Aircraft Dynamic Stability & Design of Stability Augmentation System

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Module 7

Lecture No 39

Mode Shape: Lateral, Directional Case

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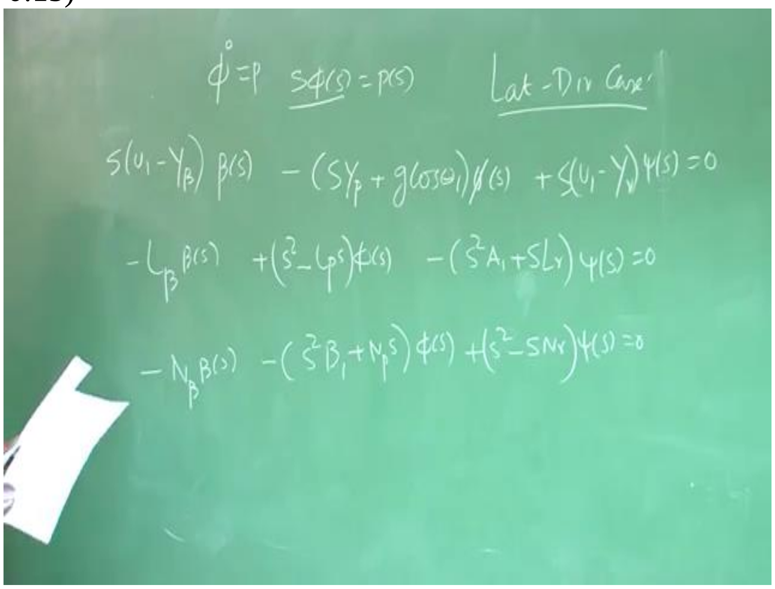

We will now talk about mode shape for lateral directional case. That is again the approach is same. Here we have to use the stability equation that was SU1 - Y beta B to of S - SYP + G Cos Theta 1 to Phi of S + SU1 - YR Psi of S equal to 0. And then, - L beta into beta of S + S square - LPS into Phi of S - S square A1 + SLR Psi of S equal to 0.

Similarly, - N beta beta of S - S square B1 + NPS Phi of S + S square - SNR into Psi of S equal to 0. So these are free response. Remember the stability matrix. This into beta, this into Phi, this into Psi, like that it is going. When we say Phi, please remember, Phi dot is equal to P has been modeled. So S Phi of S is equal to P of S. So wherever P of S is there, we can replace it by S Phi of S. That is why you could see, YP into P, for that P S Phi of S.

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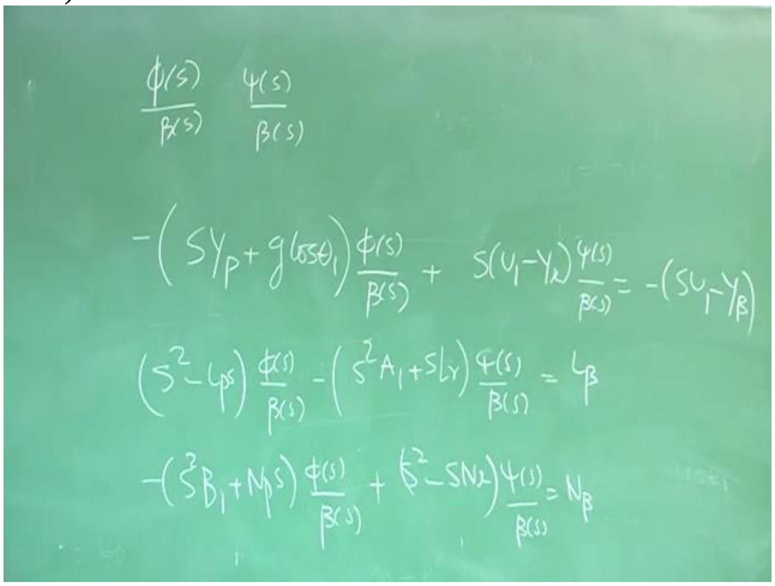

Now once I do this, I do it for a free response then I can write again, since I am looking for Phi of S by beta of S and Psi of S by beta of S, you know what you have to do, you have to divide by beta of S. And then I get equation of the form - SYP + G Cos Theta 1 into Phi of S by beta of S. It must be very boring. It is indeed boring because it is too simple.

You have to take a pen and pencil and follow the instruction. Of course nowadays you all are high-tech people. You go to the Matlab and do it. There is a danger using Matlab, you may lose the insight because Matlab, the moment it understands you have not understood the dynamics, it will fool you. Please be cautious.

Similarly, I have S square - LPS Phi of S by beta of S - S square A1 + SLR Psi of S by beta of S equal to L beta. This is the arrangement. And then - S square B1 + NPS Phi of S by beta of S + S square - SNR Psi of S by beta of S equal to N beta. So what is a work in? Our aim is to find these ratios in frequency domain. These are the algebraic equation.

So easily you can find out the expressions for Phi of S by beta of S or Psi of S by beta of S.

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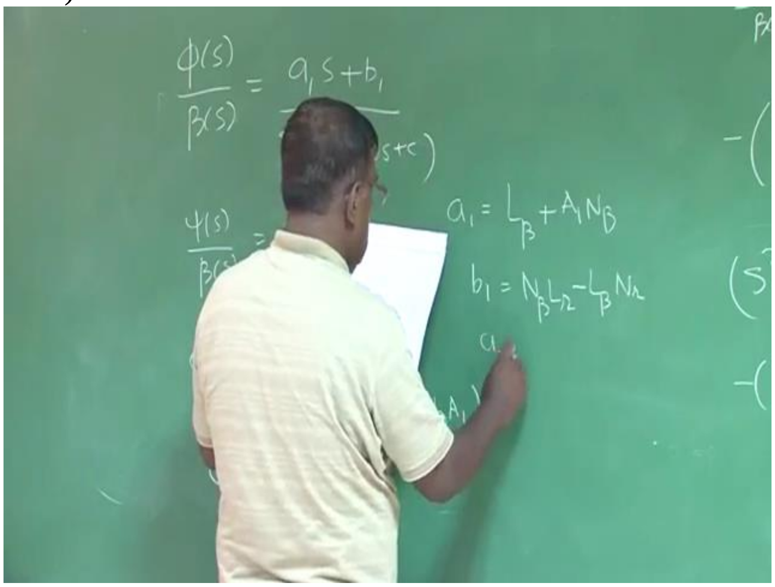

And if you do a little bit of handling, algebraic equations, simultaneous equations, you know by method of substitution, method of subtraction, by Cramer's rule, whatever it is, you can find out Phi of S by beta of S as A1S + B1 by S AS square B of S + C. And Psi of S by beta of S equal to A2 S + B2 by S AS square + BS + C where for completion I am writing, A is equal to 1 - A1B1. Remember, A1, B1, are nothing but ratio of moment of inertia which was defined earlier.

B equal to - NR - LP - B1LR - NPA1, C equal to LP NR - LR NP. Similarly we have A1 equal to L beta + A N beta. B1 equal to N beta LR - L beta NR. A2 equal to N beta + B1 L beta and B2 is equal to L beta NP - LP N beta. Like this.

What is the message? The message is very simple. That if I do find out mode shape, Phi of S by beta of S that is I want to see the relative magnitude between Phi and beta, that is Phi is this and whether it is developing beta or not on that mode, if at all, what is the magnitude, relative magnitude and what is the phase difference? What do I need to do? I need to put the value of S. A, B, all these constants are known by the, once I know the dimensional derivatives and moment of inertia, etc, etc.

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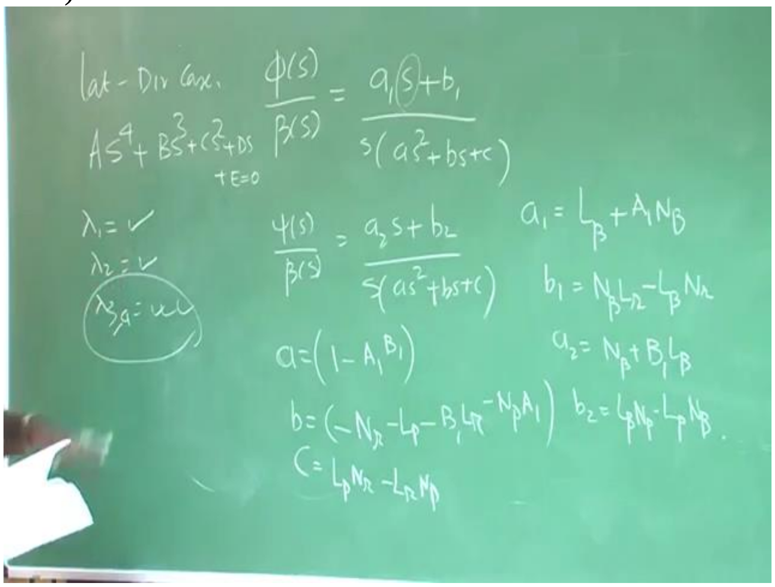

Now the question is, for lateral directional case, CV have again the equation of the form, AS square + BS + C. So for lateral directional case, you have lambda 1 some value, lambda 2 some value, lambda 34 some complex conjugate. So this is Dutch roll. If we want to know about Phi of S by beta of S, if we are looking for spiral, put the value of S here on the spiral root whatever you have got.

Similarly for here. So you will know these ratios. If you want for Dutch roll, put that value S here and get these ratios. Once you know the ratios, as a part of complex number, you will now know how to find the magnitude and the phase difference. That is as simple as that. Not a big deal.

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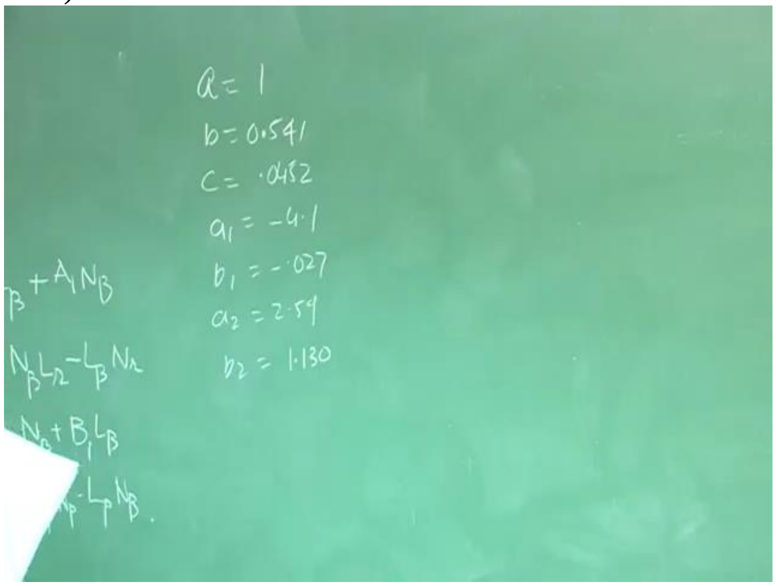

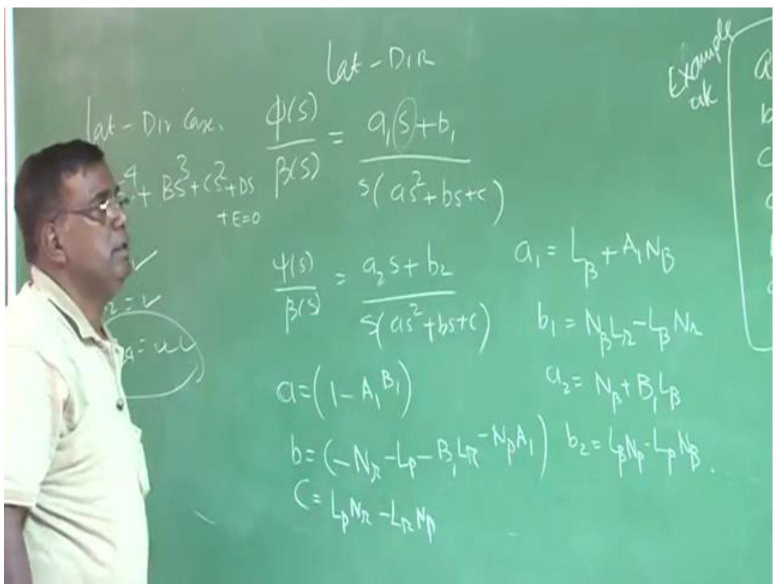

Let us say, for a particular airplane, the value of A which has come let us say around 1, B is 0.541, C is 0.0452, A1 is - 4.1, B1 is - 0.027, A2 is 2.59 and B2 is 1.130. How these have been computed? We know the expression of A, B, C and for that airplane, we know what is A1, B1, we know what is LP, LR.

For a given airplane, let us say example airplane, once a put these values, I get A, B, C, A1, B1, A2, B2, these are the values for an example airplane or aircraft and we are talking about lateral directional case. That is important. Because these values for expressions of A1, B1, A2, B2, etc or whatever we have computed here pertaining to lateral directional case, the expressions are like this.

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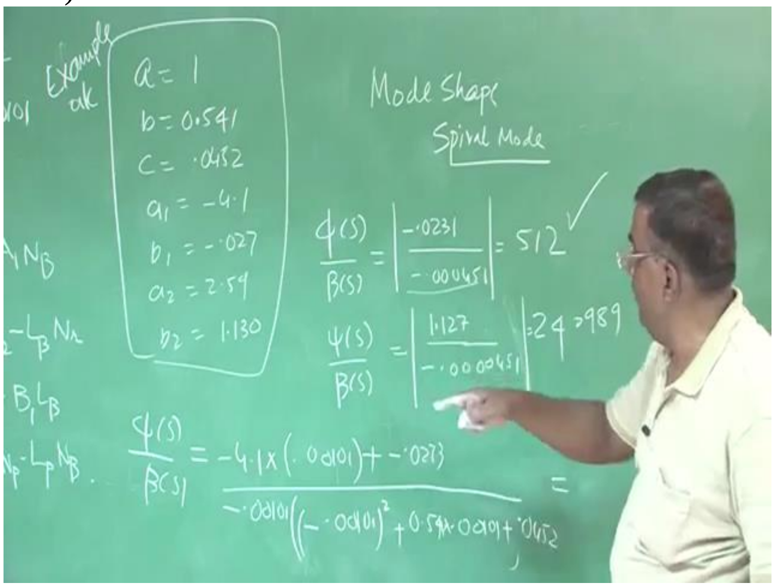

Once we do that, let us say we try to find mode shape for spiral mode. That is the one next mechanical challenge. We are trying to find mode shape for spiral mode for the example and plane and if you recall, for spiral mode, it was real. So S was a real number. Complex part was 0. And the value was - 0.00101.

Safer want to find out Phi of S by beta of S, I have to put here Phi of S by beta of S, of course we are talking in terms of ratio of 2 complex numbers. So I will put the value of A1. It this - roughly 4.1 into S. So that is - 0.00101 + B1. B1 is how much? B1 is - 0.0273 divided by again it is - 0.00101.

A value is how much? A is 1. So this is - 0.00101 square + B of S. B is 0.541. Into S is again 0.00101. Then, + C. C is how much? It is 0.452. And this, if you do that, I may commit some mistakes in putting the values. Do not blame me. A1, B1 small these values you know. S, you know for spiral mode which we have got from the exact equation of the same airplane. It is - 0.00101.

You have to simply put those values here. And let us say this comes to ratio of - 0.0231 by - 0.00451. I draw the magnitude, I get 512. For Psi of S and beta of S similar thing if I do, I get numerator 1.127, denominator is this. And this is 24989. What are the interpretations you are having from this? That is I am asking. That is the beauty of the mode shape.

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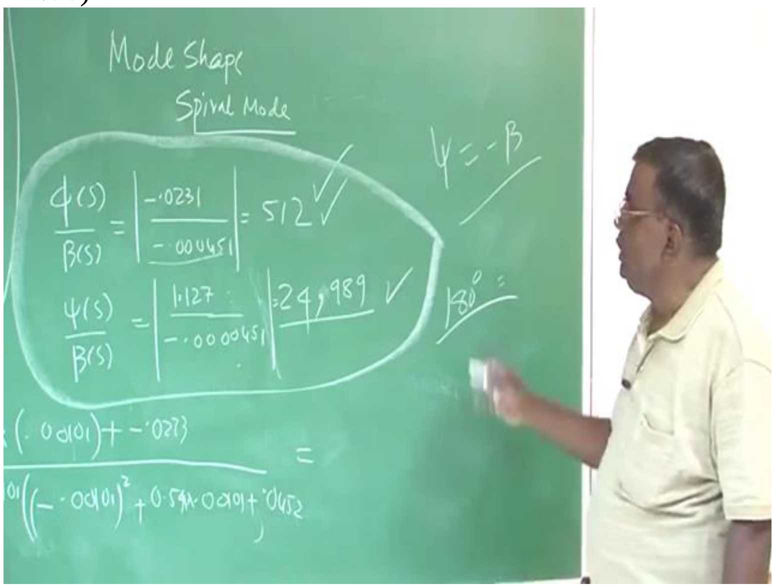

1st of all you see, Phi of S and beta of S, the numerator and denominator, they are all same sign, arc tan sign. There are no imaginary parts. So the phase difference is 0. However you can understand that magnitude of beta is very very small. That means Phi is more dominating for spiral mode. You could see bank, it starts side slipping, this, further bank. So this goes.

So this clearly tells you for a spiral mode, I am more bothered about the bank angle. And that is the information I get from this. Looking at this numerator and nominator, 2 things I understand that they are in phase when our Phi is changing, at same time beta is also in the same direction changing. However the contribution of beta is very very small as compared to Phi.

So for a controller, it is a wonderful information for him. He knows now what aircraft controlled to be used. Or which one is to be corrected 1st. There are many such decisions one has to take. If we see Psi of S and beta of S, again you see numerator here, denominator here, the magnitude is 24,989, this ratio. That means really in terms of Psi, beta is again very very small in comparison.

But one thing you remember that Psi and beta, because there is a - sign here, so they have a phase of 180 degree. Correct? 180 degree phase difference. So they were in phase and Psi and beta of S are out of phase. And you know that Psi is this. Then beta becomes negative.

Psi is equal to - beta if I am now talking about, talking about a coordinated tunnel type. So which is also giving you a correct information. What interestingly is important for you is that beta participation is very very negligible in this spiral dynamics. So you can design a controller and give a priority and which variable you have to give priority, this will help. So we have given an example of spiral mode

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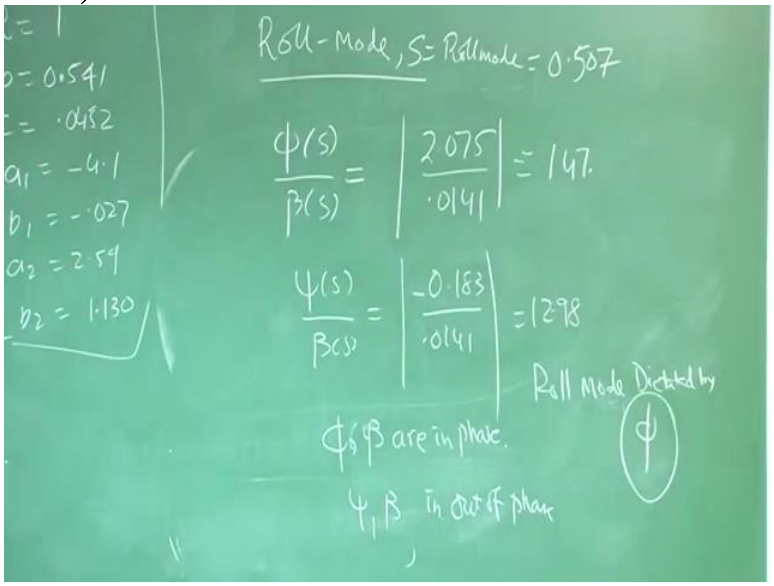

Now we talk about roll mode. What we have to do? Same equations. Here, S should correspond to roll mode. Right? From the exact equation. And you know the roots, already I have given you. And if I write, Phi of S by beta of S, we know that expression, put the values. Now I should put, S corresponding to roll mode.

And if she had example, if I recall the value was 0.507. That was the value of S or the root. If I put those values in those expressions, then I get Phi of S and this as 2.075 by 0.0141. This is equal to 147 roughly, approximately. And here, Psi of S by beta of S I get as - 0.183 by 0.0141. The magnitude will be 12.98.

Very simple. I have to just put roll mode this value in this expression. Nothing more. A1, A2, all are, we are knowing. So again here, you could see that Phi of S, Phi and beta are in phase. And again, beta is less compared to, beta domination is less compared to Phi, which is roll mode.

And for Psi and beta, we find that they are out of phase. The phase angle is 180 degree because of this - sign here. So that is, as a designer, what should I infer is the roll mode is predominantly dictated by Phi. That we know. Now we will try to get more shape for, or understanding for Dutch roll case and let us see what happens.

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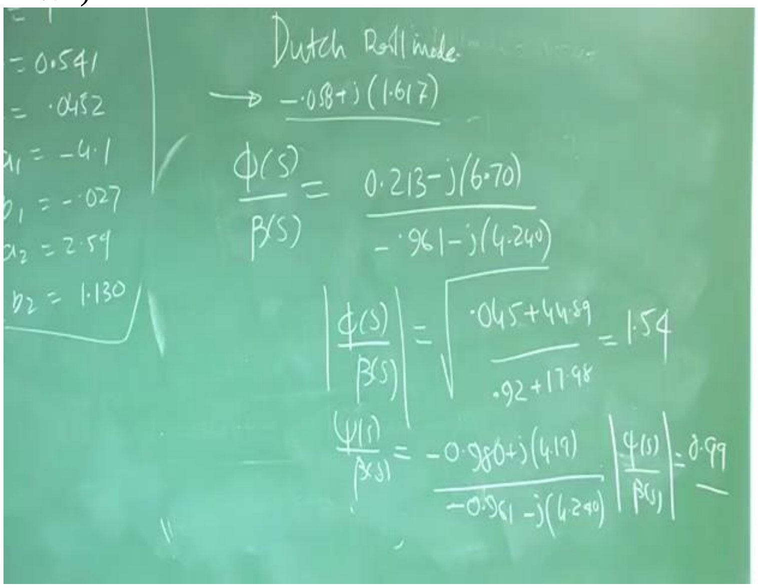

When I try to find out for Dutch roll mode, Phi of S by beta of S, my job is very mechanical. I this is the expression. I know A1, B1, all those values. What I have to do? See from the exact equation, what was the value for S and one of the root for complex conjugate is this. So this I substitute in those equations and Phi of S by beta of S I get in this form.

0.213, ratio of 2 complex numbers - j into 6.70 divided by - 0.961 - j 4.240. This is the Phi of S by beta of S in terms of ratio of 2 complex numbers. And now you know how to find magnitude. So Phi of S by beta of S, as far as magnitude is concerned, that will be good under root of you know by now the expression, 44.89 by 0.92 to 17.98, under root of this which will be equal to 1.54. That you know now.

Omega N square, numerator square, denominator square, already we have given that expression. This is the 92. Similarly, if we try to find out Psi of S by beta of S, you will find, this ratio will come as - 0.980 + j 4.19 divided by - 0.961 - J 4.240. Roughly this. And you can easily find the magnitude. This magnitude will come as the magnitude of Psi of S by beta of S. You know how to find out. This will come around 0.99.

Note down this. One is 1.54, one is 0.99. But one thing you should understand, when I try to find their phase difference between Phi and beta and Psi and beta, now also we are ready with the formula. We have already written this earlier.

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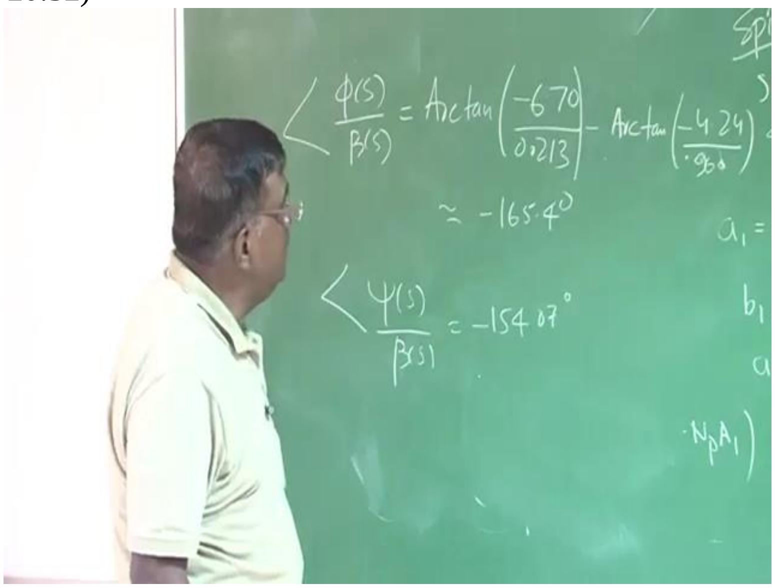

So if you find out the phase Phi of S by Peter of yes, this will be arc tan - 6.70. I am just writing it so that you can follow it. In fact, you should not see in this. You should be able to find out yourself. Arc tan - 4.24 divided by 0.960. So this will give you roughly - 165.4 degree.

Similar exercise we do for Psi of S by beta of S. We will get, - 154.07 degree. But then, how do I interpret this? Which one should I give more importance? As long as you understand, the phase difference between Phi and beta for a Dutch roll that is when Phi is becoming positive, beta is not becoming positive at the same time.

That is very important. There is a phase difference of 165.4 degree. Similarly the story is with Psi and beta. That is why I was telling, it is not this. It is this and this. If it was 180 degree, then when this man was doing like this, airplane should have gone like this. Then they are out of phase 180 degree. One is going like this positive, another is going like this.

So, swaying motion in this direction, yawing motion in this direction. That is 180 degree phase Psi and beta but it is not happening. Again we come back to this Phi of S and beta of S, you could see the phase difference is 165 degree and between Psi and beta, it is 154.

So you know relatively how Psi and beta are functioning in terms of their phase differences. They are definitely not in phase. They are out of place. Almost 180 degree out of phase. This is one.

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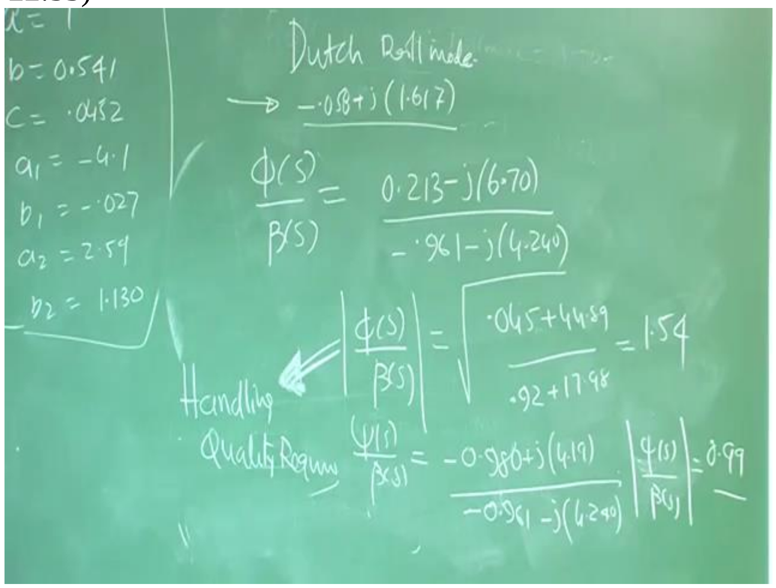

And 2nd thing you understand that for me, this is important, Phi and beta, this transfer function or this mode shape typically for handling quality requirements. For handling quality requirement, we try to have this ratio as low as possible to validate or to justify or to get a scale for acceptance for Dutch roll excitation. That is extremely important.

If the airplane is banking and side slip, this ratio is very very high, then we can understand, Dutch roll oscillation acceptance will not be approved. It should be as low as possible. So this, in a nutshell you could see, very simplistic approach. We could get so many information out of this mode shapes. You can do a lot as long as you work in linear system using transfer function approach.

But I must also caution you, today highly manoeuverable airplanes and using time, working in time domain in state space modelling, non-linear aerodynamics, this may not look very relevant apparently. But let me tell you, before you design anything big, you need to understand this. That gives a solid foundation. And bigger things are just addition to this. Basic foundation is here.

You need to solidify your understanding here before you make the next jump. Do not go for a fashion. Nonlinear aerodynamics, non-linear controller. 1st you understand this, get the right fragrance and smell of this approach. Understand, enjoy the sweat required in this approach. Then you will find, you will get the real fragrance.

And you will be ready for a controller designer. I am sure feel nice. Nowadays, all start-up companies are doing excellent in designing controls and all and a very good sign, very good contribution coming from the youth. I am sure you will be able to add value to that further.