Aircraft Dynamic Stability & Design of Stability Augmentation System
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Module 7
Lecture No 37
Revision

Good morning friends. I thought today, before I start system resources, we will try to recapitulate what are we doing so far or what we have done so far.

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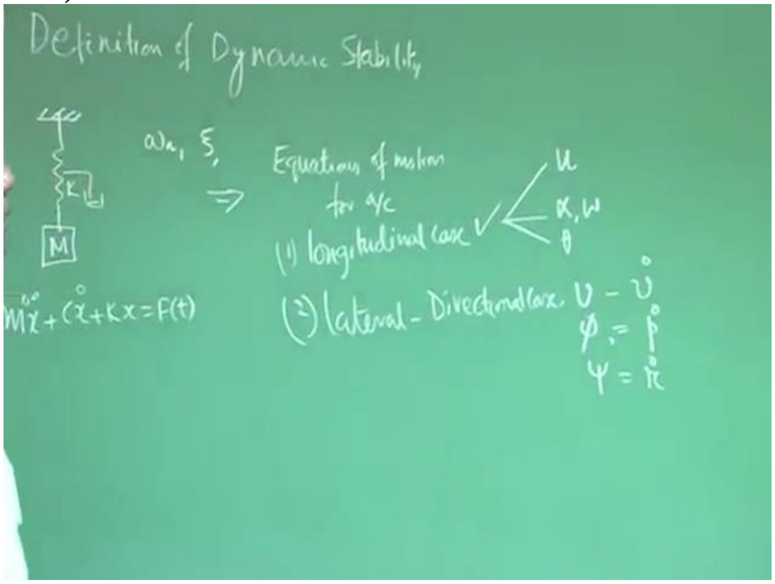

If you try to recall, we started with definition of dynamic stability. We started with mass spring damper system which is typically a 2nd order system and most of our analysis takes help of understanding whatever we have through mass spring damper system. Then we tried to understand what is natural frequency? What is damping ratio?

We also tried to understand if I stretch it and release it, how this system is going to oscillate or behave depending upon whether it is over a damped case, under damped case or critically damped case. After doing this, we made a step towards writing equations of motion for airplane or aircraft. Because here we know, the equation was MX double dot + CX dot + KX equal to F of T which describes one-dimensional motion of this mass once the disturbance is given.

We try to develop equations of motion for aircraft. Then segregate it. One, longitudinal case, another lateral directional case and we were clear that longitudinal case when we were considering, independent of having any influence on the lateral directional case, we assume that the angle of attack, the trim angle of attack as well as the disturbances are small. Because we know, if the angle of attack disturbance is large then this can cause a yawing moment as well because of 2 asymmetric vertices that generate.

Also, if I have the rates very high through the coupling, inertia coupling, it can have effect on the lateral directional case. All those things we have assumed to be negligible or not present. When we were dealing with longitudinal case, then we discussed lateral directional motion and we know that if there is a bank, role like this, like this, so the lift here will be more. So the drag here will be more here. So it will also yaw.

So lateral directional gets coupled. That is why, we have developed formulation when we are talking about stability, dynamic stability taking the lateral directional equations together. For longitudinal case, we studied how Alpha is changing, how Theta is changing. That is, U. Alpha means indirectly, I am talking about this motion.

Alpha is vertical by the horizontal speed and also pitching Theta like this. When I write Alpha, I also mean W, the vertical component. So there are 3 variables where Alpha and W are linked. And for lateral directional case, we talk about V, small perturbation along V, Y axis. Then we have Phi, we have Psi. That is this, this and this, yaw. And if you see, in terms of equation of motion, we talked about V dot, we talked about P dot, we talked about R dot, yaw rate. R is yaw rate.

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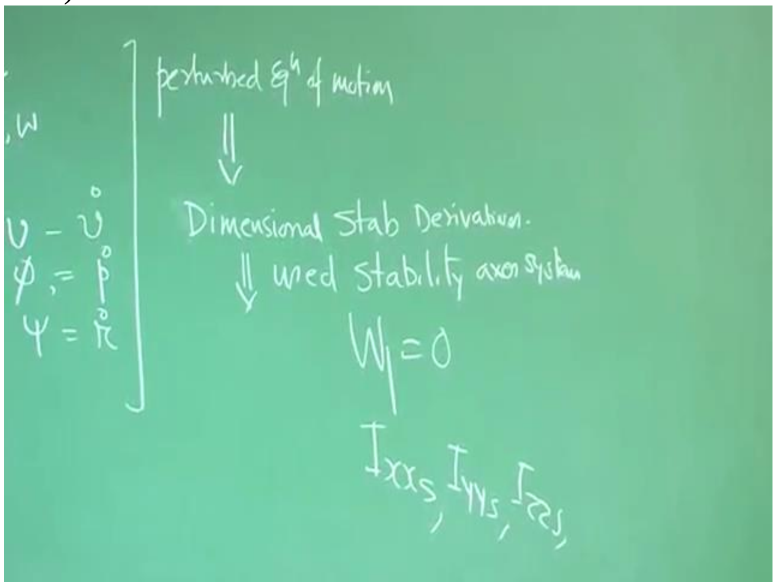

After that what we did? We developed perturbed equation of motion and defined dynamic or defined dimensional stability derivatives. We tried to understand the sign and magnitude of each derivative. And we realised that this dimensional derivative have dimensions like non-dimensional derivatives. Then we also used stability axis system and we know that once I use stability axis system, the value of W computed along Z at steady state is 0.

That simplifies our equation and also helps because lift and drag are perpendicular to each other. That helps in modelling if we choose stability axis system. But at the same time we know, the moment we use stability axis system then I must also try to find out what are IYY, IZZ, our cross moment of inertia about the stability axis system.

And we have shown, for 3 to 4 degrees angle of attack, how much variation it can have. So in preliminary study, one may even neglect it. But there is no need to neglect it. It is just if we apply one transformation, we will get this number.

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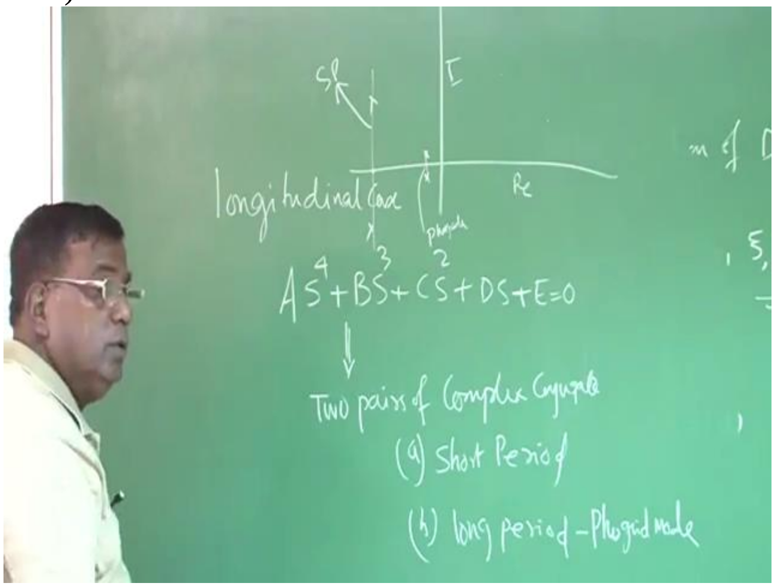

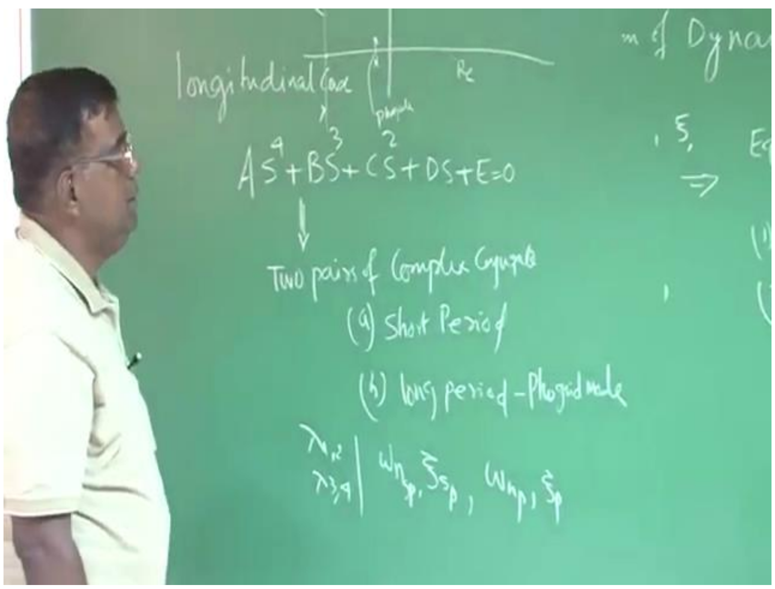

Once we have done this, then we found that if I take the longitudinal case, we get equation of the form, AS4 + BS cube + CS square + DS + E equal to 0 and A, B, C, D, E, they have the expressions based on the stability derivatives, inertia properties. More important thing here is, for most of the airplanes, we found this generates for longitudinal, 2 pairs of complex conjugate and one we identified as short period, another long period.

This is also called Phugoid, Phugoid mode. So if I try to display the roots in imaginary and real coordinate system, we will find one root is here, one complex pair is here. This is large negative. This belongs to short period and this pair belongs to Phugoid.

After knowing these roots, we know how to find, once I know Lambda 1, lambda 2, lambda 3, lambda 4, which are complex conjugates, I how to find out natural frequency short period, Zeta short period. Similarly, Omega N Phugoid and Zeta Phugoid. This is one part we have done.
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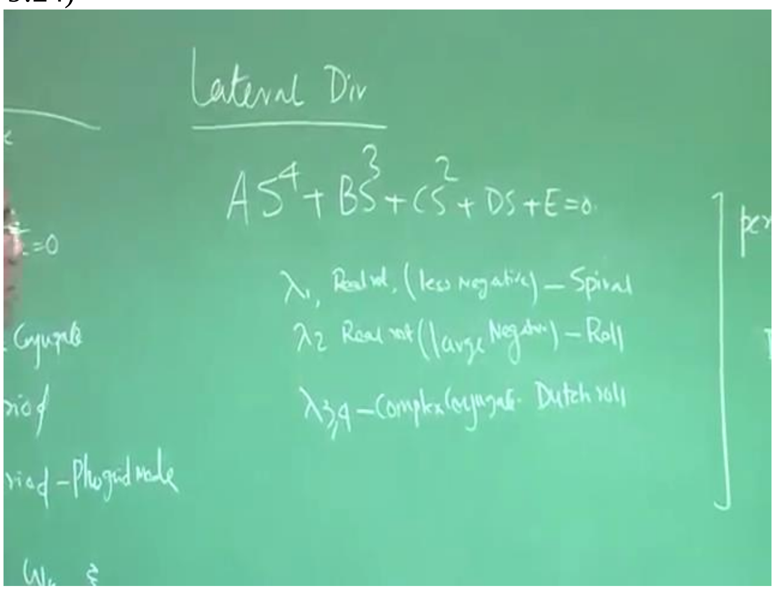

2nd we have gone to lateral directional case. There also, we got equation of this form, AS4 + BS cube + CS square + DS + E equal to 0 and interestingly, once we solve this equation numerically for most of the airplanes, roots are typically like this, one real root, another real route. This real part is very less negative or sometimes marginally positive. This is large negative. Another complex conjugate.

Complex conjugate which is in our terminators Dutch roll mode. This is roll mode, highly damped and this is spiral mode. And you know what is the spiral mode. Suppose the air plane is going like this and there is a bank and as it banks, it starts side slipping. It generates beta and CN beta is positive. So it turns like this. As it turns like this, the lift here increases.

It again banks. Unless things are properly managed, natural tendency is to go like this in spiral mode. Roll, you understand, highly damped mode because of large wing and Dutch roll is this motion, this motion if you combine them together with slight roll which can neglect, that becomes a Dutch roll mode. One motion like this, one like this.

And we also know how to control their natural frequencies of damping ratio. Because as long as you know the roots, if it is a complex pair, you know how to find natural frequency and damping ratio. If it is real, we talk in terms of time to half or time to double. Once we have done that, we realise for a designer, at initial stage, he does not have the whole configuration ready for him.

So if you ask him, you start using this equation, this quartet equation, it will be very difficult. He does not have the numbers.

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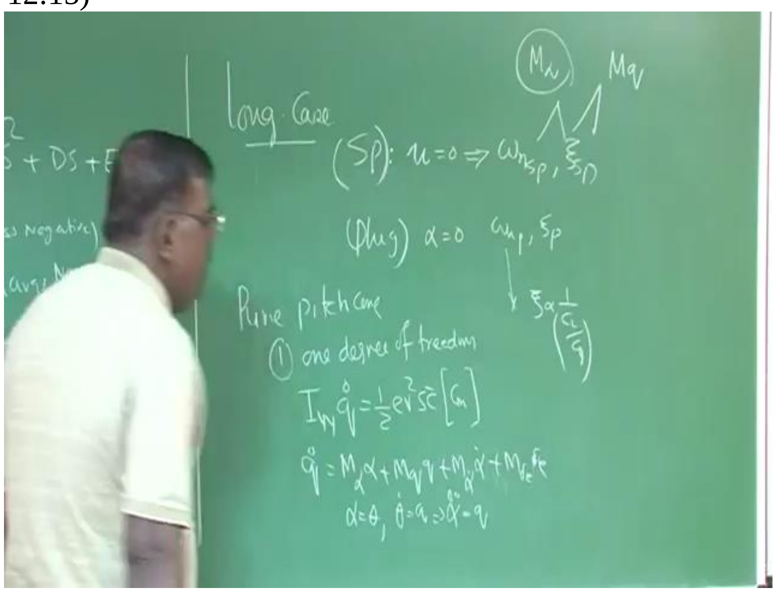

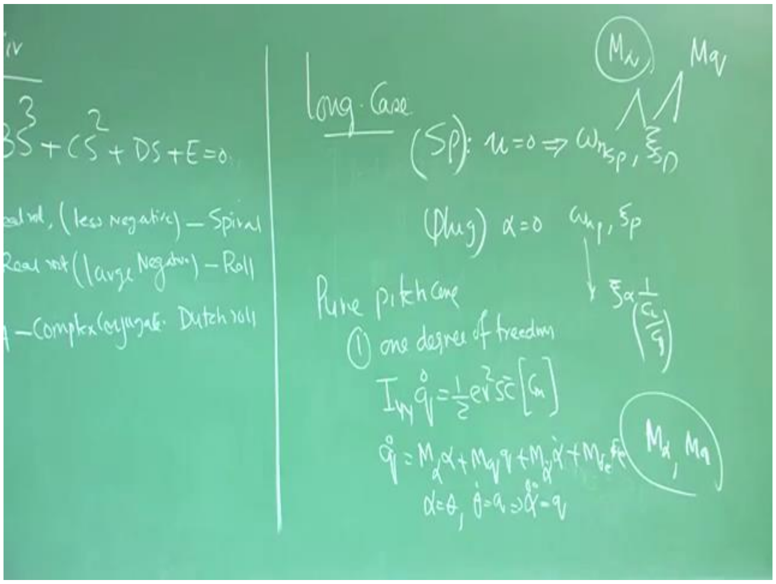

Instead, let us do some approximation and then we did in longitudinal case, we did some approximation, short period approximation and Phugoid approximation. In short period approximation we said, perturbed U is 0 and for Phugoid, perturbed alpha is 0. We have been again and again telling, Phugoid approximation is not very correct approximation. And from here, we got the expression for Omega N short period, Zeta short period, Omega N Phugoid and Zeta Phugoid.

And we said, they are approximate expressions. And we could quickly see from here that it is the M alpha that primarily decides the Omega N. And for this, it is the MQ that decides Zeta. As far as aerodynamic contribution is concerned, it is CM Alpha and CMQ. For Phugoid, we have seen, for a glider type, without any engine and all, this is, short period goes inversely with CL by CD. So if CL by CD is large, Phugoid damping is less.

After realising this, let us further do some simplification and then we went for a pure pitch case where we assumed only 1 degree of freedom and that is the pitch, only pitching is allowed, no plunging. And there we wrote the equation like this, IYY Q dot equal to pitching moment which is half Rho V square SC bar into CM and CM is function of alpha, Q, Delta A and alpha dot.

So we wrote like this, Q dot equal to M alpha into alpha + MQ into Q + M alpha dot + M Delta E into Delta E and we put an approximation that alpha equal to Theta and Theta dot equal to Q. This implies, alpha doubledot equal to Q. This is typically when we are only pure pitching.

The air plane when pitches in atmosphere, it not only does this because angle of attack is changing, so there will be a motion in vertical direction as well. From here, we clearly found that M alpha, MQ are the critical parameters which decides Omega N and damping ratio. This is for longitudinal case.

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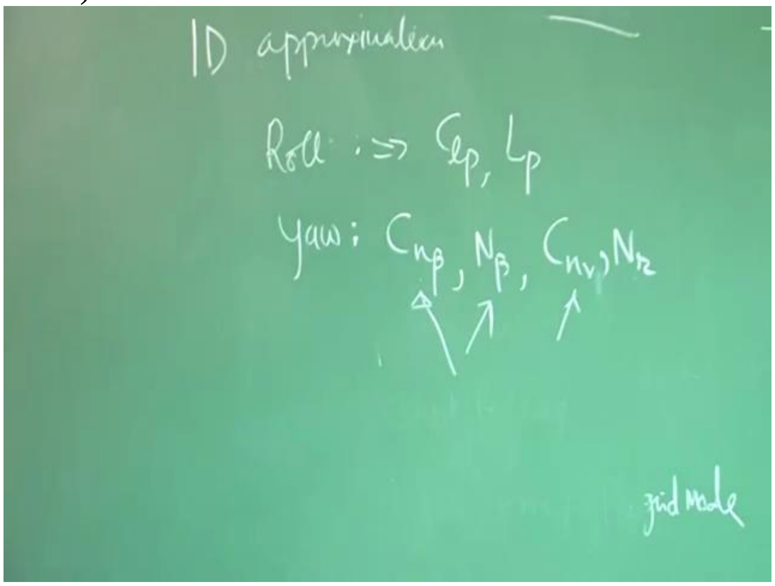

Similar, one study was carried out, 1 lecture we dedicated towards approximation, only approximation for roll as well as for Dutch roll because really not, Dutch roll when I say only approximation, it is the yaw. This is a wrong statement I gave. Dutch roll will have yaw as well as this motion, swing motion will be there. But in yaw approximation one-dimensional, we only assume, only yaw is possible.

And here we identified the parameter CLP or dimensional LP, roll damping parameter which will decide the characteristic of bank buildup or rate buildup and yaw, we identified CN beta or N beta and CNR or NR. So this is the stiffness part, non-dimensional and dimensional. This is the damping part, non-dimensional and dimensional.

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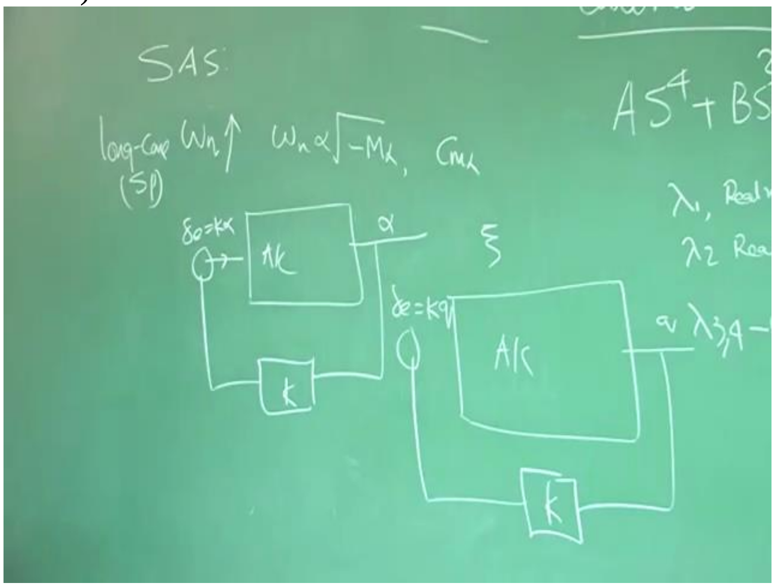

Once we have developed that background, now we made an attempt for understanding stability augmentation system and there, we talk about SAS, we talk about how to increase Omega N let us say we are talking about longitudinal case short period. If we want to increase Omega N from the one-dimensional analysis or from the approximation, we knew that Omega N is proportional to under root of - M alpha.

Also I have to tweak CM alpha. And what we did? Suppose this is an aircraft. I tap alpha, multiplied with some gain and then deflect Delta E is equal to K alpha. And that way, we enhance or we can change the value of CM alpha appropriately, we get Omega N. For Zeta we said, we tap Q, again similar logic, K and Delta E will be given as proportional to Q. So this will help us in enhancing or changing the value of Theta.

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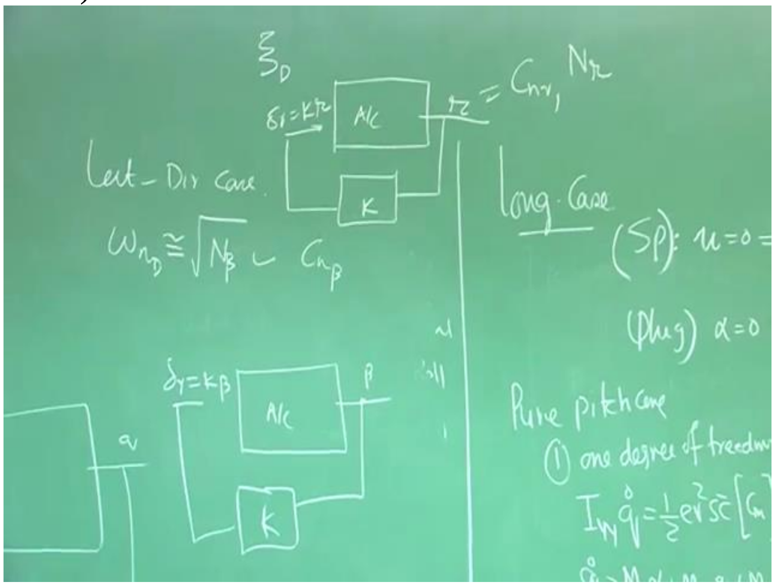

Similar thing we did for lateral directional case. We realise, if I want to change the Dutch roll frequency of the airplane, it is directly proportional to N beta or CN beta. So if I want to increase or change Omega N Dutch roll, I have to change CN beta. And for that, logic was very simple. The aircraft, you tap the beta, multiply it with K and give Delta R equal to K beta. That will enhance locally the value of CN beta or indirectly natural frequency Dutch roll in an approximate sense.

Then we wanted to increase Theta Dutch roll. Now understanding is very clear. Now you are expert. So this is the aircraft. I will tap R, this is K and this will go, Delta R equal to KR. By doing this feed, I can increase locally the value of CNR which means I am changing NR. Hence I am changing the damping ratio for Dutch roll approximately.

This is actually damping ratio for yaw, it is like yaw damping. But you know approximately that has an influence on the Dutch roll frequency. Once we do this, we also understand, we start from here for designing a stability augmentation system.

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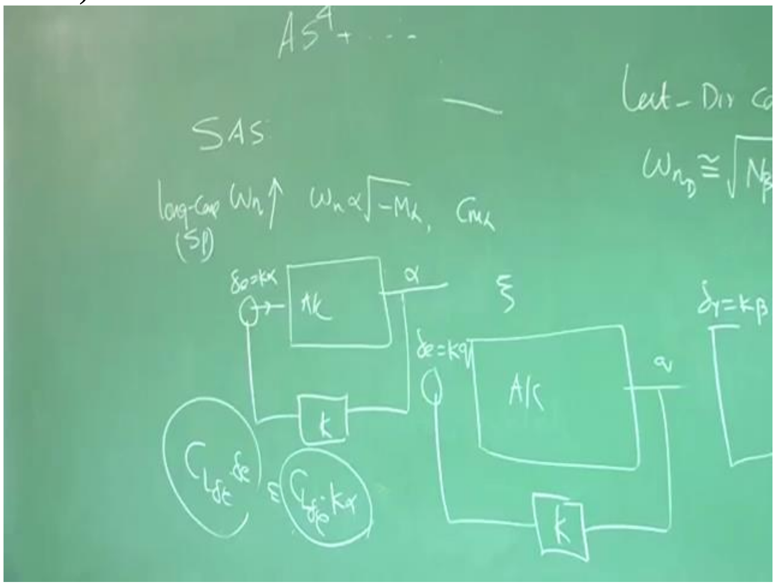

Because we should be careful when I am changing Omega N by giving alpha feedback there are other parameters like CL Delta E into Delta E. This contribution also gets altered. It becomes Delta E into K also. So effectively you could see that CL alpha of the airplane is changing for that time.

So the there may be some effects, cross effects which you need to be careful and all those things you put again back to final result with this equation and see where exactly are the roots. This is one way to handle this. But why we are doing all this things, that is also we are clear, we have spent 1 lecture on that.

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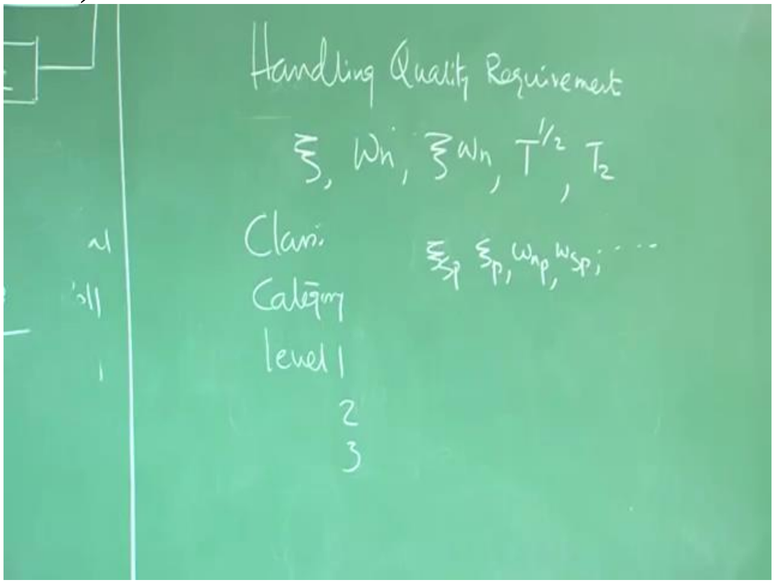

Although I have been advising, if you read and do a Google search to understand the specified handling quality requirements. And what is most interesting is the requirements are postulated using Theta, Omega N or Zeta Omega N together or time to half or time to double. Right? So whatever we have done, by doing approximation, our aim was to find out natural frequency, damping ratio, time to half, time to double.

They are all required to finally meet the handling quality requirements. In handling quality requirements, we have seen that there are various stages. One is class. Then category. What are the flight phases, classes? What is the basic type of airplane category? What are the flight phases? Handling quality is what you want? Level I, level II, level III.

For each class, for each category, we have a level I or level II or level III, specified values of Zeta short period, Zeta Phugoid or Omega N Phugoid or Omega N short period. They are specified. Either they have specified only Zeta. Sometimes product of Zeta and Omega N. You will find, why I am saying you go for a Google search. New new requirements come as the technology gets upgraded.

Similar things have been done for Dutch roll and spiral mode. What is important to notice is that for spiral mode, generally this is marginally stable. So mostly we talk in terms of time to double and must ensure the specification is stressing, time to double should be large so that the pilot has enough time to correct.

So we have done all these things and we understand that we have done sufficient as a preliminary exposure for designing the fundamentals for dynamic stability and SAS together. If you go to a control man, he starts from here. Then things become more rigourous.

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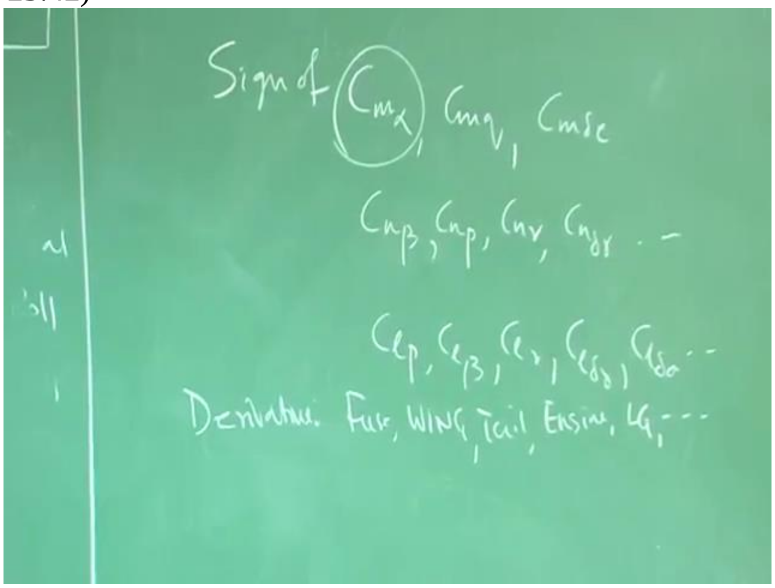

Before I conclude this summarisation please understand, you need give a lot of weightage in understanding the sign of these derivatives, let us say CM alpha, CMQ, CM Delta E. If I say CN beta, CNP, CNR, CN Delta R, like that then CLP, CL beta, CLR, CL Delta R, CL Delta A. It is extremely important that you revisit and check whether you clearly understand what are the signs of these derivatives because these derivatives will have a contribution.

All these derivatives will have a contribution because of fuselage, because of wing, because of tail, because of engine, because of landing gear, so many. You will also notice that few of the derivatives will have huge contribution for a particular components. For example CM alpha the major component which will contribute will be the tail. CLP, major contributor will be the wing.

So that, as a designer, you should clearly understand what are those components of the airplane who will contribute to these derivatives. So that you can really design appropriately as an initial guess. Suppose if you want to increase stability margin and you are going on increasing CMQ, then something is wrong because CMQ is the damping. It contributes towards damping.

If you are thinking in terms of increasing the damping ratio, we will focus on CMQ and see how CMQ can be increased. When we are talking about increasing stability margin, I look towards how to increase CM alpha and there you will find how tail volume ratio plays an important role. So you do not do mechanically whatever we have been discussing. Spend some time.

When you look for CM alpha, you see what are the components that will decide CM alpha. How can I change CM alpha without altering much change in the whole airplane. That is extremely important. And my approach for doing all these things was to give you that insight. Do not lose the sight of all these physical components which are there in the airplane and you are trying to model their effect by some sort of an aerodynamic modelling.

And finally you should know, final contributor is how good and how bad we have displayed those components. So please do not get lost into these equations or expressions. Focus your eyes on the airplane wing, see different types of wings, different types of aerofoil, different types of tail configurations.

Sometimes it will be T tail, sometimes it will be a low, tail at the midtail, it maybe canard. There could be forward surf wing. So all those things, ask yourself a question, how that is going to affect the stability? Then only you will really get the juice of this fundamental understanding. I am sure you will do a good job. Thank you very much.