Aircraft Dynamic Stability & Design of Stability Augmentation System
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Module 8
Lecture No 42
Numericals: SAS

Hello friends. In previous lectures, you saw about stability augmentation system. How the roots were to be changed? How roots are changed using stability augmentation system and what will be the benefits of using stability augmentation system. We will be seeing some numericals based on this stability augmentation system to make the concept more clear to you.

So for that, let us take an aircraft which is constrained to yawing motion. Let us see what will stability augmentation have effect on that motion. So as you have already seen in previous lectures, the differential equation for yawing motion was given by.

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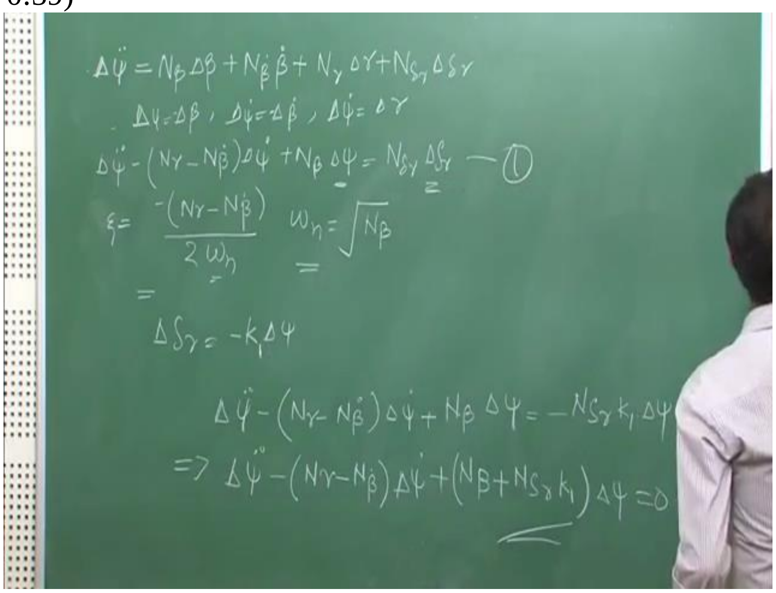

Delta Psi double dot. Do not get confused with this Delta. This represents your perturbed variable. You can write Delta Psi dot or Psi dot in small. It will be same. I prefer using notation Delta of Psi dot. Delta of Psi dot equals to, your differential equation with respect to yawing motion was Delta Psi dot equals to N beta into Delta of beta + N beta dot into beta dot + NR into Delta R + N Delta R into Delta of Delta R.

As you know, you can write, since a body is constrained for yawing motion, so you can write, Delta Psi equals to Delta beta, Delta Psi dot equals to Delta of beta dot and Delta Psi dot equals to Delta of R. So this is - beta. Delta Psi equals to - of beta and - of beta dot. Further using these values in my differential equation, your differential equation will become, Delta Psi double dot - NR - N beta dot into Delta of Psi dot + N beta into Delta of Psi equals to N Delta R into Delta of Delta R.

So this is your differential equation. By seeing this, you can sell this is a 2^{nd} order differential equation. The value of Zeta for this differential equation will be - NR - N beta dot divided by 2 Omega N where Omega N will be root over N beta. So this represents my damping ratio and natural frequency. Now suppose I want to change the value of natural frequency. So as per your stability augmentation system, I will be writing your state variables in terms of my input parameters. So let me write Delta R as - K of Delta Psi, K1 of Delta site..

Substituting this value in my differential equation 1, we will be getting Delta Psi of double dot - NR - N beta dot Delta of Psi dot + N beta Delta Psi equals to, value of Delta of Delta R equals to - N Delta R K1 Delta site. Further, this can be written as Delta of Psi double dot - NR - N beta dot into Delta of Psi dot + N beta + N Delta R K1 into Delta of Psi equals to 0.

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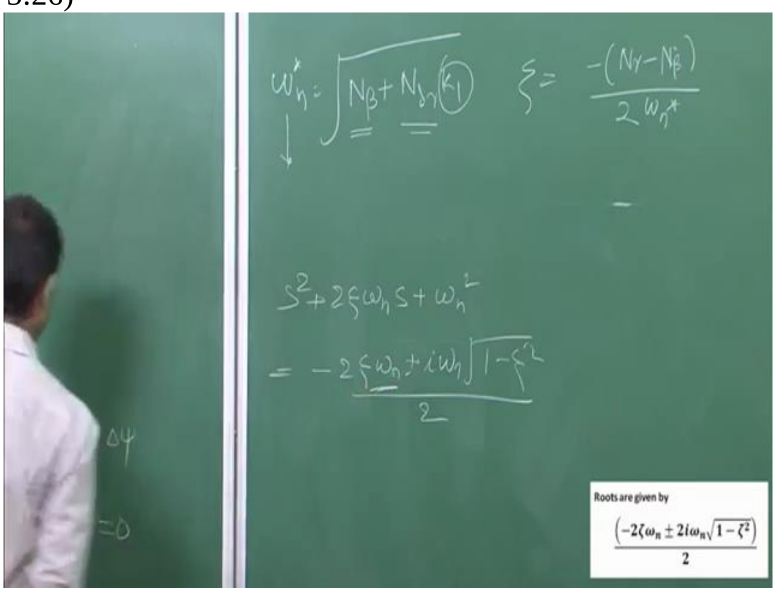

Now from this differential equation., you can see, the value of Omega N is root over N beta + N Delta R into K1. Using the required value of natural frequency, you can easily determine substituting the required value of Omega N here. And as you know, these are already given for original equations, so you can determine the value of K1.

And similarly, your Zeta will be - NR - of N beta dot by 2. This new value of Omega N star. So you can see, by substituting your value of Delta R in terms of K you want to change, you can change your system, new system's Zeta and Omega N. But see that still your real part is same.

Since you know that for any second-order system, your roots will be given as, for second-order system represented by S square + 2 Zeta Omega N S + Omega N square. Roots are given as - 2 Zeta Omega N + - I Omega N root over 1 - Zeta Square upon 2.

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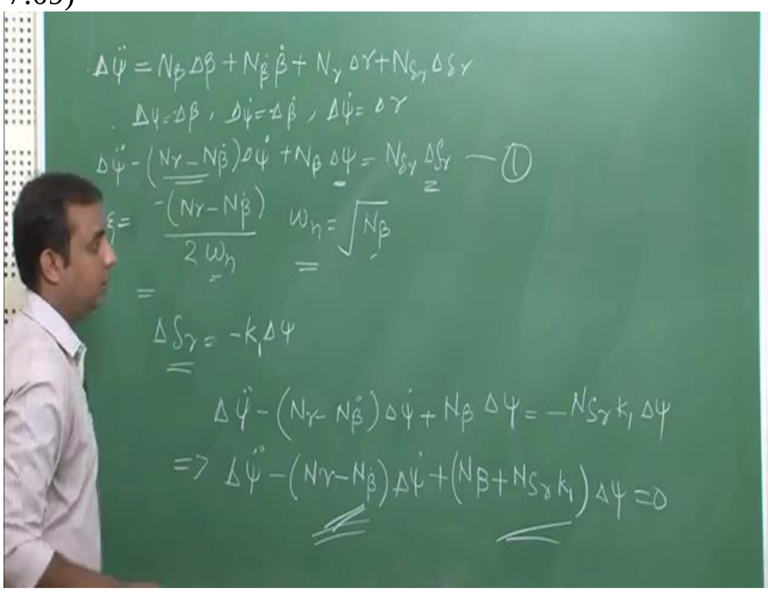

So since Zeta and Omega N term is same, in this equation in which we use stability augmentation system and Zeta, Omega N is same even in the original differential equation. So real part will be same. So you can say the decaying rate will be same. Only then frequency, the natural frequency will change in this 4^{th} equation.

Suppose I want to change both, your value of damping ratio as well as natural frequency, we will be using 2 values of K1. One will be with respect to Zeta, another will be with respect to Zeta dot. I will be showing you in this coming calculation. Suppose I wanted to change Omega and Zeta at the same time, so I will be taking 2 values.

I will be taking your input signal, that is rudder deflection that is with respect to your state variable, Delta Psi and Delta Psi dot. Because this will affect your natural frequency and this part will be for changing your Theta.

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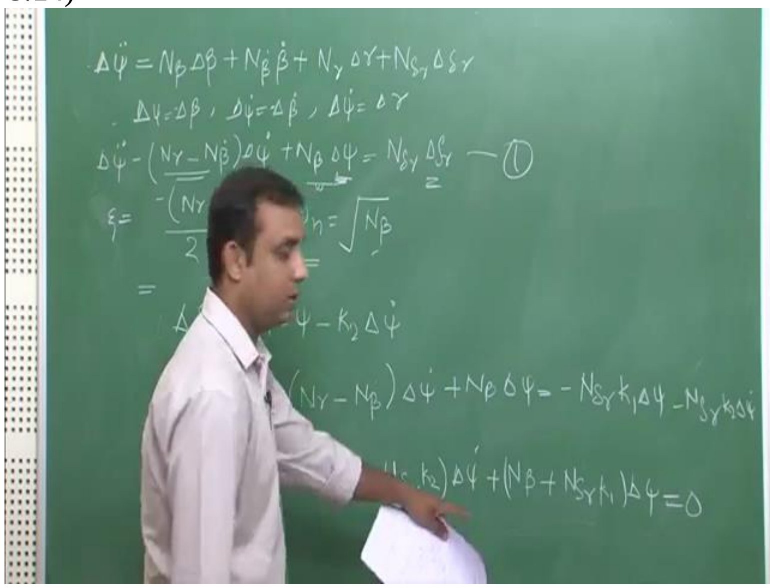

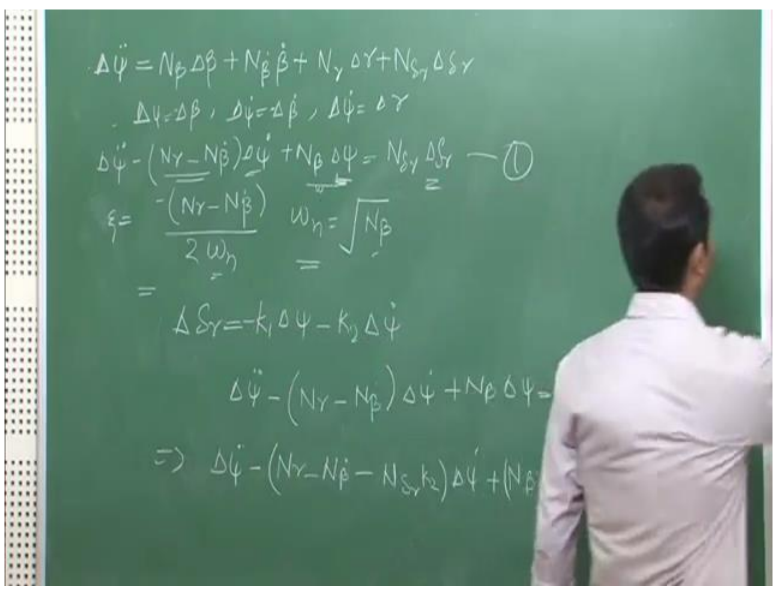

So the new value in terms of your rudder deflection in terms of state variables will be Delta R equals to - of K1 Delta Psi, - of K2 Delta of Psi dot. Now substituting this value in your differential equation 1, the new differential equation which you will get is Delta Psi double dot - NR - N beta dot Delta Psi dot + N beta Delta of Psi equals to - N Delta R K1 Delta of Psi - N Delta R K2 Delta of Psi dot.

Further solving this, Delta of Psi dot NR - N beta dot - N Delta R K2 Delta of Psi dot + N beta + N Delta R K1 Delta of Psi dot equals to 0.

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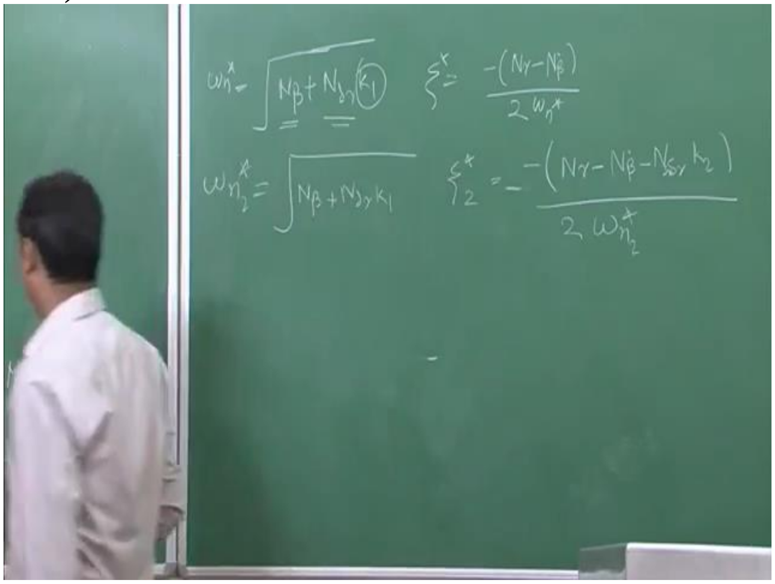

Now from this equation you can see, the new value of Omega N2 star is root over N beta + N Delta R K1 and new value of Zeta 2 star equals to minors of NR - N beta dot - N Delta R into Delta R of K2 divided by 2 Omega N2 star. Using this value of K1 and K2, we derived that the new values of Zeta and Omega N.

And similarly, as derived for this particular set of variables, you want some desired values of Zeta and Omega N, just put these values, required values of Omega N here and Zeta 2 here. 2 equations are there. You can solve and get the values of K1 and K2. Substitute that in this equation and you will get your response as per your new Zeta and Omega N. We will be taking an example based on this. So the concept will be more clear to you.

So let us take an example. Changing that value of Omega N as per the requirement of the pilot. For an aircraft which is constrained to yaw motion, as we have already derived, the differential equation will be.

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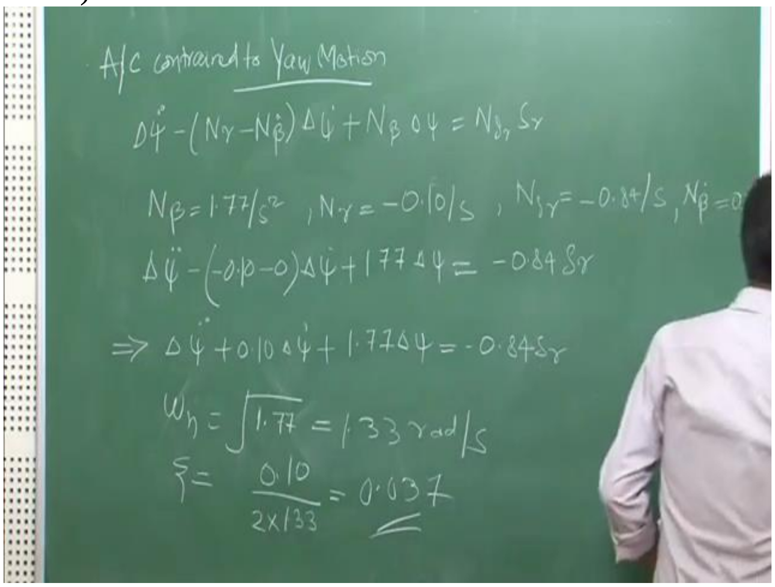

Aircraft constrained to yaw motion. The differential equation was given by Delta of Psi double dot - of NR - N beta dot into Delta of Psi dot + N beta into Delta Psi equals to N Delta R Delta R. The values for N beta equals to 1.77 per second square and R equals to - 0.10 per second. And Delta R equals to - 0.84 per second, N beta dot equals to 0.

This is a numerical from Nelson Price stability and automatic control system, chapter 6. You can go and check there. Now using these values and substituting in differential equation, I will get Psi double dot - of - 0.10 - 0 of Delta Psi dot and beta is 1.77, Delta of Psi equals to N Delta S, - 0.84 of Delta R which gives me Delta Psi double dot + 0.10 Delta of Psi dot + 1.77 Delta Psi equals to - 0.84 Delta R.

Now the value of Omega N from this equation will be, Omega N will be root under 1.77 which is equal to 1.33 rad per second. And Zeta will be 0.10 divided by 2 into 1.33 which will give me 0.037. Now you can see, the value of Zeta is very small.

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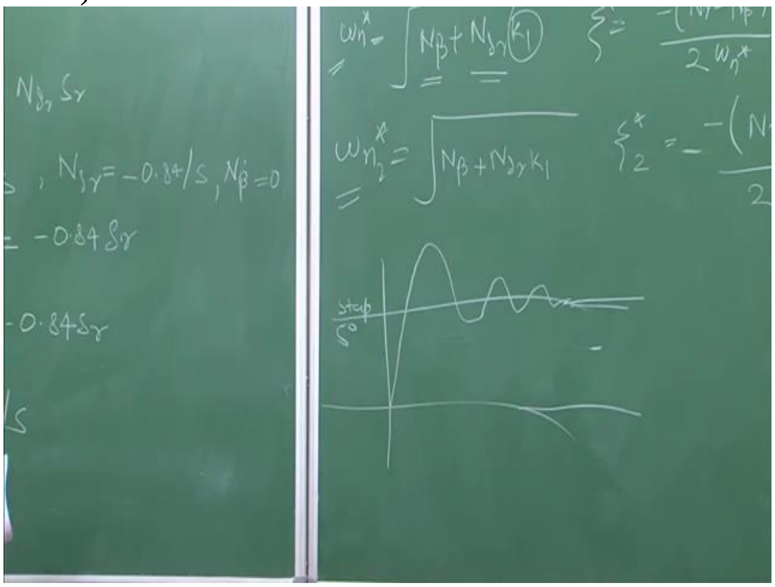

So the nature of my graph will be, since it is a second-order system, so if I give it a step response, it will be some oscillations about this, your step input, see about 5 degree, the response will be of this nature. And since your damping ratio is very small, so the oscillation will sustain for a very long time and it will take away the last time to settle down. So this is not a desire property in terms of controlling an aircraft or stability of an aircraft.

The pilot wants the oscillations to die rapidly. So what we will be doing? We will be using stability augmentation system to change the value of Zeta to some desired value so that the oscillations die quickly and the response is better. So let us design a stability augmentation system. Since we want to only change Zeta, we will be using only one variable. And to change that, you have already seen what are the fight handling qualities of an aircraft.

So pure yawing motion say, let us take Zeta equals to 0.2. So for designing stability augmentation system, I will be writing your Delta R in terms of state variable.

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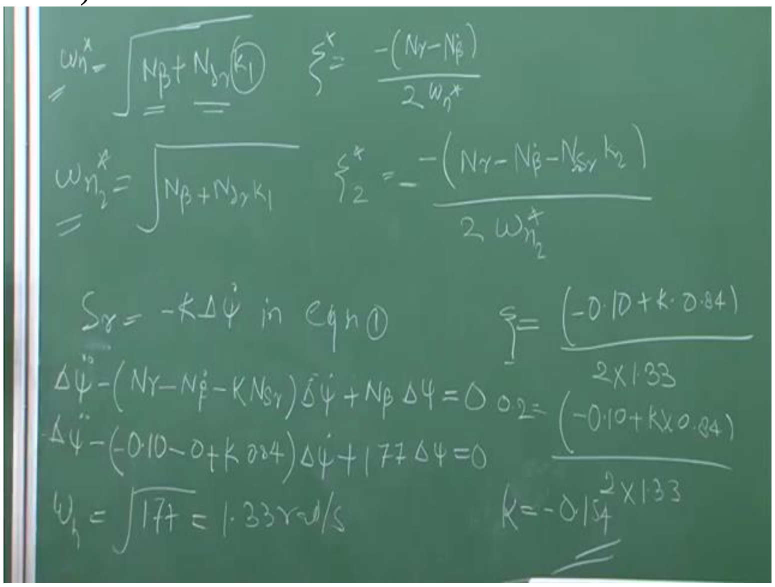

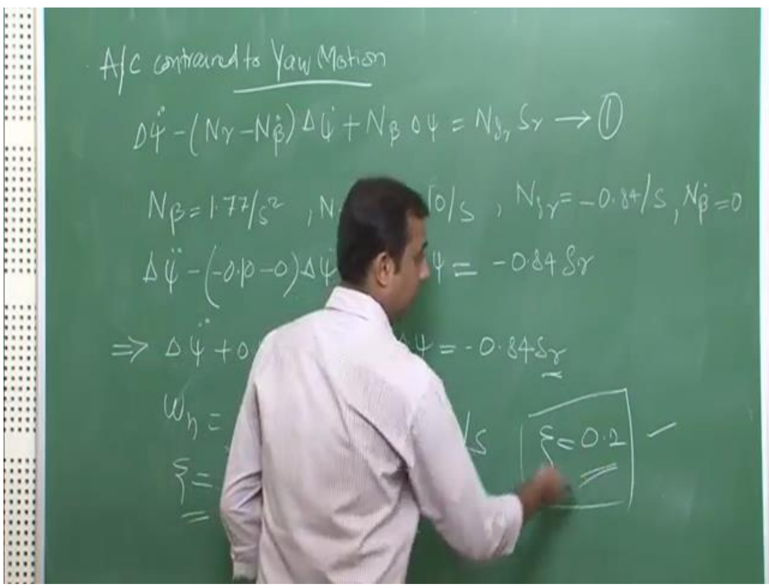

So my value of Delta R will be - K of Delta Psi dot. On substituting the value of Delta R in my initial equation, in equation 1, the new value of Zeta and Omega N will be or the differential equation will become Delta Psi double dot - NR - N beta dot - K N Delta R into Delta Psi dot + N beta Delta Psi equals 0.

Since substituting the values of NR, N beta dot, N Delta R, my differential equation will be Delta Psi dot - - 0.10 - 0 - K + 0.84 into Delta Psi dot + 1.77 Delta Psi equals to 0. Again the value of Omega N will be root over 1.77 which is 1.33 rad per second. My value of Zeta will be - 0.10 + K into 0.84 divided by 2 into 1.33.

Since the desired value of Zeta is 0.2, this value, this is the desired value, so I will substitute that desired value in desired value of Zeta which is 0.2 equals to - 0.10 + K into 0.84 into 2 into 1.33 which will give me the value of K as - 0.154. So using this value of K you can change the value of Zeta to your desired value. In this case, it was 0.2.

Now we saw an example based on stability augmentation system and previous lectures, we have seen about transfer functions, how to derive transfer function with respect to state variable. Now let us see a system, short period dynamics in which the damping ratio is very less. We have to change the damping ratio so the system (())(19:19) as per your desired requirement.

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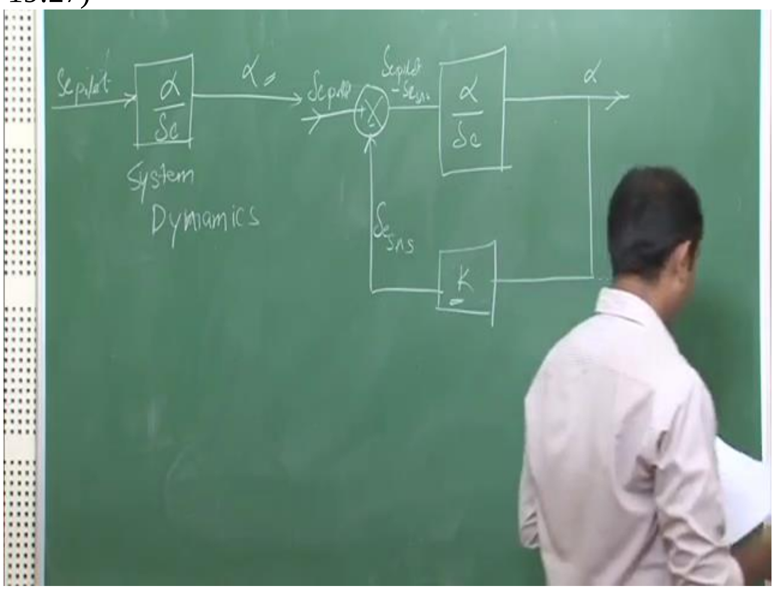

So your change of angle with respect to elevator deflection is given by transfer function alpha upon Delta E. This represents my system dynamics. Your input is Delta E which is given by pilot and your output is angle of attack. Now this is an open loop system which gives your directly angle of attack when an elevator input is given to this system.

We have seen this. This is a transfer function. So you can directly plot the response as we have seen using a step response, impulse response. Now I want to change the dynamics of this system. This can be changed using some control system. So you can say, a closedloop system. Now, SAS is nothing but a closedloop system which is represented as. This is my transfer function alpha upon Delta E.

This was mine put Delta E by pilot. This is my summer. My output is angle of attack. I will be tapping this and multiplying this with some constant, K. And this is your feedback. The net input which the transfer function of the system will be receiving is, this is due to Delta E SAS, this is Delta E pilot - Delta E due to SAS. This is your closedloop system with changes the location of your eigen values or roots.

So your natural frequency and damping ratio changes according to the value of K you provide which is the desired value of Omega N and Zeta. So let us take a numerical based on this system.

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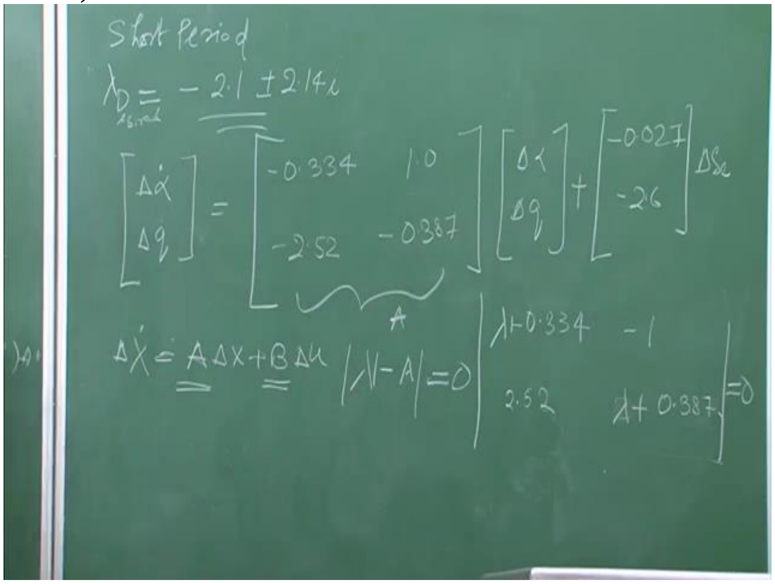

Suppose for an aircraft, the short period mode is very poor. I have to design a feedback system so that my desired root is - 2.1 + - 2.14I. These are the desired roots which is required by my system. Your system is given by Delta alpha dot by Delta Q dot. This is for short period. Equals to - 0.334, 1.0, - 2.52, - 0.387 into Delta alpha into Delta Q + - 0.027, - 2.6 Delta of Delta E.

As we have earlier seen, this is in presentation of state matrix which is given by Delta X equals A Delta of X + B Delta of Q. This is known as system matrix, this is known as control matrix. So for this particular state representation, what will be the eigenvalues of this state space matrix? You can easily determine using lambda I - A. This is your A matrix. Determinant of this is equal to 0 which will give me for this A matrix, lambda + 0.334, - 1, 2.52, S, Lambda + 0.387, determinant of this equals to 0.

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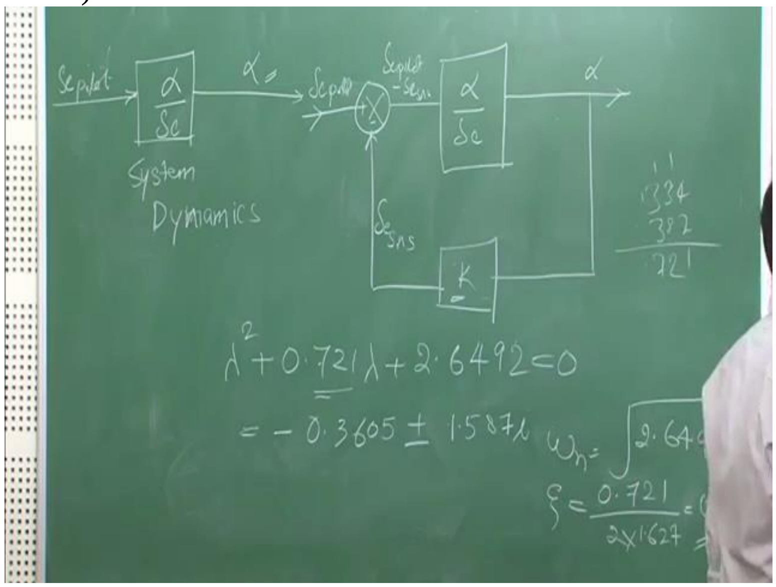

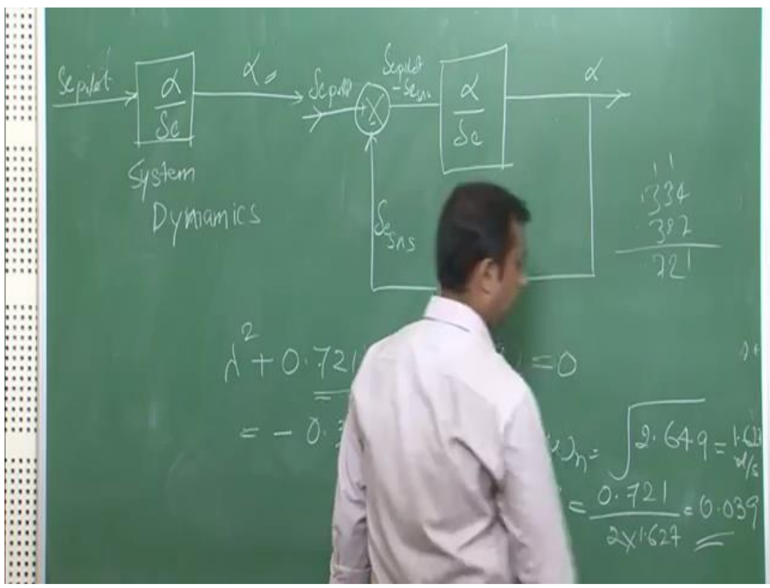

We will get characteristic equation as Lambda Square + 0.721 Lambda, + 2.6492 equal to 0. The roots of this characteristic equation equals to - 0.3605 + - 1.587 I. From this characteristic route, you can find the value of Omega N which is root over 2.649 which will give me 1.6 to 7 rad per second. And Zeta will be 0.7 to 1 divided by 2 into 1.627 equals to 0.039.

You can see from this the value of Zeta is very small. Now I have to change this value so that the roots of my new characteristic equation is given by this. So I will be writing my value of Delta E in terms of state variables alpha and Q.

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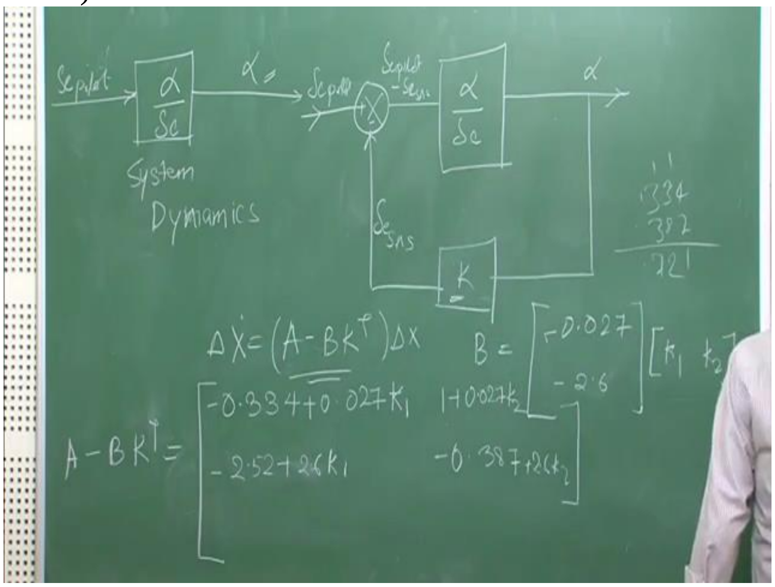

As derived earlier, for stability augmentation, the new matrix which is given as, it is X double dot equals to A - BK transpose or K as already told into Delta of X. So in this question, your B matrix is given by - 0.027 and - 2.6. Your value of K transpose or is given and K2.

Now substituting this value, your new system matrix will be A - BK transpose equals to - 0.334, + 0.027 into K1, 1 + 0.027 K2, - 2.52, + 2.6 K1 and - 0.387 + 2.6 K2. The characteristic equation for this new system matrix will be Lambda I - the value of this characteristic matrix or the new matrix.

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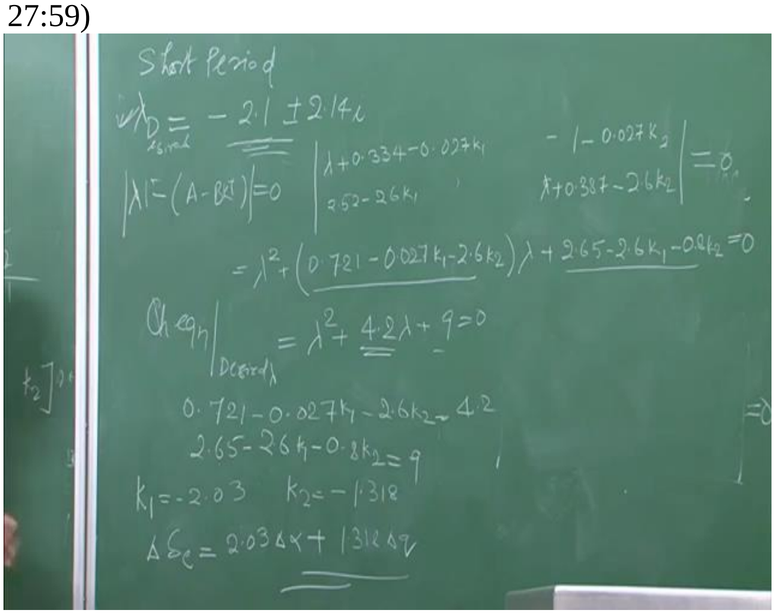

So my new characteristic equation is, Lambda I - A - BK transpose, determinant of this equals to 0 which will give me Lambda + 0.334 - 0.0 27K1, 1 - 0.027 K2, 2.52 - 2.6 K1, Lambda + 0.387 - 2.6 K2, determinant of this equals to 0. Further solving this determinant, you get Lambda Square + 0.71 - 0.027 K1 - 2.6 K2 into Lambda + 2.65 - 2.6 K1 - 0.8 K2 equals to 0.

This is the characteristic equation of my new matrix and since the roots of this new characteristic equation should be as per desired, the value - 2.1 + - 2.14I. So the characteristic equation using this as your root will be, characteristic equation for desired Lambda is given by Lambda Square + 4.2 Lambda + 9 equals to 0. This is the characteristic equation using the roots as - 2.1 + - 2.14I.

Now comparing the coefficients, 4.2 equals to this coefficient and 9 equals to this coefficient. 0.21 - 0.027K1 - 2.6K2 equals to 4.2. And 2.65 - 2.6K1 - 0.8K2 equals 29. Solving these equations, you will get the value of K1 as - 2.03 and K2 as - 1.318. So your Delta E in terms of K-1 and K2 and state variables, alpha and you will be written as Delta E equals to 2.03 Delta alpha + 1.318 Delta Q.

So you can see, using your Delta E and value of K1 and K2 with state variables, you can change the roots of particular equation, the value of Zeta and Omega N as per your desired system. So, SAS is a very useful technique to change the value of natural frequency and Zeta just using a small gain in terms of your state variables. So using this system, you can change the response of an aircraft according to your convenience or according to your needs. I would like you to practice more problems on stability augmentation system. Thank you.