Aircraft Dynamic Stability & Design of Stability Augmentation System

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Module 8

Lecture No 41

Stability Augmentation System

Good morning friends. By now, you must be very close to asking yourself a question, how much you have really understood about stability augmentation system.

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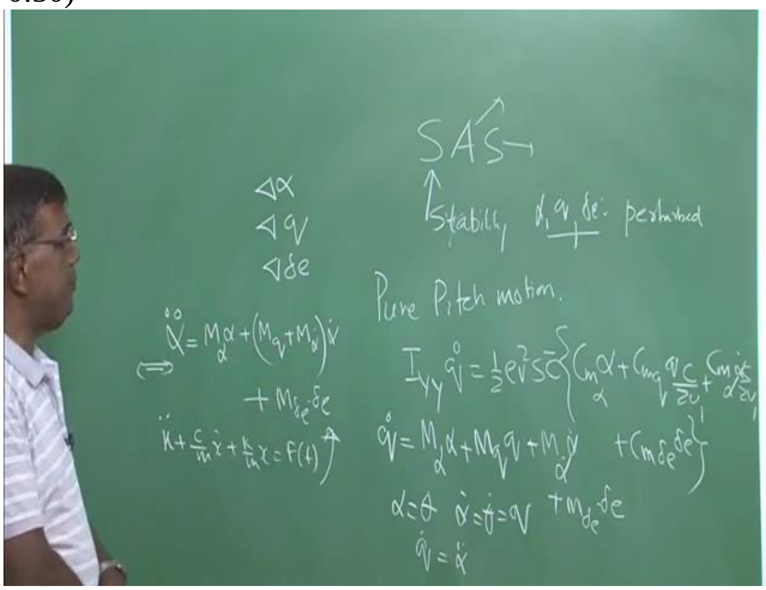

And if I take you that when I wrote SAS, this S was for stability. This is augmentation this is the system. The system here is aircraft. What is the meaning of stability augmentation? That also dou know. For example, if I take a pure pitch motion and able equation of motion will be IYY Q not equal to pitching moment which is half Rho V Square SC bar into CM alpha into alpha into CMQ into QC by 2U1 + CM alpha dot into alpha dot into C1 + CM Delta E by 2U1 + CM Delta E into Delta E.

And please understand, we are writing perturbed equations of motion. If we have alpha, Q, Delta, these are all perturbed quantities. In the lectures by 2 of my (())(1:47) scholars they have been using Delta alpha for perturbed alpha, Delta Q for perturbed pitch rate, Delta Delta E for perturbed elevator deflection. So you should be very very clear that we are talking the same thing.

And here, we also know that we can write this as Q dot equal to M alpha into alpha + MQ into Q + M alpha dot into alpha dot + M Delta E into Delta E. Then for pure pitch, we have made the approximation that alpha equal to Theta and alpha dot is equal to Theta dot equal to Q. And since Q dot will be equal to alpha double dot.

And by substituting here, we got equation of the form, alpha double dot equal to M alpha into alpha + and Q + M alpha dot into alpha dot. + M Delta E into Delta E. So all are in now alpha. And you could immediately see, they represent similar form of X double dot + C by M X dot + K by M X equal to F of T where F of T, force, in this case it is the moment. So we know that we are comfortable, we are in 2^{nd} order system.

And what does this stability means? We want to check whether the airplane after being disturbed from the pitch like this, whether it is coming down like this or it will go on doing, diverge like this. So who decides that?

For understanding that, what did we want? We want something like this. For understanding that, we try to find out what is the natural frequency and what is the damping ratio. You could understand also. By now, you are experts. If I go on increasing this Zeta, then it damps very fast.

Depending upon the pilot flying the machine, he would prefer a particular value of Zeta and omega N which is given by the handling quality requirement. So there is a need to change Zeta or omega N in flight because we know, this Zeta and omega N this change with altitude also, with speed also.

So you have to be very very smart so that depending upon at what altitude, what is happening to Zeta and omega N, I should be able to online augment they stability meaning we have to augment these values, M alpha, MQ, so that finally Zeta and omega N are what we look for. That is the stability augmentation part. And what we have done?

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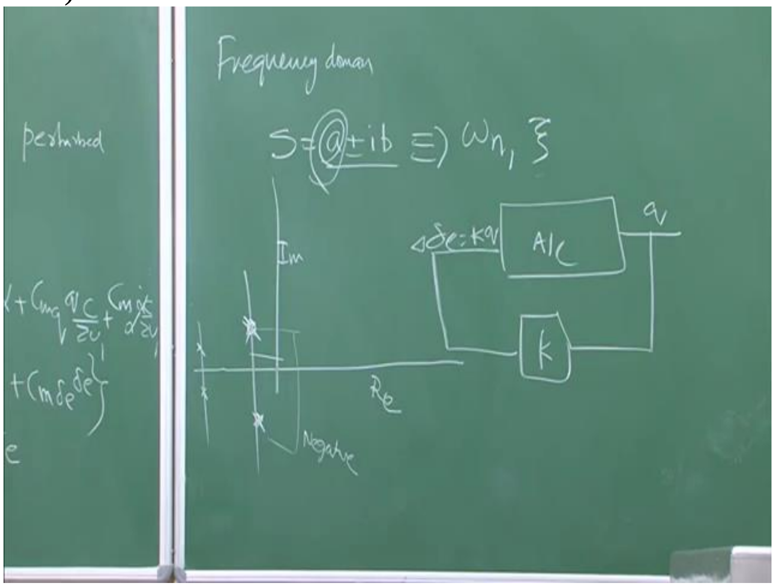

We have done it in the form of, in frequency domain or using Laplace transform, what we did? We found out the root S and let us say for a particular case, it has real and complex part with conjugates, so immediately I know that is going to oscillatory motion. But that is going to diverge or converge, depends upon the value of A. If A is negative, we know that it will converge.

And pictorially, what does it mean? It means that if this is the imaginary plane, this is the real plane, if I want A to be negative as a necessary condition for the air plane to go dynamically stable so that the oscillation gets damped out, naturally I expect that this pair should be on the left hand lane of this real imaginary surface. So that this part is negative, this real part is negative.

But the question is, do I want this root to be here or do I want root to be here? Both are satisfying this condition but real part is negative and it is a conjugate. So naturally there will be an oscillation that will damp out, behave like a 2nd order system. The bigger question is, whether I want the real part to be this much or this much or this imaginary part to be this much or this much. That is an extremely important question.

And why? Because depending upon this root, these values of omega N and Zeta are determined. Right? You all know this, how to do that. So in my exercise, when I taught you, what we did?

We used the SAS. This is the aircraft. Suppose we want to increase Theta. So the model was like this. And this Delta Delta E was equal to KQ, proportionately, I am deflecting the elevator, proportional to Q.

And the moment I do it like this, the CMQ gets augmented. And CMQ gets augmented means the Zeta gets augmented. So depending upon whether you want Zeta more or less, you can play around here. But how much? That you can play around with the value of K. So this is what we did in frequency domain. And we also understand, my younger friends, they love to work in time domain.

Because they always caution me, frequency domain analysis is strictly valid for linear system and they want to work in time domain. So what they will be doing?

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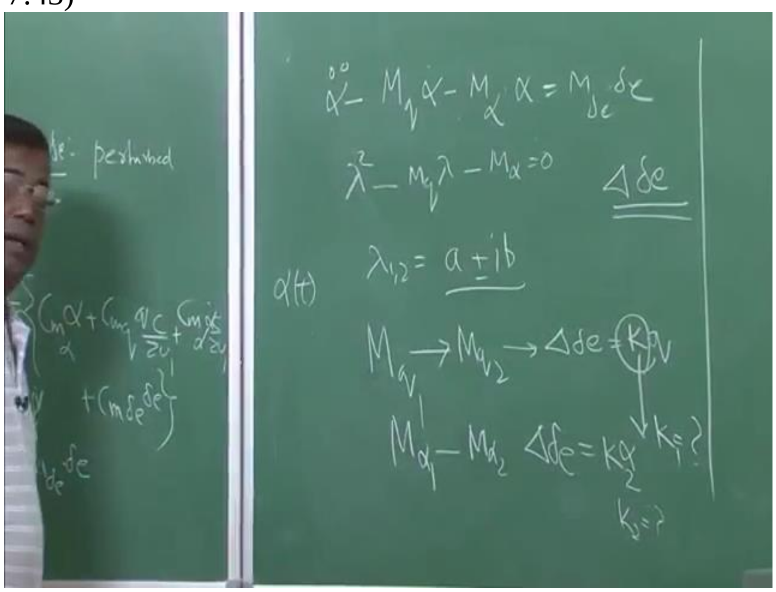

They will write, suppose equation is alpha double dot - MQ alpha dot - M alpha into alpha equal to M Delta E into Delta E. This typically, again 2nd order system. And you all know, the characteristic equation for this case will be lambda square - MQ lambda - M alpha equal to 0. And they will find out the roots, lambda 1 and lambda 2. Please understand, this alpha means alpha T, Potter quantity in time domain.

We have not taken any Laplace transform. So once they find out the roots, again they find out whether the root is real, imaginary or complex conjugate and accordingly they will now tell if I want to change MQ from MQ1 to MQ2, how much Delta Delta E is report. Let us say this is KQ. The question is, how much the value of K?

And simultaneously if you want to also change the natural frequency, then you know I have to change M alpha1 to M alpha 2. Then Delta Delta E, we have K alpha, K2 alpha let us say K1. The question is what is K2? Right? So they will be solving it in time domain and make your life very simple and you will be almost like a machinist will be working to design a stability augmentation system.

But remember, a word of caution, you can do all these things mechanically but please understand, the moment you talk about Delta Delta E, how much I can deflect, there will be a limitation on the amplitude of Delta Delta E to be given which will make the flow the way dynamics will lie. For example, (())(9:45) separate also the rate, at what rate I am going to change, that also decides to dynamics.

So those are the finer points. So please watch out Mr Prashant and Mr Dwivedi, Vijay Dwivedi who have been solving examples and adding value to this course. Both of them will be giving you hint how to handle such a design of SAS in time domain. Again I caution you, their notations are different. They are typically control persons, we are typically aeronautical persons.

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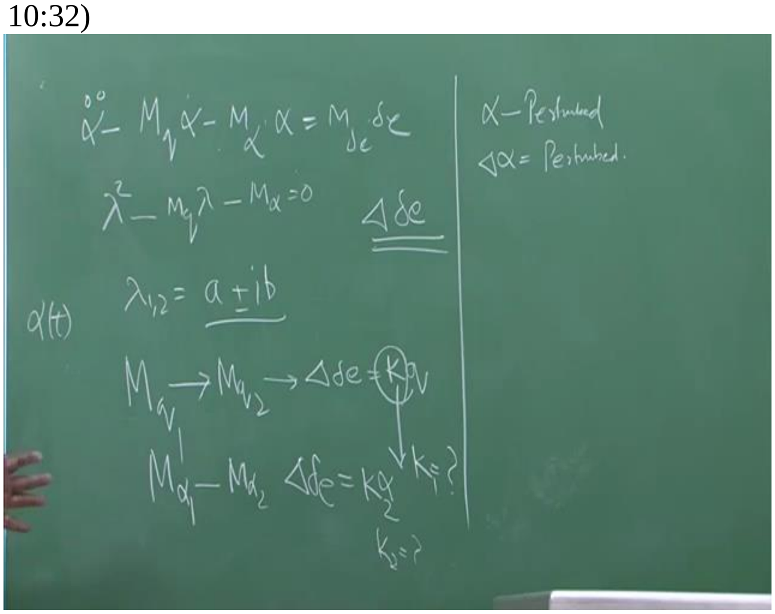

Although now they are doing Ph.D. in aerospace, there is a problem in notation. When I write alpha, small alpha, it is my perturbed quantity. For a typical control person, he writes Delta

Alpha as perturbed quantity. So those notations, I will replace them to discuss with you and enjoy this. I am sure, in your assignments they will give some problems.

Do not worry about exams and all. You understand this so that you can use it in the industry. Thank you very much. Now I request Prashant and Mr Dwivedi.

So far we have studied about the characteristic roots of the equation, the longitudinal mode and your lateral directional mode. And the nature of the roots which we derived from these equations were somewhere of similar nature which I am drawing.

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This was the root equation for full Eigen values. These were roots of the short period mode. These were the roots for Dutch mode. These were for your Phugoid mode and these for spiral mode. Now during our lectures, we saw that the roots should be in the left half of this complex plane the presented by imaginary axis and this will be your real axis.

Now during this course, while designing an aircraft, the basic requirement was that my plane should be in stable mode. And as you can see from the roots of this full 6 (())(12:38) equation of motion, the roots were all in the left half of the plane. So we can say the plane was stable.

But what about the requirements of pilot? Or you can say, we have studied the lecture on flight handling qualities. Now as you know that real part of these roots give us time constant. Whereas this complex conjugate gives us the natural frequency and damping ratio. But there are many instances where your damping ratio or natural frequency are not as per your requirement.

For instance you are sorted out, the damping ratio is very less. Whereas, as for your flight handling qualities, it should be a greater value so that the pilot does not have to (())(13:27) modifying your control or giving an additional control your aircraft. So in that case, there are out since to change your eigen values or roots of this characteristic equation.

When way is, either you can change the parameter but it is a difficult way because we have to modify the dimensions, the geometry of the paragraph. Another way is known as stability augmentation system. So today what we will be covering is known as stability augmentation system. As in the name itself it is mentioned augmentation.

Augmentation means to add something to a system so that it becomes more stable or more efficient to operate. So what we will be doing in stability augmentation system is, we will be adding feedback to your aircraft dynamic so that the value of this root changes according to your desired value which are easy to operate for a pilot.

Let me recall, the 6^{th} of equation of motion for an aircraft which was given by X dot equal to some function of your state variables and the input signal where your X is your list, your drag, your moments about X axis, Y axis and Z axis. So when we apply the stability augmentation system, what we will be doing? We will be modifying the state variables with respect to your input signal.

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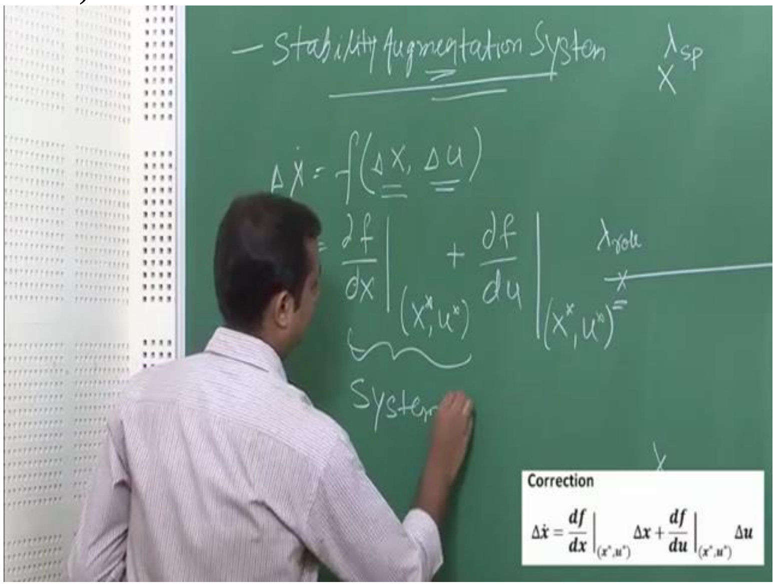

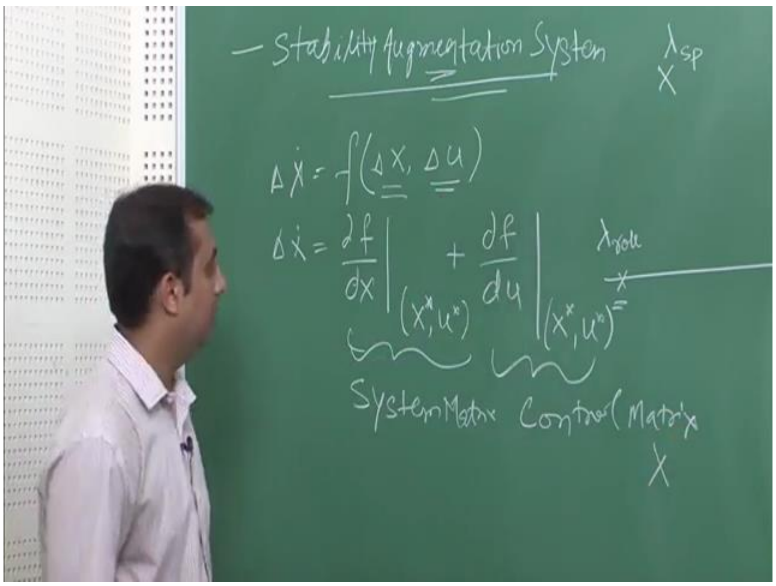

And this 6th of equation of motion can be written as Delta X dot equal to DF by DX at, as you know that while deriving 6 degree of freedom, we used equilibrium position or equilibrium state about which the equation of, 6th of equation of motions were derived.

So I can write this, 6th of equation as Delta X dot equal to DF upon DX at some steady-state value represented by X star, U star + DF upon DU at steady-state value, X star, U star. My this term is the known as system matrix and this term is known as control matrix.

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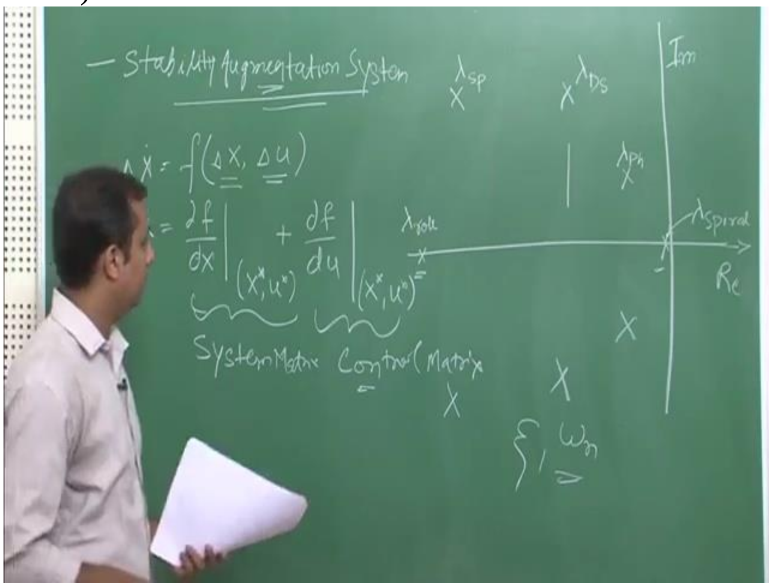

As I already told that using this control matrix, we will be changing the location of this Eigen values in the complex plane so that we can use desired value of Zeta or natural frequency or both. Before doing that, let me tell you why we require such stability augmentation system.

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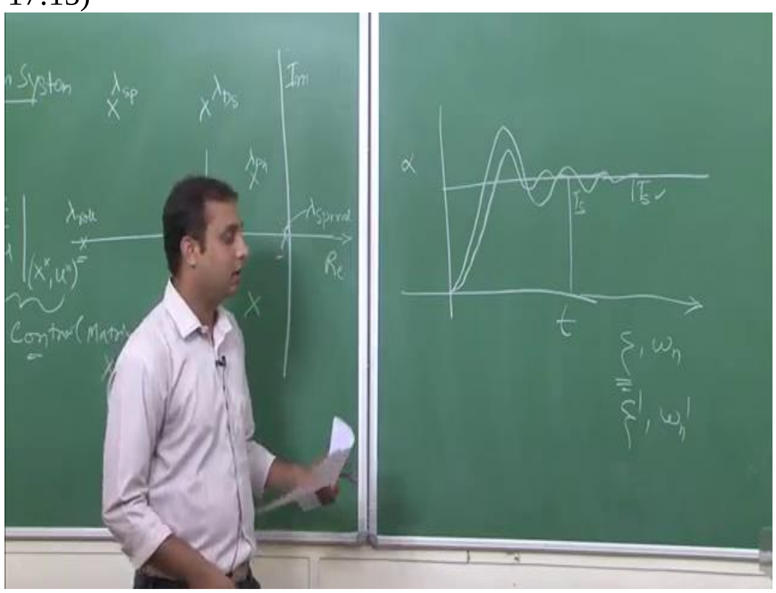

Suppose for an aircraft must oppose for Phugoid mode, my angle of attack with respect to time changes, since it is a 2nd order equation, so we will be getting nature of time response such as, response of this nature. Now you can see, as it is a 2nd order system, so it will be having an overshoot and settling time. This is settling time, TS.

Now this was my nature of your short period mode, angle of attack with respect to time. It will be having some value of Zeta and omega N. Now suppose this Zeta is very small, that damping is very small. So my obviously the time required to settle, afflation to die out will be very large. But for my aircraft behave smoothly, the damping ratio should be sufficient enough so that my oscillation dies rapidly.

For instance, I want my nature of graph to die out very quickly. I want my settling times to be this. So for that, the value is Zeta will be Zeta dash and value of omega N will be omega N dash. So I already told you, there are 2 ways of doing that. Either you can change your aerodynamic coefficient. But it is a difficult process because you have to vary dimensional and geometry of the aircraft.

Another way we are talking is using stability augmentation system. So let us see what will happen, when we use stability augmentation system, what will happen to your state space matrix?

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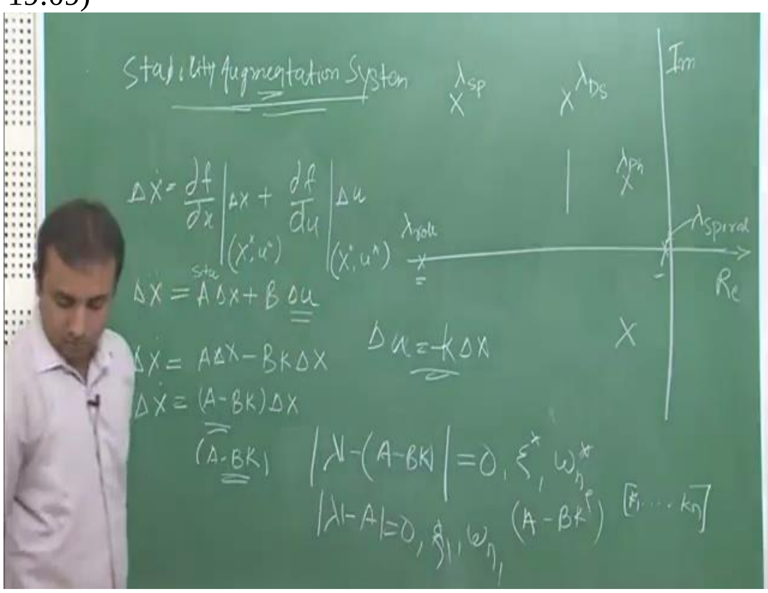

I already told, your state space matrix was given by X dot equals to DF upon DX at some steady-state value of your state variables and input + DF upon DU at steady-state value. I call this as standard representation in state space will be given as X dot equal to AX dot Delta X + B of Delta U. My state space matrix can be represented by Delta X dot equal to A Delta X + B Delta U.

Now for stability augmentation, I will take a feedback which will be in terms of Delta U equals to some K times some state variable. Now let me substitute this value in your state space matrix. My state space matrix will become A Delta of X - BK Delta of X. So Delta X dot equals to capital A - B K of Delta X.

Now as you can see, when we use the stability augmentation system, now the new matrix will be A - BK. As you can see, from this new differential equation or state space representation, the roots of this matrix representation, state space representation will be lambda I - A - BK, determinant of this equals to 0. So you can say, my state matrix representation, the A matrix or state, we call this as state matrix has changed to A - BK.

Whereas for original system, your characteristic equation was lambda I - A, determinant of this equals to 0 which gave me the value of lambda 1, Zeta 1, omega N. These were the Zeta and omega N for original system and Zeta and omega N which we required for which we preferred over the old Zeta and omega N will be given by this equation, the value of K will determine the new Zeta1 and omega1 given by Zeta Star and omega N star.

Now some book, the notation of this K is given as A - BK transpose. Do not confuse with that. This K can be a single value or in it is, given by K1 to KN depending on number of states which you are giving as a feedback. Thank you.