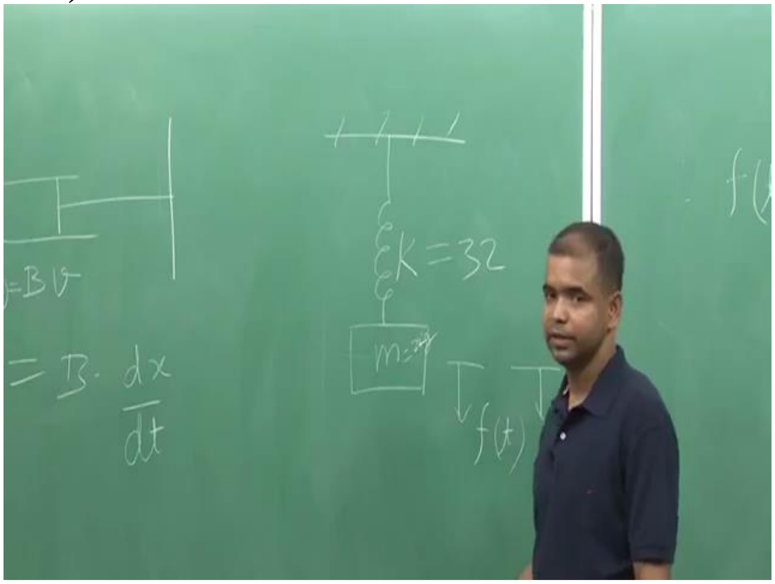

Suppose I have a spring constant of value 32 and mass of 2 kg, you can see its response plotted in this plot. Amplitude of the oscillation is not decaying with the time. So this, we call as undamped motion. The time difference between 2 consecutive peaks is constant throughout the oscillation. So its frequency is constant and this frequency, we call as undamped natural frequency. Now let us see if we attach a damper in the system, then how it is going to change the dynamics of the system?

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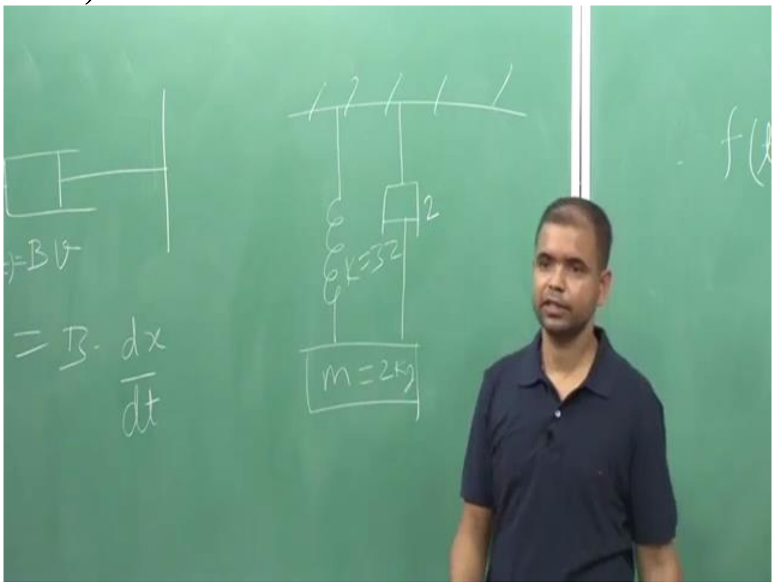

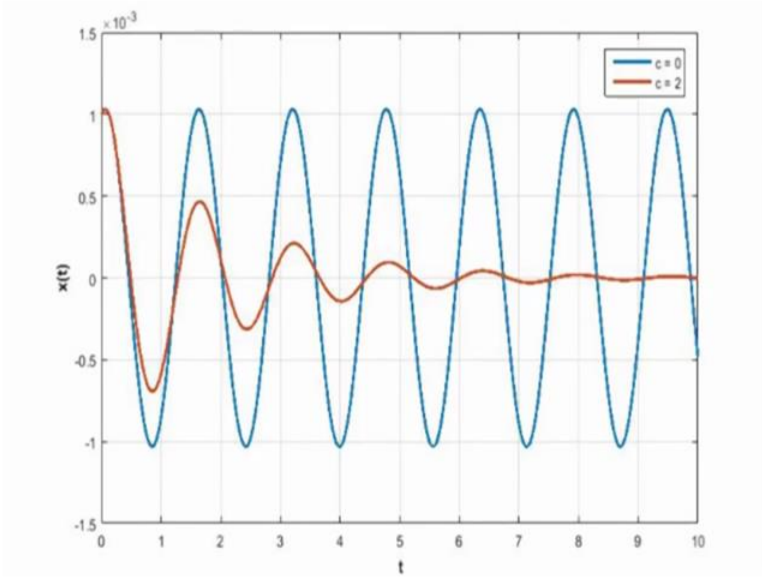

Suppose this is the spring and one mass is attached to this. It is 2 kg and the spring constant K is 32 and there is a damper and its value is suppose, it is 2. The amplitude of oscillation is decaying with time and it is due to, the damper has an inherent tendency to dissipate the energy of the system to the environment. That is why the energy of the system is decreasing and the amplitude of oscillation also is decreasing with time. And here also, you can see the difference between 2 consecutive peaks is constant and this we call as the damped natural frequency or damped frequency.

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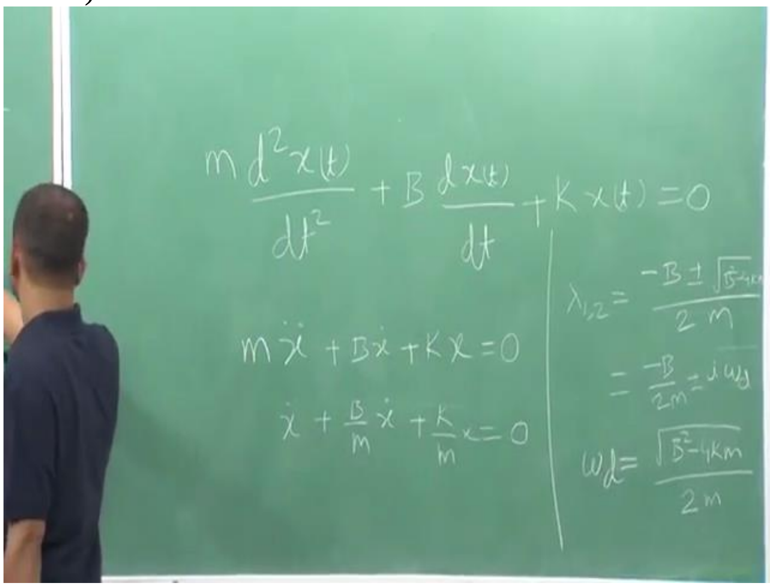

Now I can write its equation of motion. This is B. You can write C also. Both are same. If I ride its characteristic equation, it will be MX double dot + BX dot + KX equal to 0. This is the characteristic equation of this differential equation. Or I can write, X double dot + B by M X dot + K by M X equal to 0. If I see the roots of this equation, the roots are given by - B + - B square - 4KM by 2M. And this will be - B by 2M + - I Omega D. And this Omega D, we call the damped frequency. Omega D will be given by under root B square - - 4KM divided by 2M.

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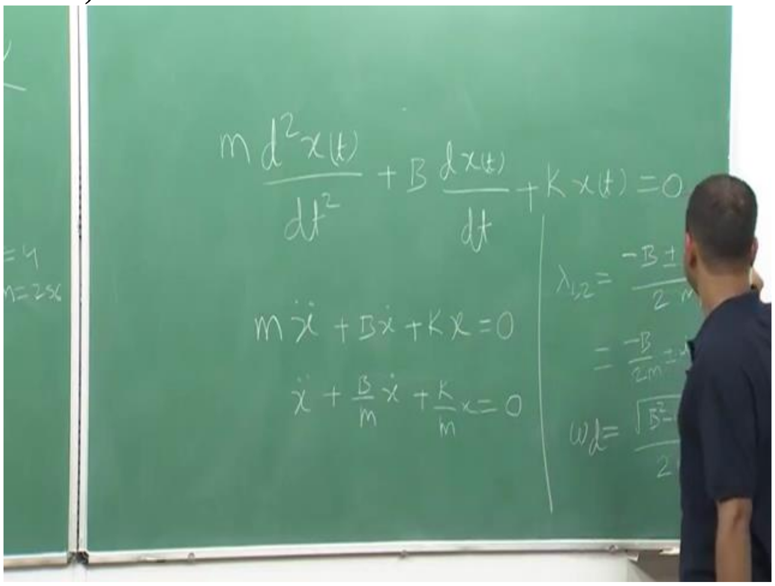

Now for B equal to 2, our B square - 4KM, B square will be 4 and 4KM is 256. So B Square is less than 4 KM. This 4KM is negative. We get imaginary roots and in that case, this is damped frequency.

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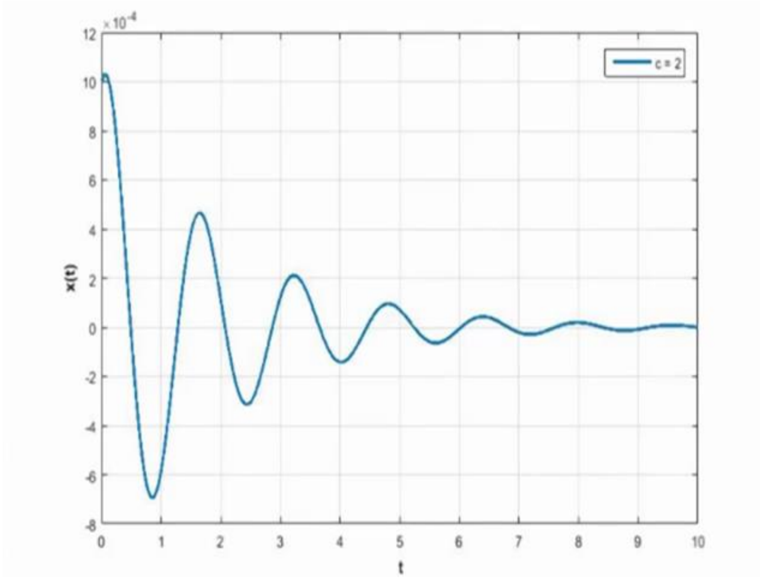

Here, in this curve also you can see, the amplitude is decreasing with time and the difference between 2 consecutive peaks and the inverse of that will give us this Omega D, the damped frequency. We can calculate by putting this value also.

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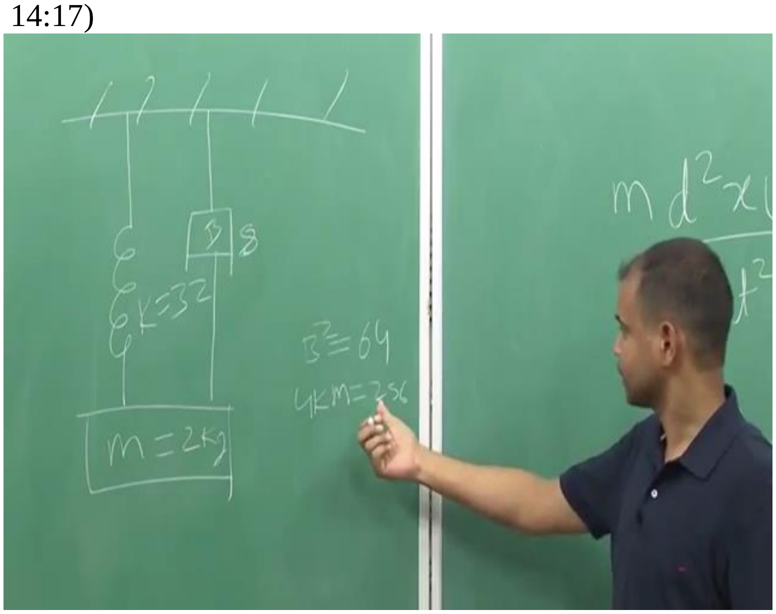

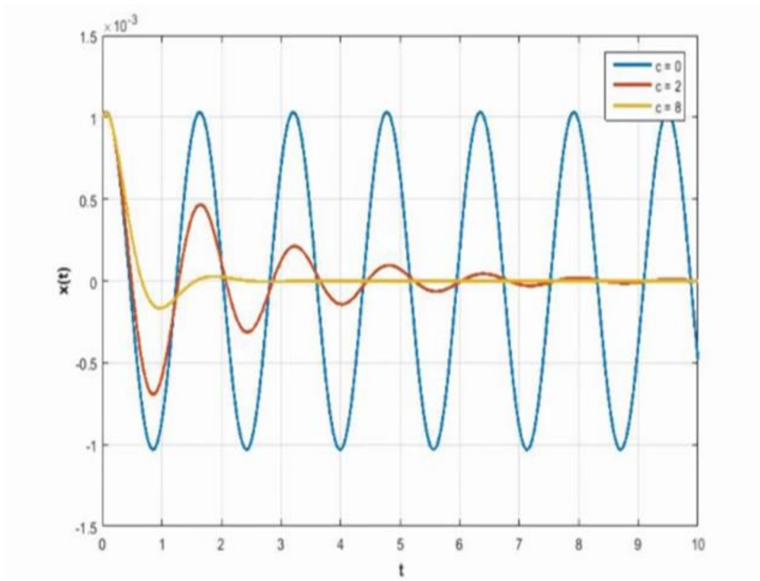

Now if I increase the value of this damping coefficient from 2 to 8, in this case, this will be 64. In this case also, this term, B square - 4KM will be negative and these roots will be imaginary and we will get under damped motion and here also you can see the damping is more but still it is under damped and the curve is overshooting its mean position and coming back to its mean position.

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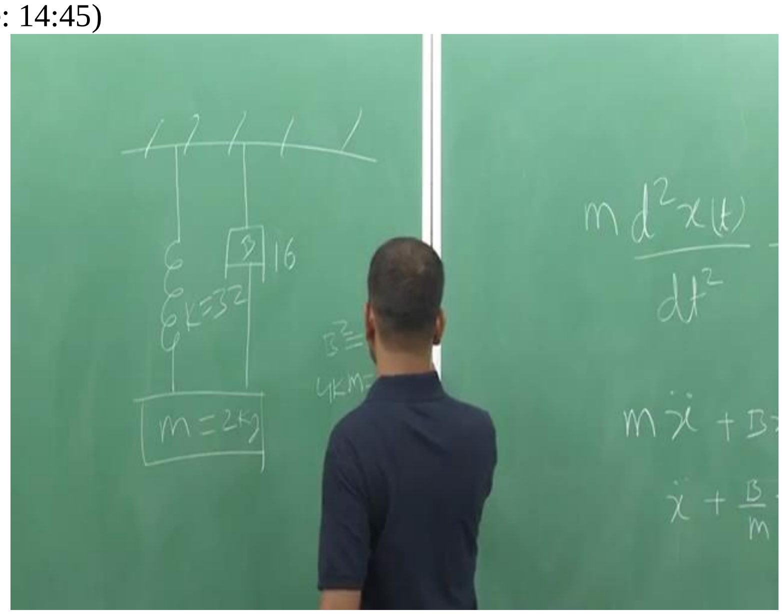

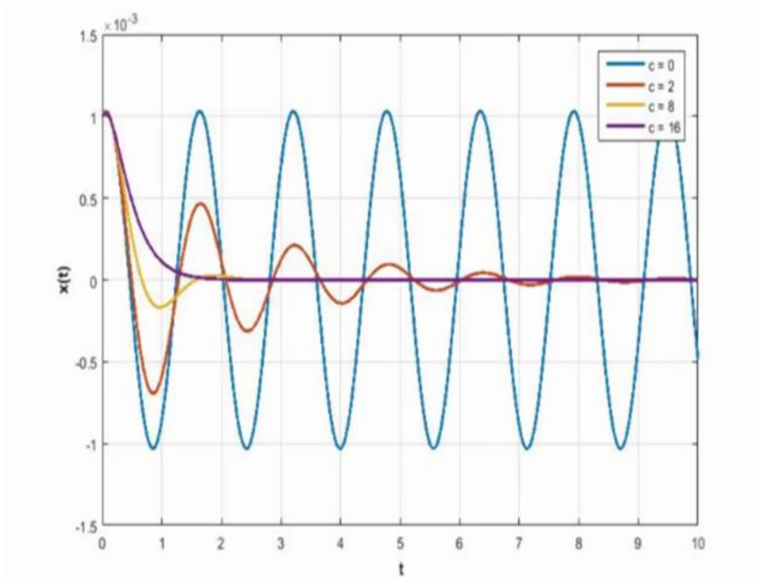

And if I further increase my damping ratio from 8 to 16, in this case, this will be 256 and this part will be 0 and this will be a negative term. In this case, we call it critical damping and here you can see, there is no overshoot. And with the increase in damping, the first time when the overshoot disappears, we call it critical damping. There is no oscillation in the response.

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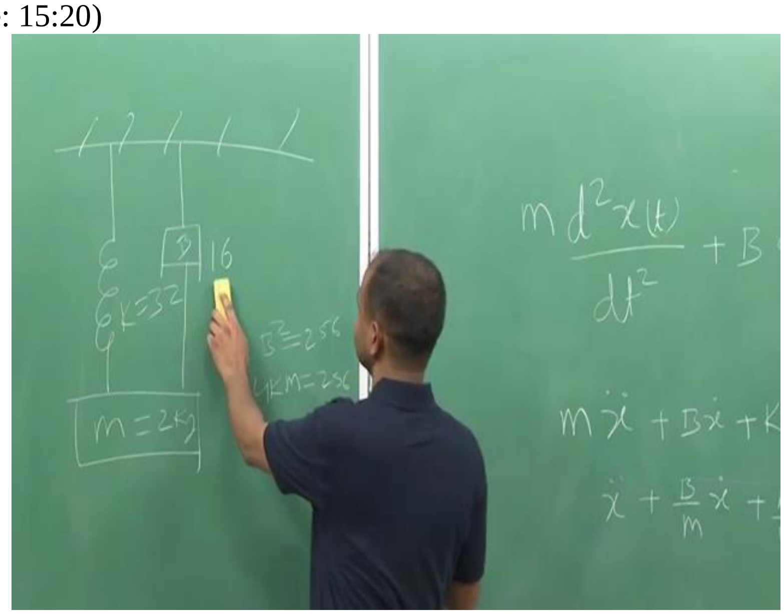

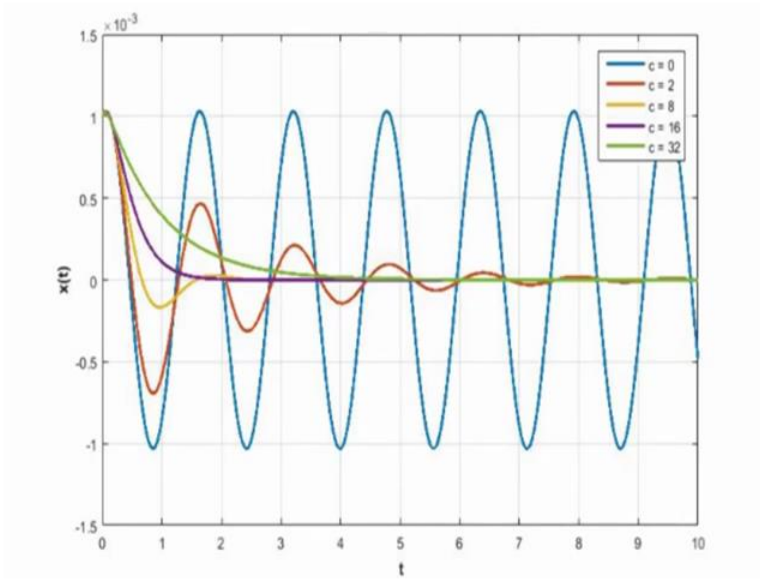

And if I further increase from 16 to 32 and so on, then you will have higher damping and this situation we call it as over damped response.

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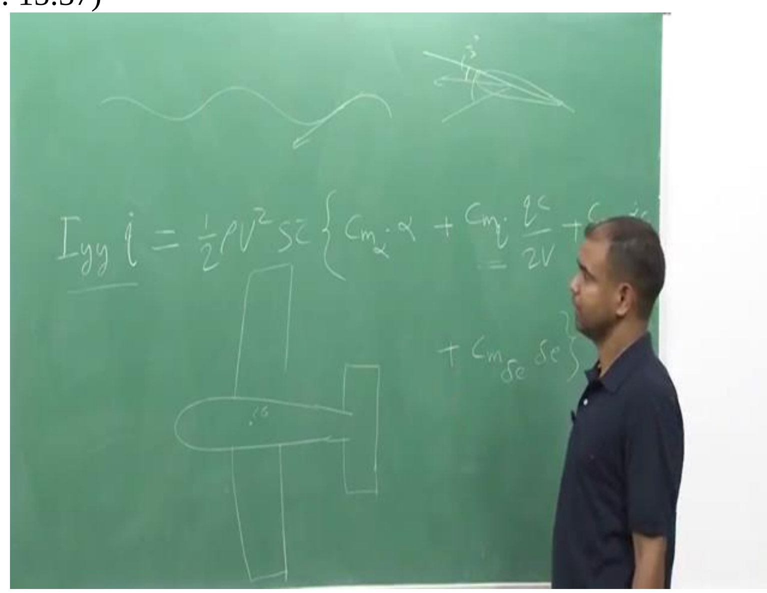

If I try to visualise my longitudinal motion or the pitch only motion of the aircraft, if you remember the equation, IYY Q dot, Q dot equal to half rho V square SC bar multiplied by CM alpha into alpha + CMQ into QC by 2V + CM alpha dot into alpha dot C by 2V + CM Delta E into Delta E. And here, you can see, this is basically mass into acceleration, this is inertia and this is rate of change. So this is the force if you see on the right-hand side, if I multiply this half rho V square SC bar with this term, the CMQ will have a velocity term into the multiplication because this square will be cancelled this velocity. And similarly with CM alpha dot also.

So you can see, CMQ and CM alpha dot, these 2 terms producing a moment on the aircraft which is proportional to the velocity and that is why ultimately this will act as a damper. This will be generating the damping effect in the pitching motion. If I see the top view of airplane, when there is a pitch rate, this horizontal stabiliser will have a tendency to oppose that change in the pitch and due to this force, we get this CMQ. And similarly when my aircraft is moving like this and suppose I am here, and suppose this is the velocity of the aircraft, when my aircraft was moving in this direction, at that time, if I had Angle of attack 3 degree and when it will be moving here, then angle of attack will be increased to some higher value.

And due to this increase, a moment will be realised on the aircraft and this will try to oppose the change in the pitch. And this is proportional to the velocity. So ultimately, it will try to damp that oscillatory motion about the mean position of the aircraft. Now let us solve one numerical.

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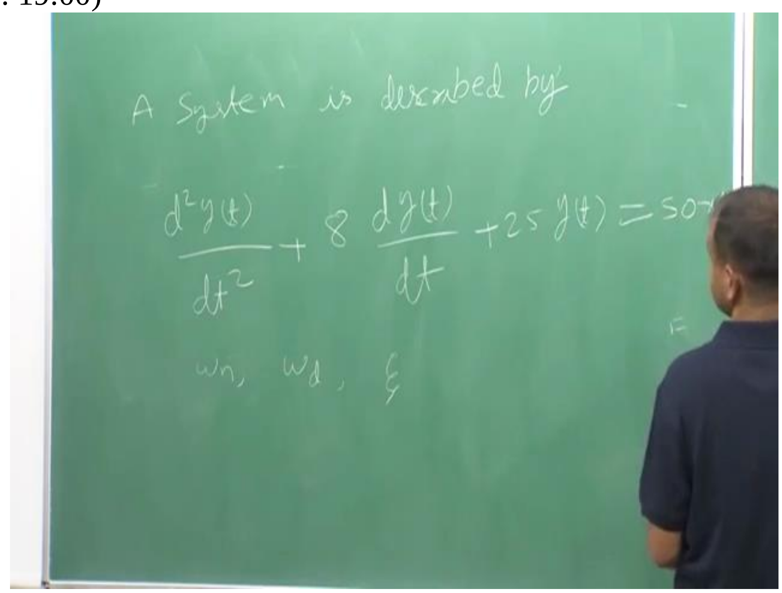

A system is described by a differential equation, D square Y of T by DT square + 8 DY of T by DT + 25Y of T is equal to 50 X of T. Now we have to find Omega N, undamped natural frequency Omega D, damped natural frequency and damping ratio Zeta.

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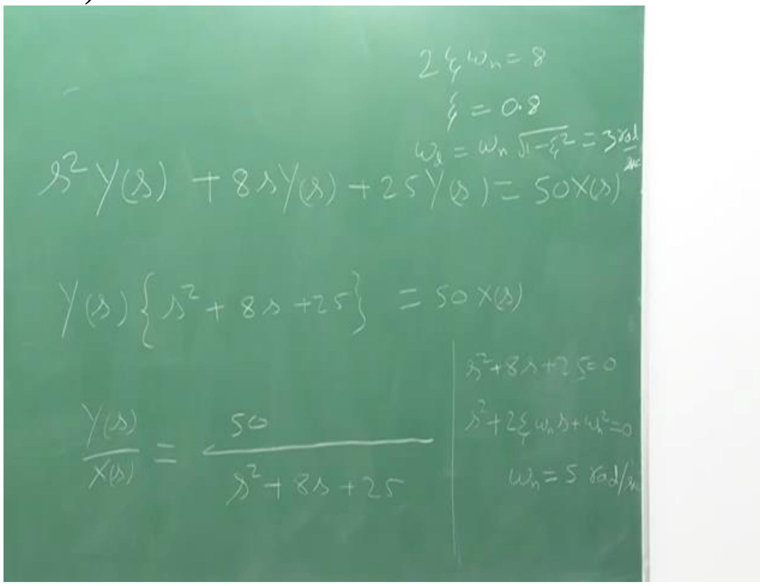

If I take the Laplace transform of this equation, I can write S square Y of S + 8 SY of S + 25Y of S is equal to 50 X of S. And I can write S square + 8S + 25. And from here, Y of S by X of S can be written as 50 by S square + 8S + 25. I can write from here, the characteristic equation, S square + 8S + 25 equal to 0. And now if I compare with the standard equation, S square + 2 Zeta Omega N S + Omega N square equal to 0. Then if I compare these 2 equations, I find Omega N square is 25. So Omega N equal to 5. So Omega N will be 5 radians per second. And 2 Zeta Omega N is equal to 8, Omega N is equal to 5 radian per second.

So I write Zeta is equal to 8 divided by 10. So it will be 0.8. Omega D, you can write Omega D equal to Omega N root over 1 - Zeta square. And from here, Omega N is 5 radian per second and rule over 1 - beater square. Zeta is 0.8. So 1 - 0.8 square. So it becomes 1 - 0.64 which is 0.36 and its root is 0.6. And Omega N is 5. Then 5 into 0.6 is 3 radian per second. Now coming to our next numerical.

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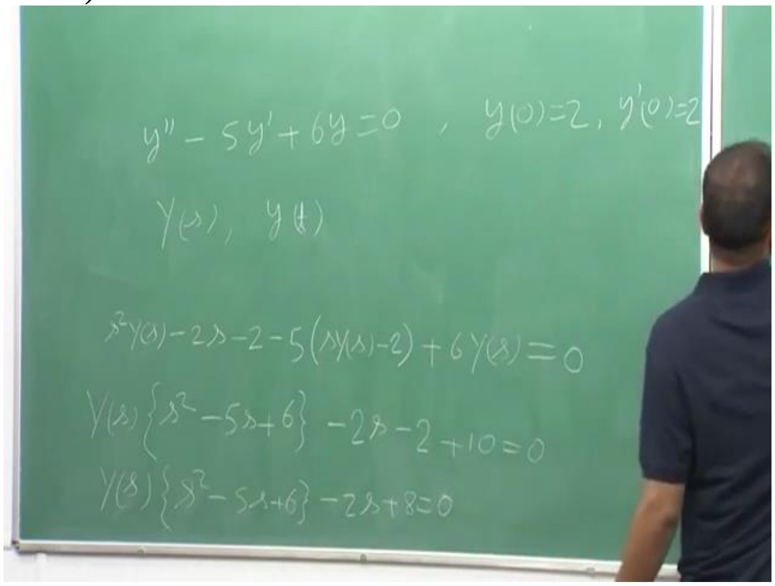

A system described by Y double dot - 5Y dot + 6 Y is equal to 0. And we are given the initial conditions, Y0 is equal to 2 and Y dash of 0 is 2. And we have to find Y of S and Y of T. So by taking the Laplace transform, we can use the formula. The Laplace transform of D Square Y of T by DT square is given by S square Y of S - SY of 0 - Y dash of 0.

And the Laplace terms form of DY of T by DT is given by SY of S - Y of 0. So by using these 2 formula, we can write S square Y of S - 2S SY of 0 and Y dash of 0 is also 2. So - 2 - 5.Y dash 0 SY of S - 2 + 6Y of S equal to 0. And if I separate Y of S from this equation, I will get S square - 5S + 6 and the remaining term, - 2 S - 2 + 10 is equal to 0. And from here, we can get Y of S, S square - 5S + 6 - 2S + 8 is equal to 0.

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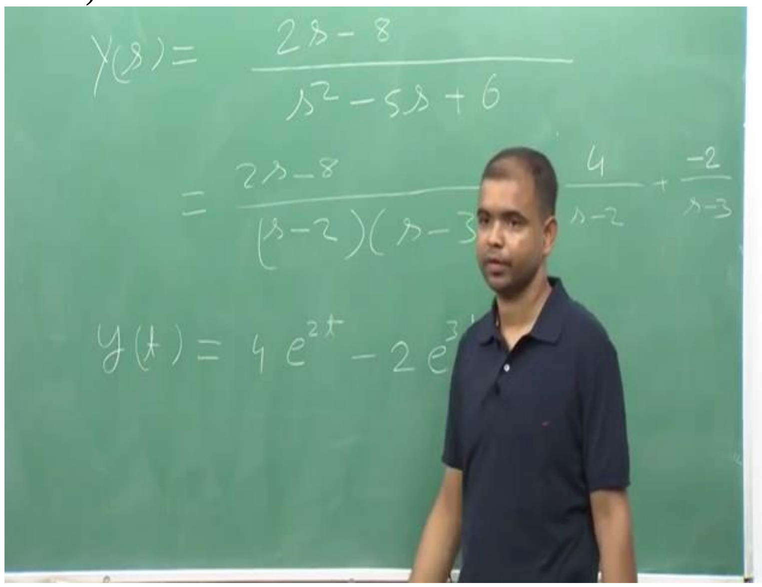

So finally we can get Y of S 2S - 8 divided by S square - 5S 6. And now if we break it with partial fractions, then S square - 5S + 6, we can write it, we can write this term, S - 2 into S - 3 and if I break it into partial fraction, I write it is equal to A by S - 2 + D by S - 3 and if I determine the value of A by putting S equal to 2 and eliminating this term, I will get 2 into 2 - 8 divided by - 1.

So this will be 4. So I write 4 by S - 2. And now B. I put S equal to 3 and I eliminate this term. Then I will get 2 into 3 - 8 and 3 - 2 this will give me - 2 divided by 1. So this will be - 2. So this is my expression for Y of S. And the next part is, what is Y of T? So if I take inverse Laplace transform, I will get Y of T is equal to 4 E to the power 2 T - 2 to the power 3T. Thank you