Aircraft Dynamic Stability & Design of Stability Augmentation System
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Module 1
Lecture No 05
Pitch Dynamics – 1D

Yes, dear friends. We have gone one more step and we have seen for a Mass Spring Damper System of second order in nature, how do I realise what is the mathematical background or trick or technique required to find out the damping ratio and natural frequency. It is more important to make our understanding very clear that a second order system need not oscillate. It will oscillate or it will come to your, return without oscillation that depends upon what is the value of C, the damping constant. Or if I tell in terms of zeta, if the damping ratio is less than 1, it will be oscillatory return.

If it is zeta greater than 1, it will be over damped case, no oscillation. And zeta equal to or damping ratio equal to zeta critical or damping ratio equal to 1, then it will also not have oscillation. It will come back and the return will be fastest. All those things you need to analyse before you apply all this trick. Yes? Do not do it blindly. But preparing ourselves with this sort of a trick and skill, we need to see how to put our 1st foot forward to analyse an aircraft and aircraft, dynamics. We will not rush. We will take a simple case.

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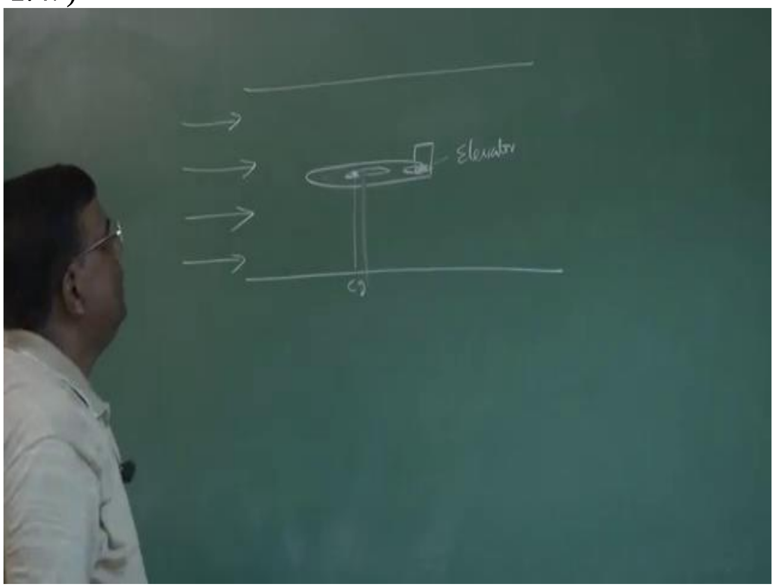

We will take a case where, say this is a wind tunnel and I mounted aircraft. This is the elevator and I mounted in such a way that. I mounted in such a way about the hinge point that it has the pitch degree of freedom. It can only do pitching.

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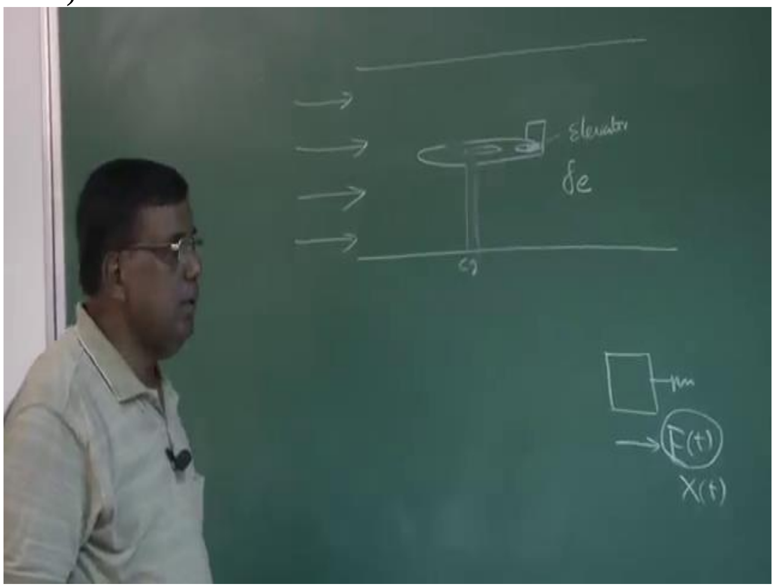

No motion like this. Only pitching. No plunging motion. Only pitching motion. No joying motion, no lateral motion. Only pitching motion is permitted. I want to analyse this. What I do? I give it a small disturbance. I give some elevator input, Delta E and then release it. I will have to study how this oscillation and how its transient will behave. And I would like to use that whatever you have understood for a mass spring damper system, can I use those things correctly or not?

And when you are using it, what more understanding is required which will make our life easier to transform the knowledge of a mass spring damper system to an aircraft system? Remember, for a mass spring damper system, we are talking about ST and we are talking about motion, linear motion along X direction. But here, the difference is, this is an angular motion. So whatever this forcing function was doing as a member of force, here the force is replaced by the moment.

What type of moment? What is this moment called? Moment about Y axis which is pitching, this is called as pitching moment.

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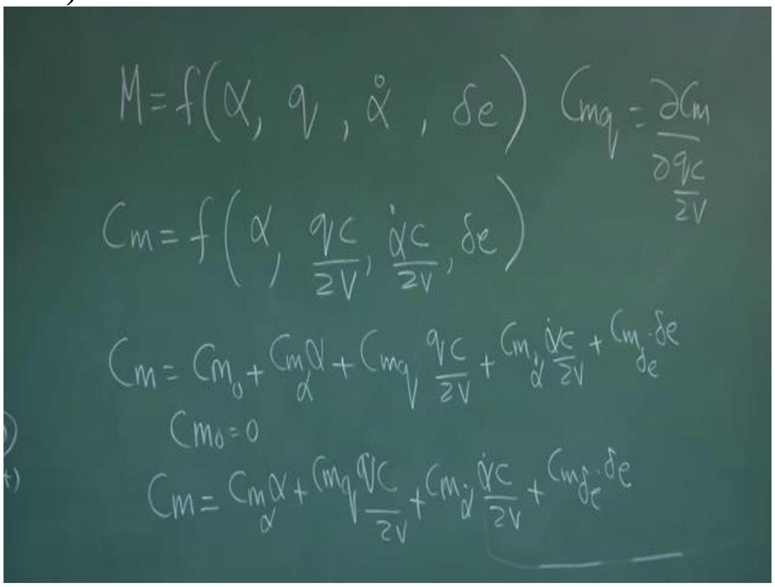

And if you recall, this pitching moment will be function of alpha, angle of attack. We know what is angle of attack? Angle of attack is the angle between the chord line and the velocity vector. Then, it will function of Q that is the pitch rate, what pitch rate it is oscillating. Function of alpha dot and I am putting Delta E. We have done all these things in our last course, alpha dot because you know, there is a delay in the down wash.

So whatever time you are analysing, actually, those down wash has not happened because it takes finite time for studying what it takes to travel to the tail. So there is a delay. And on a quasi-study approach, we try to approximate moment through its non-dimensional from this F alpha or Q, we used to write QC by 2V. Why? Because we want to keep all this argument in non-dimensional form.

You see, alpha is in radiant, non-dimensional, Q is radians per second which is dimensional. So we take this Q to QC by 2V. Similarly, alpha dot also, we will do like that which all these things, you are familiar and those who need more clarification, they follow my last lecture on an aircraft stability. If this is the form, then using Teller study, assuming a linear ergonomics, I can write CM, expand CM as CM not + CM alpha into alpha + CMQ into QC by 2V + CM alpha dot into alpha dot C by 2V + CM Delta E into Delta E. What is CM alpha?

This is DCM by D alpha. What is CMQ? CMQ was D CM by DQC by 2V. Q CM alpha dot is DCM by D alpha dot C by 2V. Q at CM Delta. It is at partial derivative. What is the meaning of

CM alpha? It is the change in CM per-unit change in alpha holding other variables constant. Right? So this is partial derivative.

Let us take a simplified case where CM not equal to 0, then CM will be equal to CM alpha into alpha + CMQ into QC by 2V + CM alpha dot into alpha dot C by 2V + CM Delta E into Delta E.
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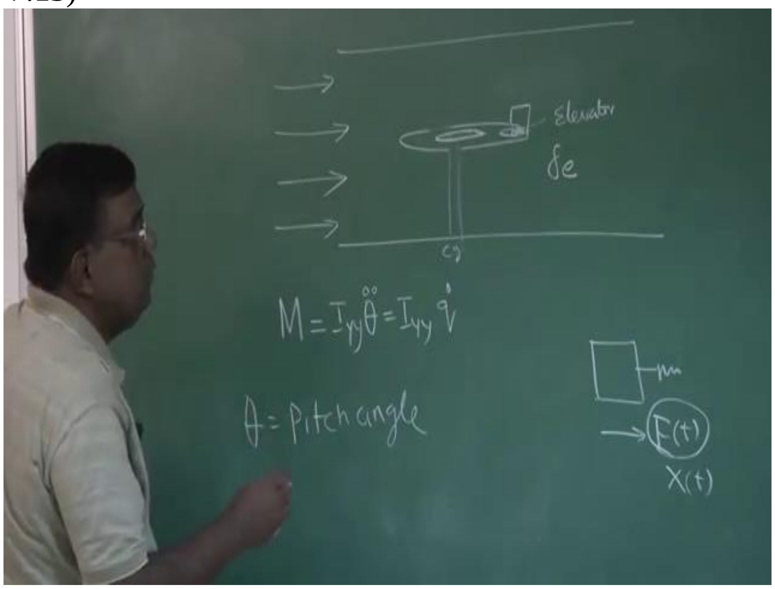

Once I put CM not equal to 0, come back here, if my wing is symmetrical, fairly symmetrical, the body is (())(7:21) symmetrical, everything is symmetrical, then naturally, alpha equal to 0. There will not be any CM. So I can always put CM not equal to 0. Now come back. When we were doing mass spring damper system, our first step was to write the equation of motion. That is how we wrote X double dot equal to this this this or X double dot + C by M or X double dot K by MX equal to F of T by M.

That was an equation of motion. But here also we have to write an equation of motion but we have to understand that this is now angular motion. So that will become one by M equal to IYY Theta double dot or I write it IYY Q dot. Please understand what is Theta. Theta is the pitch angle.

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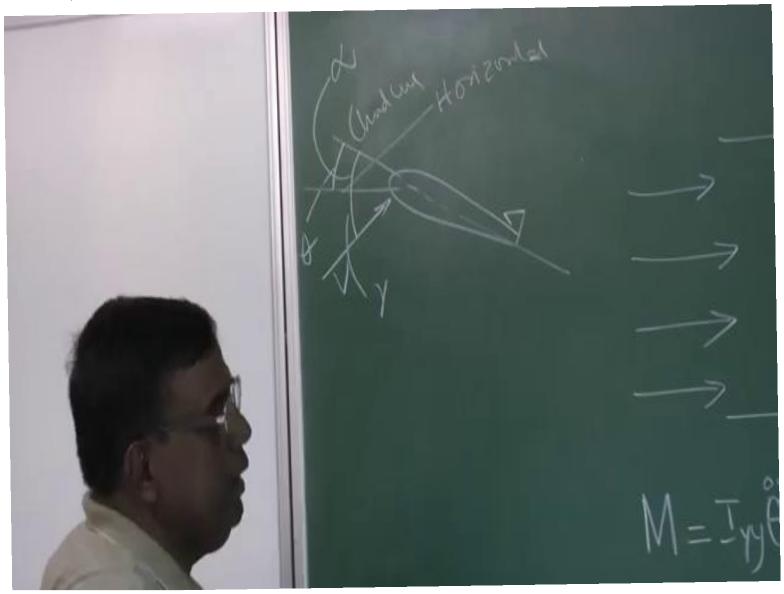

Plus to make you more comfortable so that this is the airplane and this is the chord line. This is the velocity vector and this is the horizontal, then the angle between velocity vector and chord line is alpha and how much this chord line is working with horizontal is Theta and how much the velocity vector is making with horizontal is gamma. I repeat, angle between chord line and the velocity vector is alpha, angle between the horizontal and chord line is Theta, that is pitch angle and angle between plus director and horizontal is gamma.

We are talking about Theta, pitch angle. That is, if this is the airplain, this is the horizontal, then this angle, what is the pitch angle? How much angle it is? Altitude is changing. And the rate with which it is changing which is Theta dot which is Q and the acceleration is Theta double dot or Q dot, angular acceleration. So this is typical, moment equal to I into alpha, angular isolation. So I am writing M equal to IYY Q dot. I am trying to develop the equation of motion because I know if I want to solve it like spring mass damper system, my first approach should be to write equation of motion.

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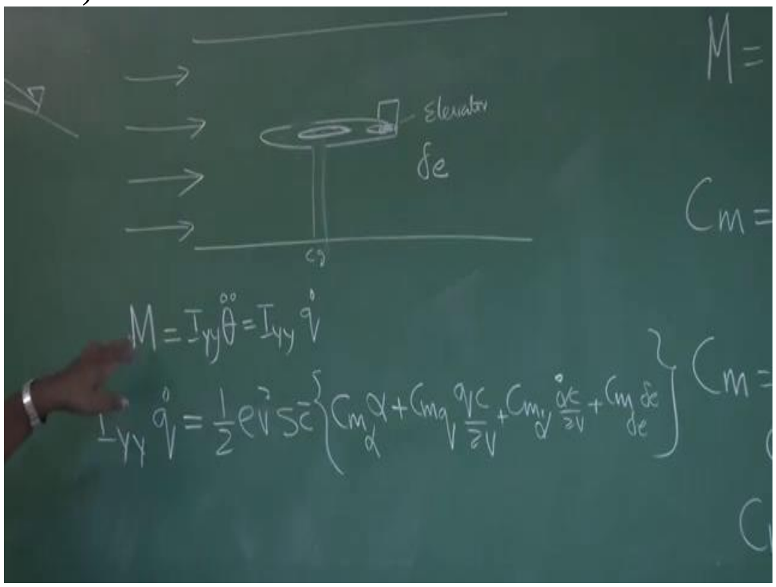

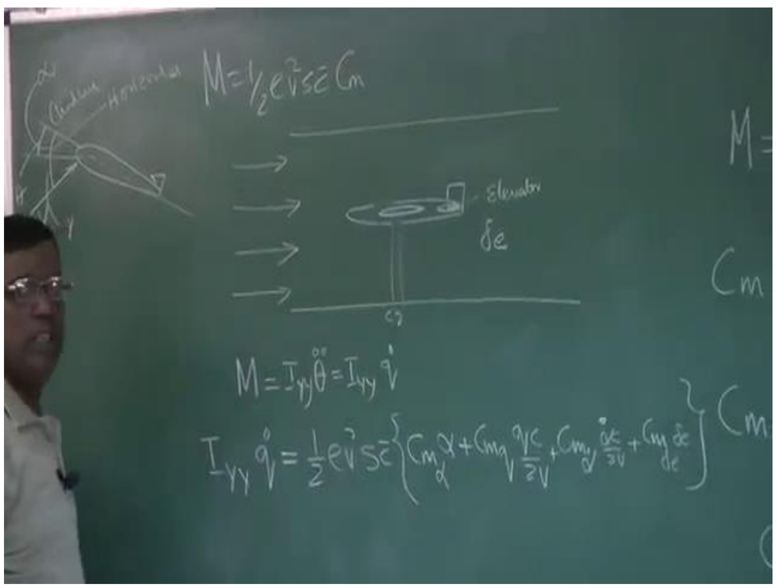

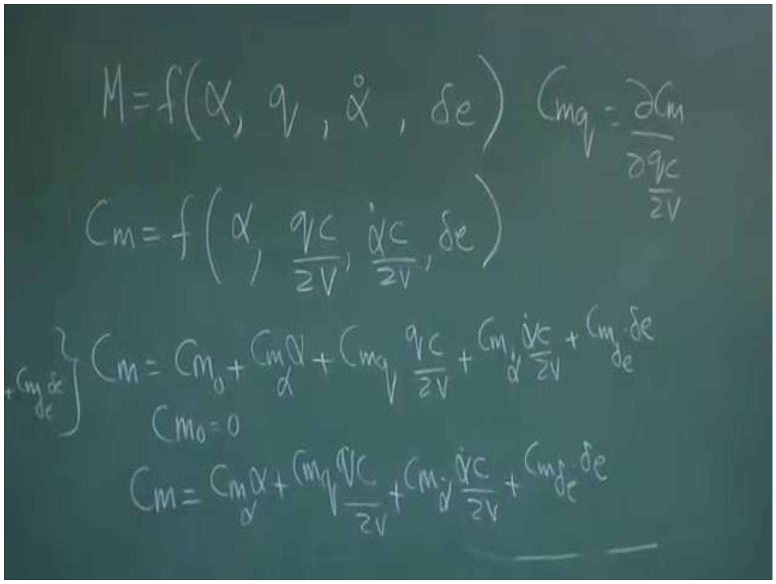

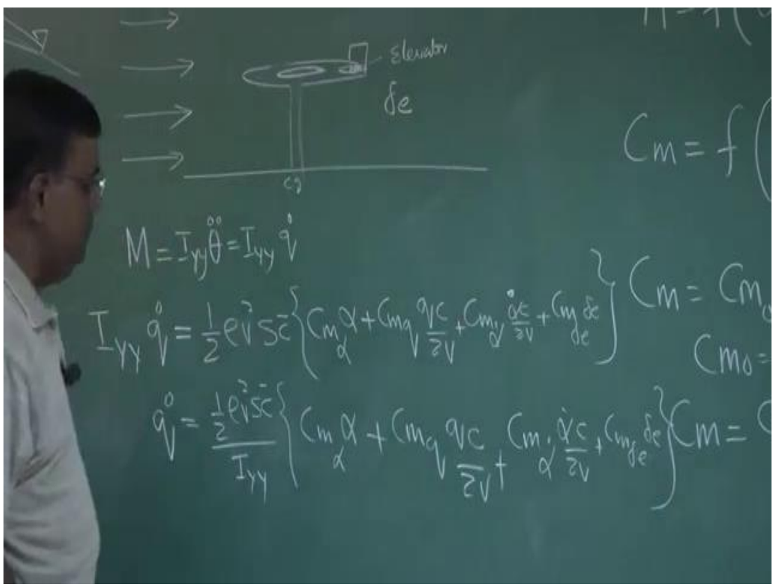

So what do I do now? I write IYY Q dot equal to moment. Moment will be what? Half rho V square SC into CM. For CM, I write CM alpha into alpha + CMQ into QC by 2V + CM alpha dot into alpha dot C by 2V + CM Delta E into Delta E. Correct? This is the moment. Moment is nothing but half rho V square SC CM. Remember, how do I write moment? It is half rho V square SC bar into CM. That is by definition. For capital CM, I am using this expression.

And why I am doing all these things, I want to write the equation of motion and the equation of motion here is regarding the angular motion. So now I can write, Q dot equal to half rho V square SC bar by IYY. IYY is the moment of inertia about the y-axis. Into CM alpha into alpha + CMQ into QC by 2V + CM alpha dot into alpha dot C by 2V + CM Delta RE into Delta T.

Now let me write in a neater form so that there are no confusions. Do not forget, we are developing equation of motion, angular motion and try to see how best we can use whatever we were understood through mass spring damper system.

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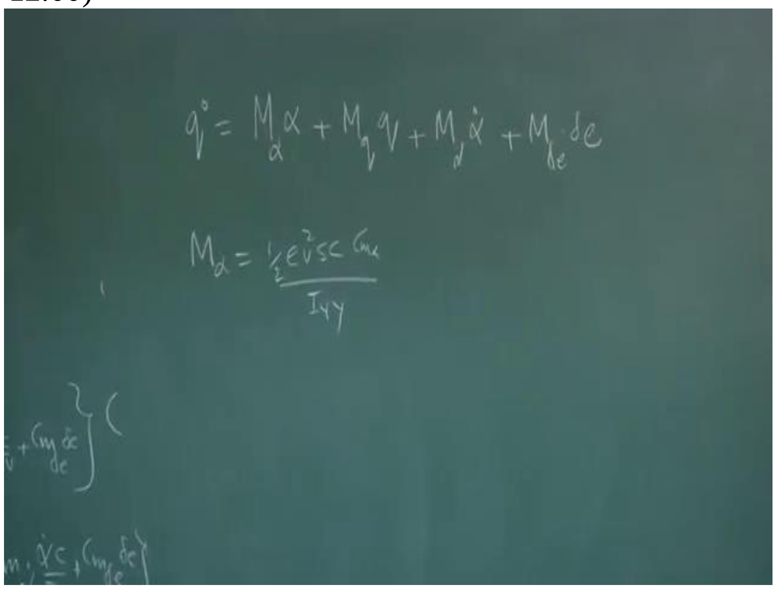

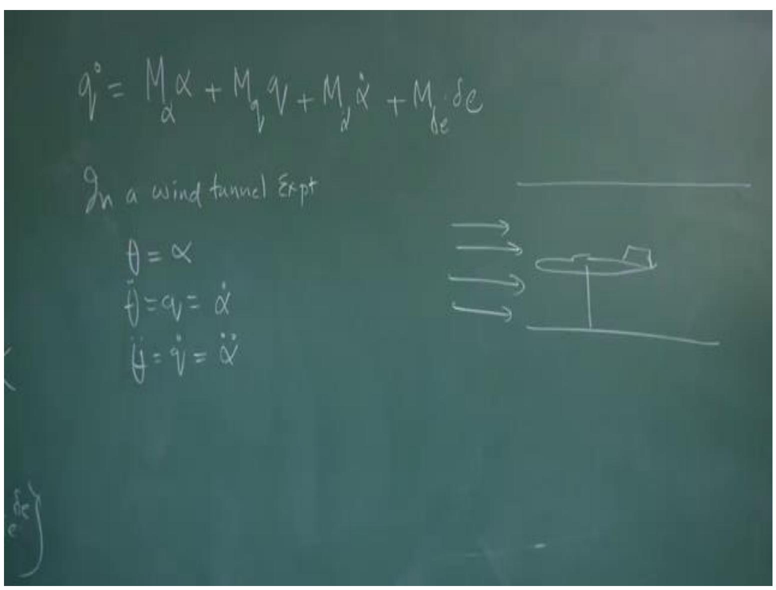

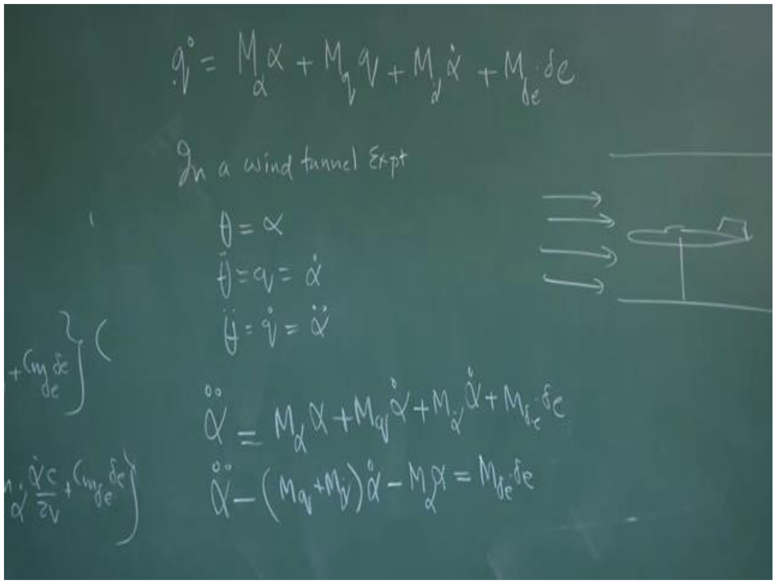

So I write Q dot equal to M alpha into alpha + MQ into Q + M alpha dot into alpha dot + M Delta E into Delta E. From here, I can write in this form. Then what will be M alpha? M alpha would be half rho V square SC bar CM alpha by IYY. Very simple manipulation. The whole of this into CM alpha is M alpha. The whole of this into CMQ into C by 2V will be MQ. Like that, you can always find out. I will erase this. That is for your understanding.

Now you know that in a wind tunnel experiment, when I am drawing it like this, this is the wind tunnel, this is the body or airplane and the velocity vector remains unchanged. That is governed by the pressure ratio. So what is happening? In a wind tunnel scenario, when an airplane is not supposed to plunge like this only those days, in that case, whatever Theta is there, that becomes alpha.

Similarly, whatever Theta dot equals to Q becomes alpha dot. And whatever Theta double dot equal to Q dot, that becomes alpha double dot. This is clear? In a tunnel, in actual practice what will happen for and airplane? If there is a disturbance, it will not only do this but it will also change the height because there is a lift force. It is allowed to move but here because it is having only 1 degree of freedom, it can only do this. The velocity vector, the gamma of light path angle remains constant.

So Theta equal to alpha, Theta dot equal to Q equal to alpha dot and Theta double dot equal to Q dot equal to alpha double dot. So I will use it here. So for Q dot, I will write alpha double dot equal to M alpha into alpha + MQ into alpha dot + M alpha dot into alpha dot + M Delta E into Delta E. That becomes my modified equation of motion under a wind tunnel approximation where it assumes, the model can have only 1 degree of freedom. It can do only like this. No plunging motion.

Now, I will do little manipulation because I want this, I want a mass spring damper type equation of motion. So I will write it like this, alpha dot - MQ + M alpha dot into alpha dot - M alpha into M alpha equal to M Delta E into Delta E. Okay? What do you now see? Do you feel delighted? We have conquered our writing skills and we have come exactly same as the second order system.

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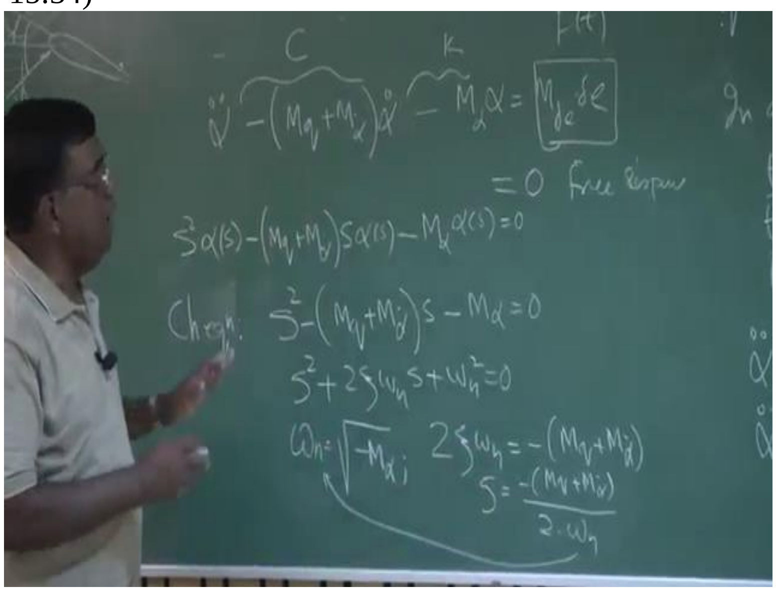

So this is alpha double dot - MQ + M alpha dot into alpha - M alpha into alpha equal to M Delta E into Delta E. For free response, I put this to be 0. The forcing function is like F of T. Here, this is like C and this is like K. Did you see? X double dot + CX dot + KX equal to F of T. Exactly same form. Now the problem is solved.

So now once we have this sort of equation, how do you find what is zeta and what is Omega N? What you do, you take Laplace transform here and you get S square alpha of S - MQ + M alpha dot S alpha of S - M alpha into alpha of S equal to 0. I have taken Laplace transform of this essential equation. So I have got a characteristic equation now as we got for a mass spring damper system as S square - MQ + M alpha dot S - M alpha equal to 0. This is my characteristic equation.

What is our aim? Our aim is to find zeta and Omega N. Now we will compare with the standard equation which we have developed already for a second order system and that is typically like this for a free response. So now I compare, what is Omega N? Omega N is under root of - M alpha and what is zeta? I will find like this, 2 zeta Omega N equal to - MQ + M alpha dot. So if I put Omega N value here, 2 goes down. So I can find out zeta.

So zeta will be - MQ + M alpha daughter divided by 2 Omega N and Omega N I can (())(17:56). How beautifully see we got the value of natural frequency and the damping ratio using whatever we have learnt for a second order mass spring damper system. Apart from the mathematical trick or skill, we also realise one thing that when I write - M alpha and this is under square root, so this becomes a real number only one M alpha is negative.

That means, this analysis is for statically stable airplane. Please understand. That is why I am telling you, you need to understand the basics very clearly rather than applying it blindly. Also here, we understand that zeta prime depends upon damping derivative MQ and for a mass spring damper system, zeta prime will depend upon C, damping constant, C. But here it is, damping constant equivalent is MQ.

And naturally CMQ means moment because of rate. That is the damping characteristics. So it is all consistent, handy unless you will only be using this understanding when you analyse the dynamic stability of an airplane. Thank you very much.