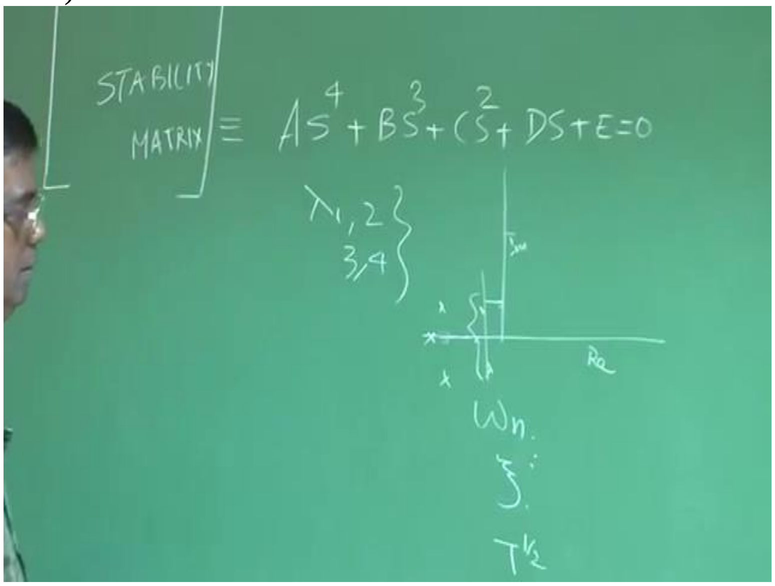
Aircraft Dynamic Stability & Design of Stability Augmentation System
Professor A.K. Ghosh
Department of Aerospace Engineering
Indian Institute of Technology Kanpur
Module 6
Lecture No 33
SAS for Longitudinal Dynamics

Today we will be discussing very introductory understanding about stability augmentation system.

(Refer Slide Time: 0:27)



And the basis is you understand that when we took the determinant of this matrix, which was stability matrix and the exact solution as equation of this form, AS4 + BS cube + CS square + DS + E equal to 0. Determinant of this matrix and also we realise, roots, Lambda 1, Lambda 2, Lambda 3, Lambda 4 for longitudinal case, for lateral directional case, they depend on the value of A, B, C, D, E, their magnitude and their sign and in turn it depends upon the dimensional derivatives

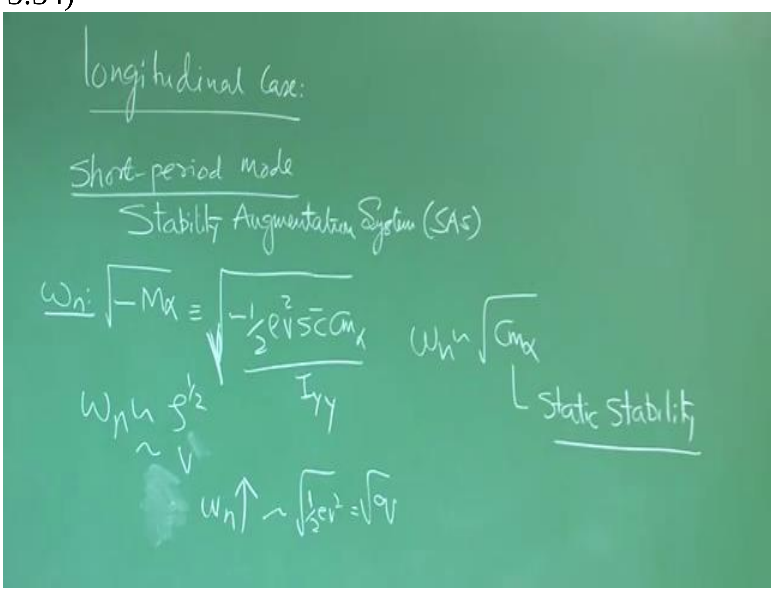
And dimensional derivatives in turn depend upon the non-dimensional derivatives and of course inertia properties. And we also realise that if we plot imaginary and real part of the root, we wanted them either this way or maybe on the real negative side. What was important that roots would lie on the left half of the plane to ensure that there is a convergence.

But the question comes, how far from the real and what is the magnitude of this if it is a complex conjugate, that depends upon what? Whenever we think of complex conjugate, we know there is a natural frequency and then there is a damping ratio. And we need to change Omega N and Zeta depending upon our requirement which is decided by the handling qualities of an airplane.

Today we will be discussing about how do I change Omega N and Zeta to satisfy the handling qualities of an airplane. This handling qualities, we will be talking about but to begin with, you understand that they are primarily characterised or benchmarked through the values of Omega N, Zeta or their product for different different modes, for longitudinal, short period, for Phugoid, for lateral directional, roll mode, for Dutch roll frequencies.

But all are focused towards Omega N and Zeta or sometimes time to half amplitude, T half. These are all the concepts which were there when we analysed the second order system.

(Refer Slide Time: 3:34)



So we will take first suppose we are dealing with longitudinal case. I will give one example, let us say for short period mode. And what we are discussing now? We are trying to see how stability can be augmented during flight. So we say SAS, stability augmentation system or in short, we say SAS. How do I approach? I said our target is handing Omega N and Zeta.

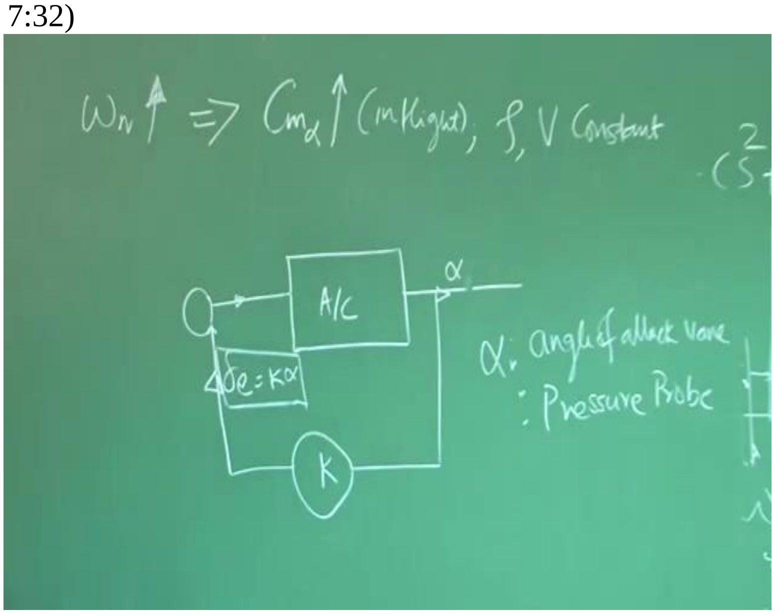
Let us say, if it is Omega N and if it is a short period, by now we know we have analysed pure pitch, we have analysed an approximate short period mode by simplifying the exact equation. Also we know the exact solution as well but for a designer, will always try to start with a simpler thing. So Omega N if you see for pure pitch, it will be root - - M alpha and this you know is equal 2 - half Rho V square SC bar CM alpha by IYY the whole under root.

You could see that this Omega N will go on changing with Rho. It will change with V. V square and Rho, I can try it because it is under root. Root recall half and V. The point is, as I am going higher and higher, the natural frequency in short period mode will go on reducing. As I increase, the short period natural frequency will go on increasing or in totality if I say, Omega N changes with, increases with dynamic pressure that is half Rho V square under root, this is under root of Q.

So either I can realise with attitude. As it goes higher and higher, Omega N reduces. With increase in speed, Omega N increases. With higher altitude, Omega N decreases because with higher altitude, density goes down and with increase in speed, Omega N goes on increasing. But also in totality, I can say Omega N changes with dynamic pressure the airplane is experiencing. That is primarily the environmental condition.

However from aerodynamic really, you could see that Omega N changes with CM alpha. And what is CM alpha? CM alpha if you recall, it is a measure of static stability or in turn, it is a measure of stability margin.

(Refer Slide Time: 7:32)

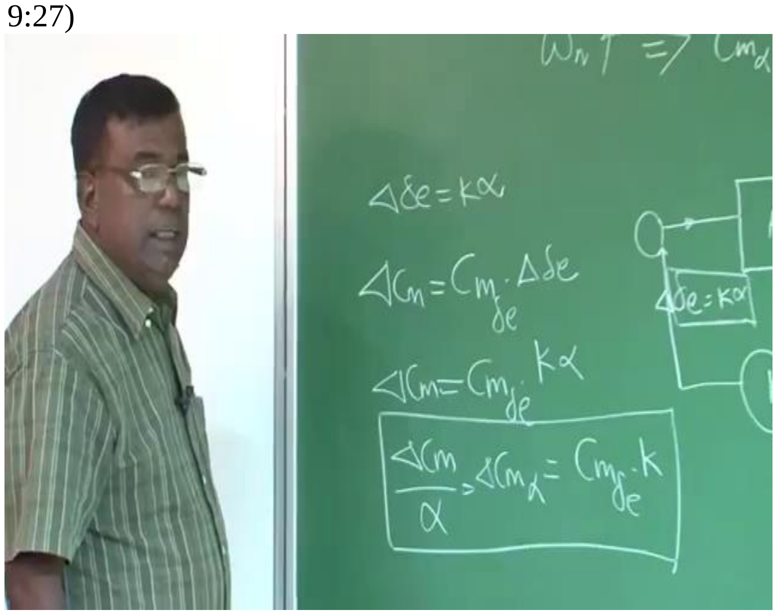


Let us say, if there is a requirement that Omega N be increased at some point in the fight. What is the way to do it? Omega N to be increased, and that means, I need to increase CM alpha in flight keeping Rho, V constant. That is, at a given altitude, at a given speed, I want CM alpha to be increased so that Omega N is increased. How do I do in flight?

So let us say, the approach is simple. This basic understanding should be clear. This is the aircraft model and here there is a elevator input going here and output and getting as alpha, angle of attack. What I do? I tap this alpha and multiply with again, K and then feed such a way that Delta E is K alpha. So this is the Delta Delta E which is proportional to K alpha. Somebody may ask, how do I measure alpha?

Alpha can be measured by using angle of attack vane, it could be measured by pressure probe. All these things we have discussed in the last course. How CM alpha is changing, let us see.

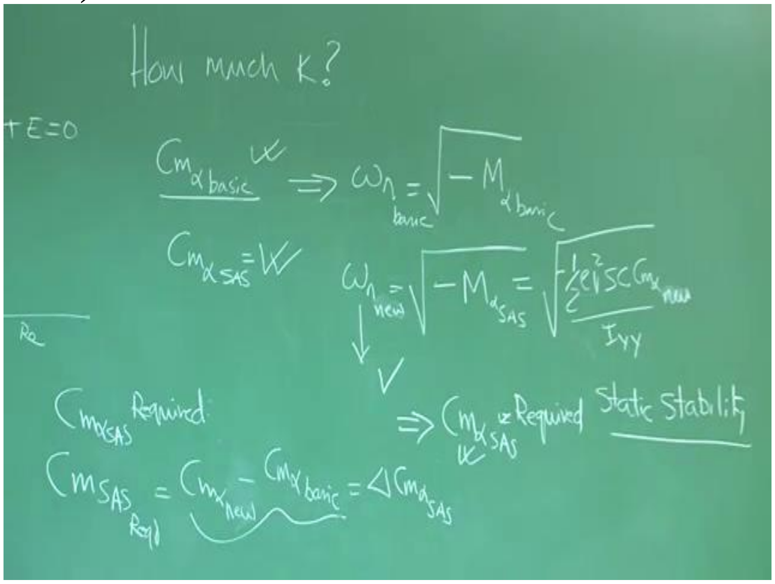
(Refer Slide Time: 9:27)



Because of this Delta Delta E which is given as CM alpha, what is that Delta CM we are introducing in the airplane? That you know, CM Delta capital E into Delta Delta E which will be equal to CM Delta E into K alpha. So this is Delta CM into this by giving Delta Delta is equal to K alpha. And then from here, Delta CM by alpha which is Delta CM alpha which is nothing but CM Delta E into K.

So this understanding is there. If I want to introduce additional Delta CM alpha or artificially I am changing the stability matrix for that time, I can easily do it by feeding alpha and deflecting elevator at that time proportional to K alpha.

(Refer Slide Time: 10:36)



How much K? Let us say we have CM alpha Basic. That is, which has been designed from the beginning depending upon when location, tell location, CG, etc, etc. This is available for the airplane. Now, we want CM alpha SAS. Some value which I am targeting. And how do I get that value? I get that value because this corresponds to some Omega N basic which is nothing but - M alpha Basic.

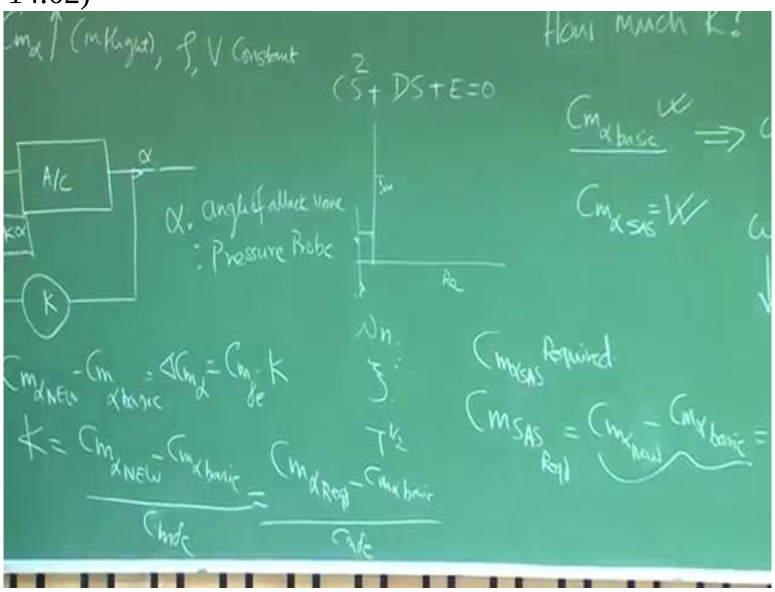
Now I want Omega N SAS. That will be under root - M alpha SAS. And this is I know equal to under root - Rho V square SC CM alpha by IYY and this is CM alpha SAS. So if I know this value, how much I need, this is known. And if I know at what altitude I am flying, I know SC, I know IYY. So from here, I can easily find out what is CM alpha SAS required?

Do you understand this? We know what is CM alpha Basic. That is Omega and from there I know what is Omega capital N basic. That is given by M alpha basic. What is the CM alpha required through SAS? Why do I require it? Because I want Omega N new. Correct? So this CM alpha would be new and from there, I need to find out how much CM alpha SAS is required. And what will be CM alpha SAS?

CM alpha SAS required will be CM alpha new - CM alpha basic because already CM alpha Basic is there. And that is what is the role of the SAS which I write as Delta CM alpha SAS. Clear? I repeat again. CM alpha Basic you know from the design which is given by at the root of M alpha Basic. And if I for the responded, I find, for Omega N new. For a given altitude and speed, what is the CM alpha new required.

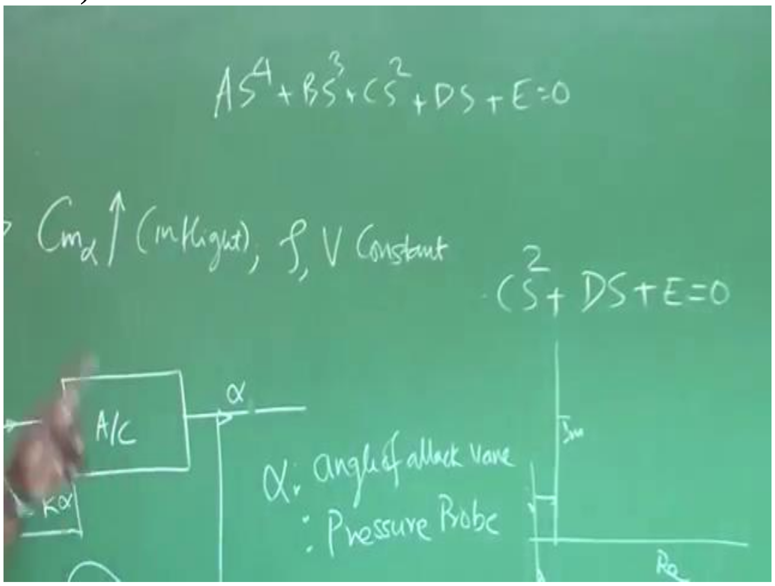
That means, next question is, how much is CM alpha SAS required so that Delta CM alpha SAS gives the additional Delta CM alpha which is nothing but CM alpha new - CM alpha Basic.

(Refer Slide Time: 14:02)



And then you know this CM alpha new - CM alpha Basic which is Delta CM alpha, that is equal to CM Delta E into K. So what is K? K becomes CM alpha new - CM alpha Basic divided by K oblique divided by CM Delta E. And textbook, you will find this is also written as CM alpha required - CM alpha basic divided by CM Delta E. That gives the value of K.

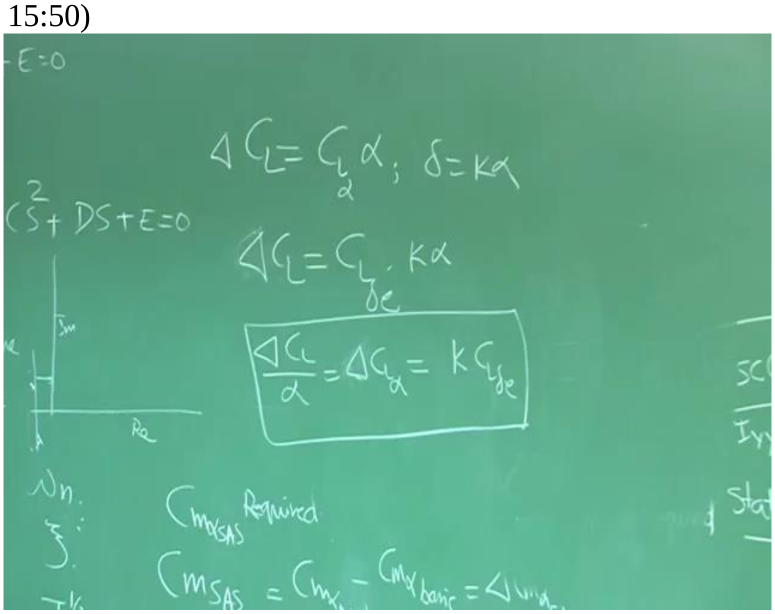
(Refer Slide Time: 15:08)



But there are restrictions on K. Because do not forget, finally whatever CM alpha new you are getting, that has to again go back to this. That has to be plugged into that equation and see whether the roots are what you are looking for or not. So that restricts. You cannot take too high-value off K, you cannot take to low value of K.

So those are of engineering aspect but fundamentally, the approach is this. This is one. Then you have to also see that by doing this, whether any other parameters are changing or not, specially aerodynamic parameters.

(Refer Slide Time: 15:50)

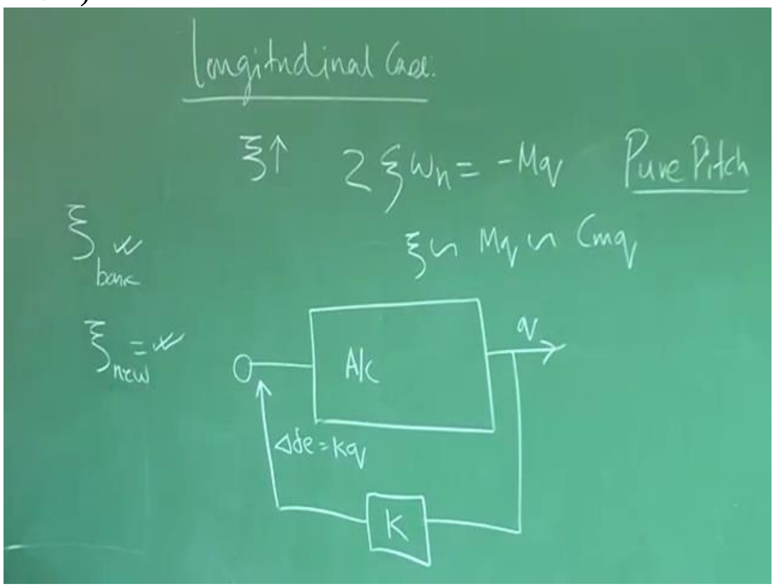


I have to see that CL, there will be change in CL also because CL alpha is given by CL alpha into a alpha. The additional CL and if you see Delta CL because of Delta Delta E which I have given as Delta equal to K alpha. So this will be CL Delta E into K alpha. So Delta CL by alpha which is nothing but Delta CL alpha which will be given as K into CL Delta E.

So by giving a SAS deflection using Delta Delta equal to K alpha, it is fine, you have been able to change CM alpha Basic to CM alpha new or CM alpha required so that Omega N is what you look for. But do not forget, it is also changing the value of CL alpha. So that also, the input will go here and we check whether still you are getting the roots on the left-hand side and to the desired location so that you have the correct value of Omega N.

Okay because this we have started using a very first approximation, we have taken a pure pitch case. And that is the way to start. Otherwise it becomes very complicated. So this sort of an iteration goes on. Now if we want to look for how do I change damping ratio, let us also see. Once you have understood this, how to change damping ratio should also become very simple for you.

(Refer Slide Time: 17:42)



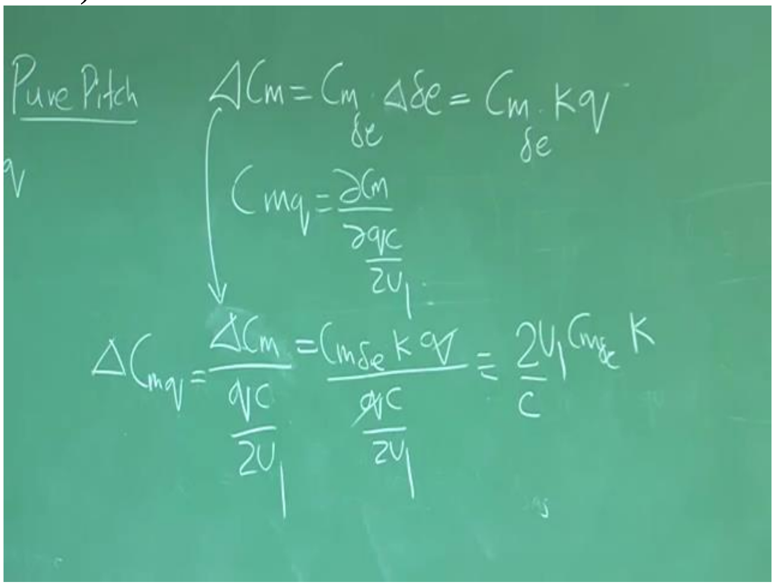
Let us not forget, we are talking about longitudinal case. Let us say now we want to change Zeta. Again we will take help of pure pitch for longitudinal case to start. And you know, 2 Zeta Omega N is equal to - MQ and Zeta is proportional to MQ or C Zeta is proportional to CMQ from the pure pitch equation. Now you see. Let me write pure pitch. We are talking about longitudinal. We are starting with this.

So suppose the airplane has Zeta basic. Now we want to increase the Zeta to Zeta new. That is the question. Because we are not happy. We find that as I am going higher and higher, the Zeta is also reducing because Zeta if you see here from MQ is nothing but half Rho V square SC bar CMQ by IYY. And other terms are there. So you know that very clearly, Zeta will change with altitude as well as dynamic pressure.

So we have a Zeta basic which you design the airplane for a given altitude but you are now operating at a different altitude let us say. So now you want a Zeta new so that your airplane has the right type of damping ratio as dictated by let us say handling qualities. How do I approach? Again the method is simple understanding wise.

If this is the airplane and this is the Q, pitch rate and this is what I do. Now I tap Q, pitch rate using a gyro and feed Delta Delta E equal to K into Q. Proportional to Q, I feed elevator deflection. Now as long as Q is there, I require this. So now what will happen?

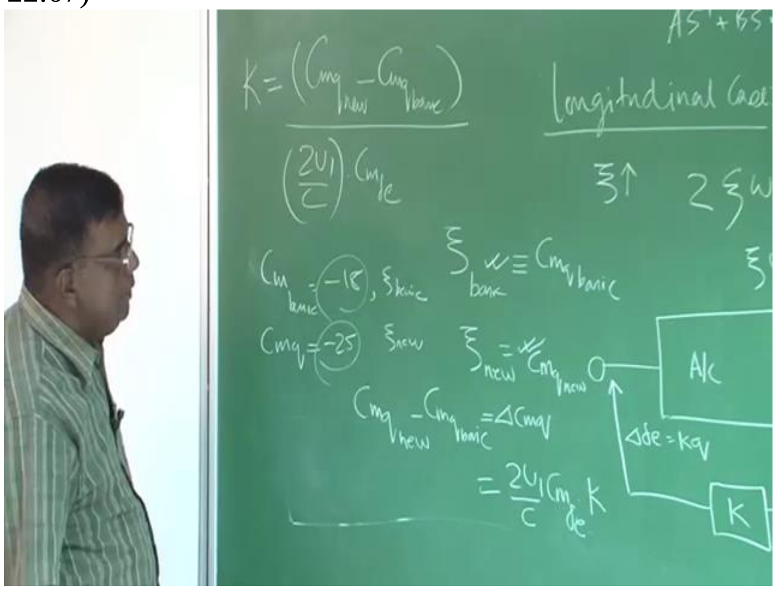
(Refer Slide Time: 20:04)



If I do this, again what is Delta CM? That will be CM Delta E into Delta Delta E. That will be equal to CM Delta E into KQ. That is how I have deflected. And CMQ is defined as DCM by DQC by 2U1. So I divide both sides by QC by 2U1. So I get CMQ equal to CM Delta E into KQ. I write it Delta or for your clarity, let me write Delta CM divided by QC by 2U1 to get CMQ by definition.

So I divide it by QC by 2U1 and that gives me, Q and Q gets cancelled. This gives me 2U1 CM Delta E K by C. Clear? I hope I have not done any mistake. CM Delta E into KQ. So I am dividing CM Delta E into KQ by QC by 2U1. 2U1 goes up, C is here, K and CM Delta E. So this is Delta CMQ which I am generating by feeding Q which is measured through a rate gyro and deflecting the elevator proportionally with Q.

(Refer Slide Time: 22:07)

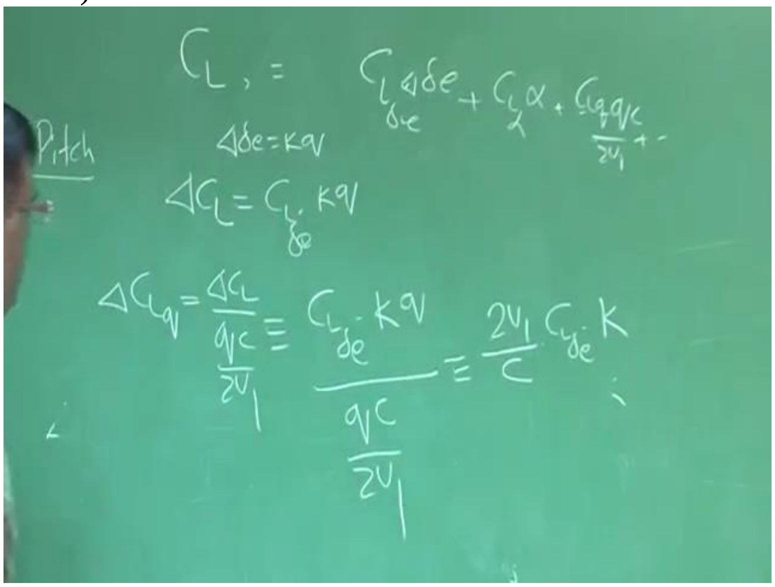


Now the question is, I know Zeta basic, it has corresponding CMQ basic and for Zeta new, I need CM Q new. So how much additional CMQ I require will be CMQ new whatever is required - CMQ basic. That should be equal to Delta CMQ and that should be 2U1 by C into CM Delta E into K. Or I can write K equal to CM Q new - CM Q basic divided by 2U1 by C into CM Delta E.

This CM Q new, many textbooks will tell CM Q required. Purposely I have put CM Q new so that you know that suppose CM Q basic was - 18 for Zeta basic. But for CM Q, it is - 25 for Zeta new. Then this - 25 - 18 is Delta CM Q. That comes here. And the rest is here. Again, whatever value of K you get, you have to again feed it back to that AS4 + BS cube + CS square + DS + E equal to 0 and check whether the aircraft is still than equally stable or not.

And what sort of root or pair you want? What exactly you want those roots so that you get the correct value of Zeta and Omega N altogether. So you have to go collectively, with everything together you have to go.

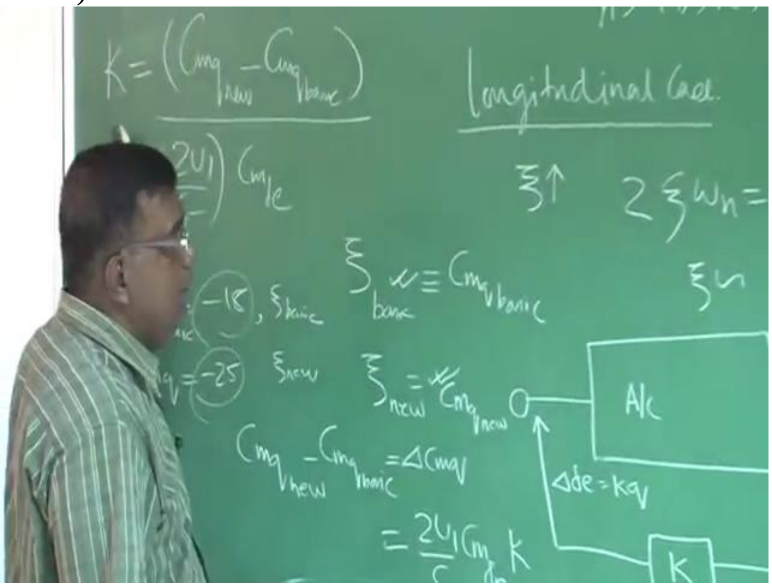
(Refer Slide Time: 24:23)



But at the same time, you must also understand that as I am giving a feedback on Q using a gyro so you know that CL also has a component, CL Delta E into Delta E. Like it has CL alpha into alpha blah blah. So Delta CL because of Delta Delta E. Delta Delta E equal to KQ. What will happen is, here also I have CLQ into QC by 2U1. Like this, all terms are there. So this will tell you, CL Delta E into KQ.

So again you find additional CLQ, Delta CLQ be seen by the airplane and that will be Delta CL by QC by 2U1. That is equal to CL Delta E into KQ by QC by 2U1. But this becomes 2U1 by C CL Delta E into K.

(Refer Slide Time: 25:34)



So whatever K we have identified from here, make sure this CLQ which was basic that also gets altered by this Delta CLQ which you have to also correct once you are solving this exact equation. This is in a nutshell I have given you the approach. We will solve 1 or 2 examples on this so that you understand. And similarly we will build for directional case, lateral case.

And if you understand this much, you yourself will be able to extend this understanding to lateral directional case. We will do that in the next class. And I am trying to talk about handling qualities soon. But I thought, before I talk about handling qualities, we should demystify all this SAS and all. These are basically this, this is the concept. That is all.

The rest is solving (())(26:33) equation of motion or finding a transfer function. How to find your solution? Understanding is this. Okay. Thank you very much.