**Aircraft Dynamic Stability & Design of Stability Augmentation System**

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**Module 6**

**Lecture No 32**

# Pure Yawning and Pure Rolling Motion

Good morning friends. After exhausting ourselves with huge huge expressions we decided that we will try to see what other information we get from one-dimensional analysis of pitching motion, yawing motion and rolling motion. And you will see how important are these in designing stability augmentation system. We have already done longitudinal case.

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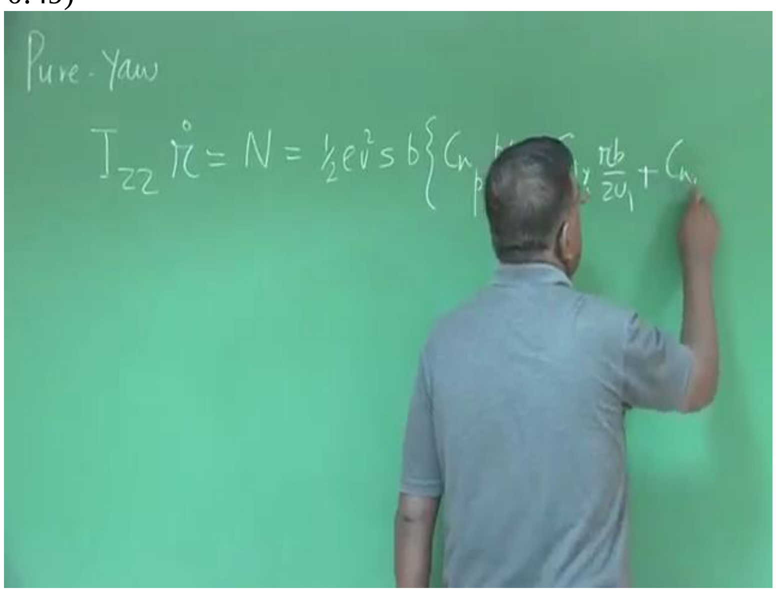

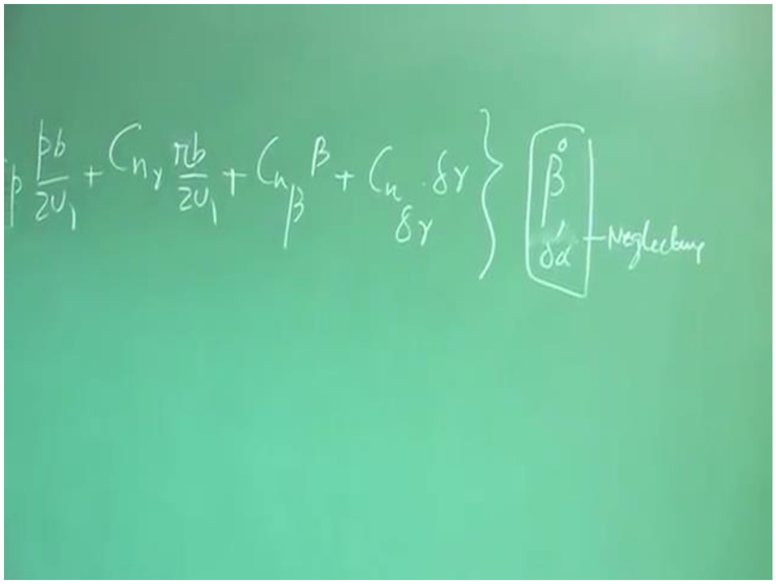


If I take pure yaw, that is only this yaw motion is allowed and as you know, as far as yaw is concerned, when the right-wing goes back, that is a positive yawing moment. When you are flying like this and the right-wing goes back, that is a positive yawing moment. And we also know that IZZ R dot is equal to yawing moment N where R is the yaw rate. MN I can write it as half Rho V Square S B into CNP into PB by 2U1 + CNR into RB by 2U1 + CN beta into beta + CN Delta R into Delta R.

I am neglecting any contribution because of beta dot or Delta A. This I am neglecting. Both beta dot and Delta A contribution I am neglecting. You know, beta dot is like alpha dot. Alpha dot for longitudinal plane and beta dot is because of side wash, rate of change of side wash specially on the vertical tail which I am neglecting.

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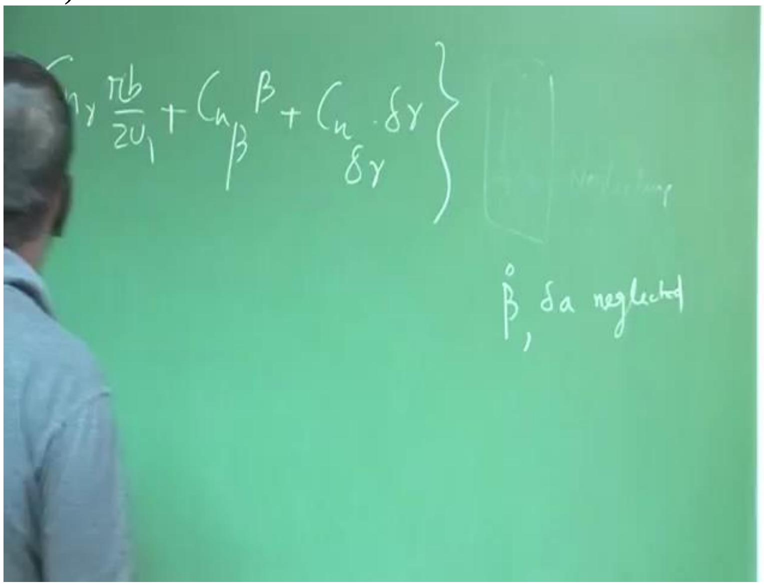


It should not confuse you. So let me erase this part, I write it here that beta dot and Delta A effects are neglected. So this is the equation I have got. Again, you are now expert.

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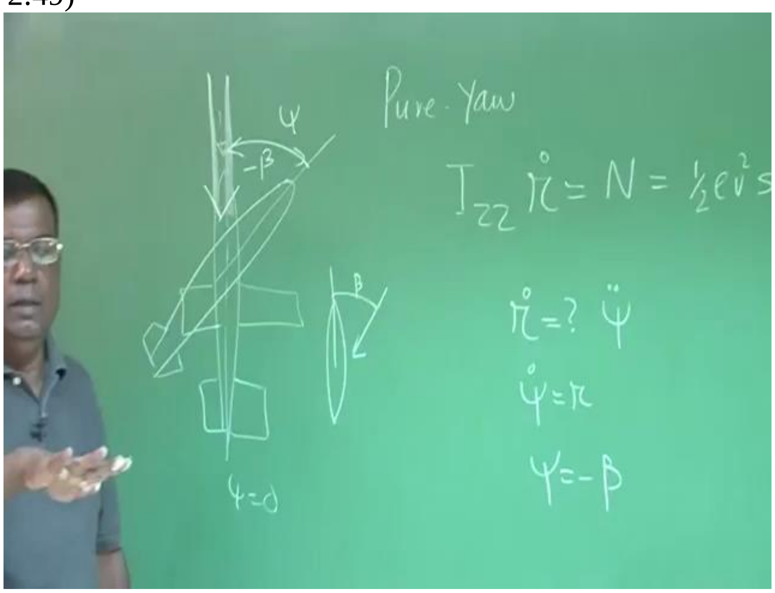


I write, R dot equal to what? R dot is nothing but psi double dot. Remember? Because psi dot is basically R. The yaw rate if there is no bank just we are talking about is. So psi dot is basically R and R double dot is psi double dot. Also we know that psi is equal to - beta.

Remember? If this is the airplane, this is psi equal to 0, relative wind is coming like this. If I turn the airplane like this then this is the psi and relative air is coming like this. So this is also - beta.

Because positive beta, we have defined. Relative wind coming from the right hand side. So if there is no bank, it is yawing in the horizontal plane then psi is equal to - beta.

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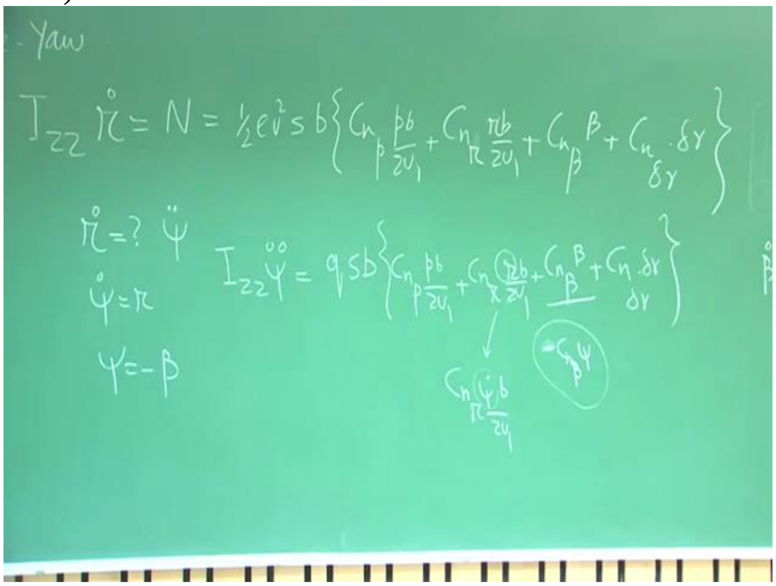


So now if I translate this equation using this, then I get IZZ psi double dot is equal to, I can write this QSB where Q is the dynamic pressure. Into CNP into PB by 2U1 + CNR into RB by 2U1 + CN beta into beta + CN Delta R into Delta R. Right? But what we are looking for? We are writing the equation in site. We know psi is equal to - beta. So here I will write, this term I will write as + CN beta into psi with a - sign.

And here, this R, I will write CNR into psi dot B by 2U1. Is it clear, what I have done? Psi dot is R. So for R, I have written psi dot and beta is equal to - site. So this -. Okay. Very simple?

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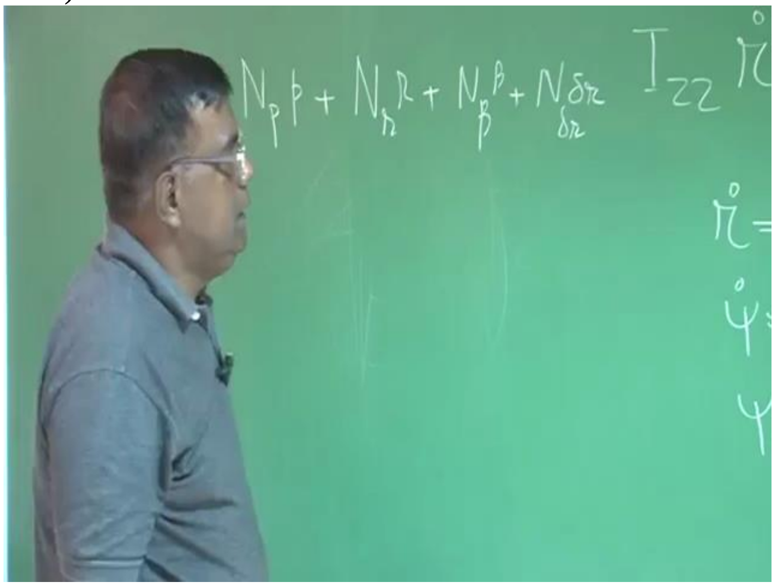

So finally the equation takes the form, psi double dot equal to, I can write NP into P + NR into our + N beta into beta + N Delta R into Delta R. So what is NP?

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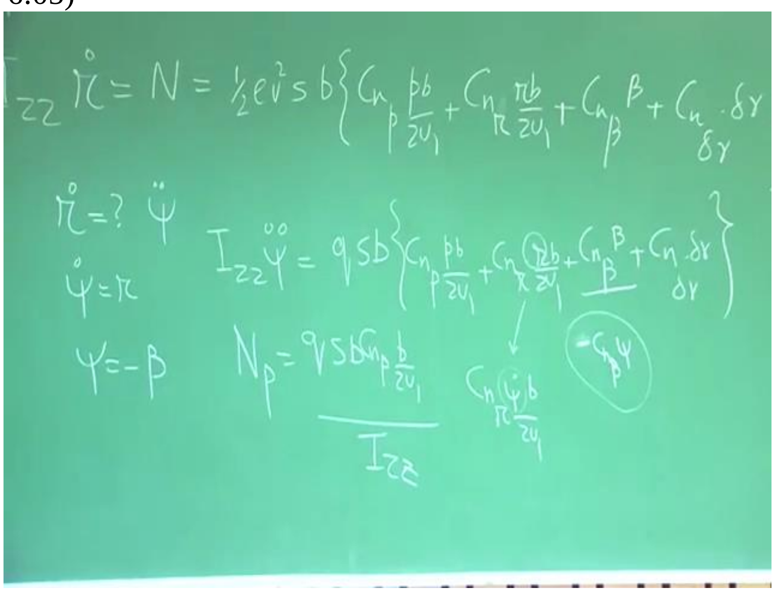

NP is nothing but from here you can see, NP is nothing but QSBCNPB by 2U1 divided by IZZ. Okay, similarly CNR oh, sorry, edit, similarly NR will be CNR into by 2U1 into QSB divided by IZZ. Like that you can find this expression which already you know.

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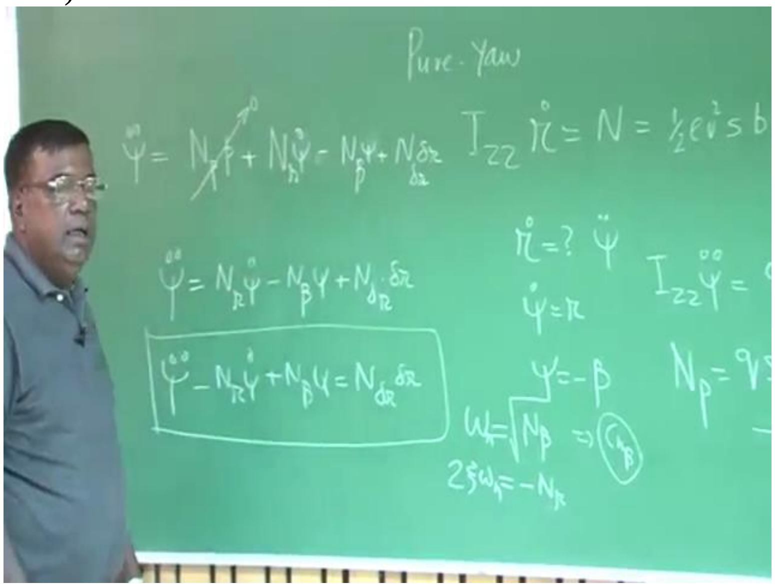


And of course here I have to modify it, I write it as psi dot and I write - this. What I have done? From this equation, beta is equal to – psi and psi dot is equal to R. So all those things, I have converted into psi dot. Now if the airplane is doing only pure yaw like this, then there is no P. We are not doing this motion. We are doing only this motion.

So naturally I will say thank you very much, this term goes. So I have got psi double dot is equal to NR into psi dot - N beta into psi+ N Delta R into Delta R. So I can write this as psi double dot - N R psi dot + N beta into psi is equal to N Delta R into Delta R. Again you see, pure yaw motion when we are modeling, we are again getting a second order differentially equation and which is same, the form is same as the mass spring damper system equations of motion and we know how to handle that.

From this equation, I can write Omega is equal to under root of capital and beta. Similarly 2 Zeta Omega N will be equal to - NR. Remember, for longitudinal case, it was, Omega N was - M alpha. So now see the beauty, if you want to change Omega N, this clearly tells you, you have to handle CN beta. And how can you handle CN beta? How can you change CN beta?

You know, it is through vertical tail volume ratio. That means, either increase the vertical tail area or the increase the length of the vertical tail from the or distance of the vertical tail from the CG of the airplane. Once this is done, then we have to do the pure roll. Let us see what we understand from pure roll and how can the origin formations I can use for designing a stability augmentation system.

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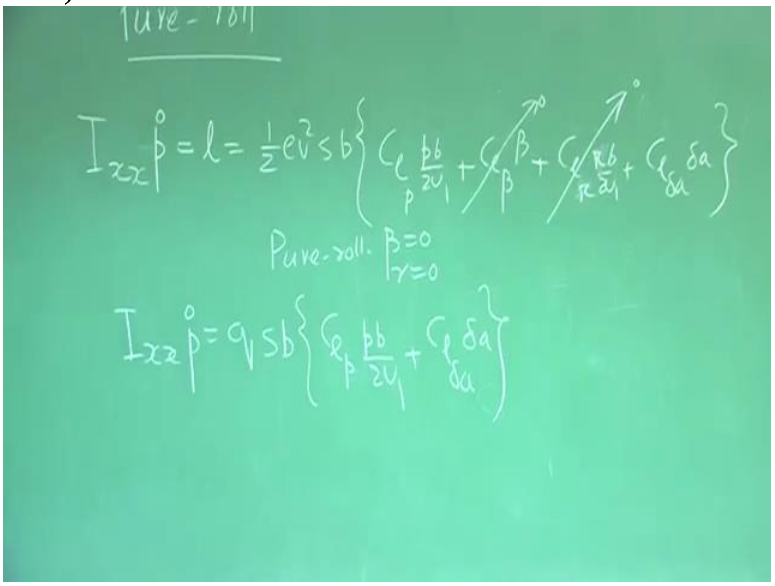


So let us discuss about pure roll and before we discuss this, please understand, pure roll we are meaning thereby this is the pure roll. But you know, as I roll like this, the angle of attack here will increase. So drive also will increase here. That will give me a yawing moment also. Like, when you are doing a yawing moment like this, the list here will increase. It will give a roll also. But, we are seeing only one-dimensional motion.

That is we are only talking about pure roll. No yaw motion. So again if I write, equation for that, this is rotation about X axis. This is equal to rolling moment. That is equal to half Rho V square SB into CLP into PB by 2U1 + CL beta into beta + CL R into RB by 2U1 + CL Delta A into Delta A. This is a generic form. But we know it is a pure roll.

Since it is a pure roll, beta is not there, R is not there. Only I am doing pure roll, this. No beta, or no yaw rate. If I do this then I get IXXP dot is equal to QSB into CLP into PB by 2U1 + CL Delta A into Delta A because this goes. This is R. This is also R. This also goes. So this is the form.

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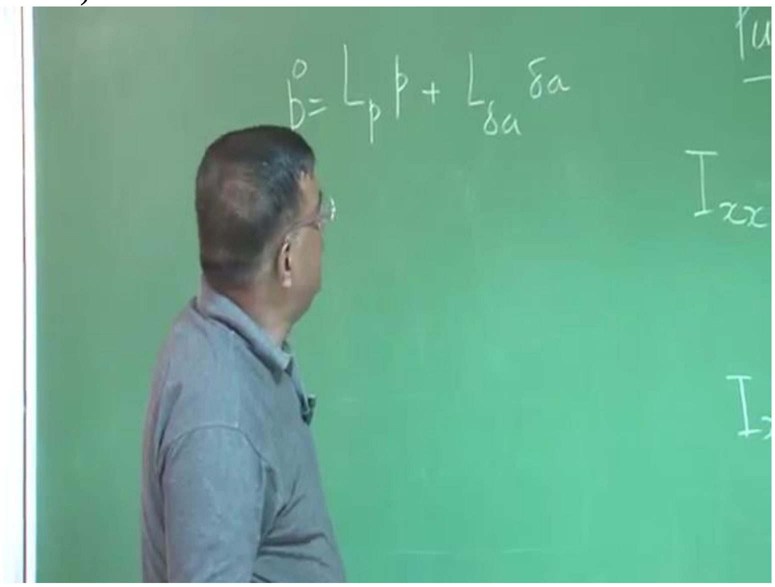


And I can easily write, P dot equal to LP into P + L Delta A into Delta A. So simple. And what is LP? LP is, what will be LP?

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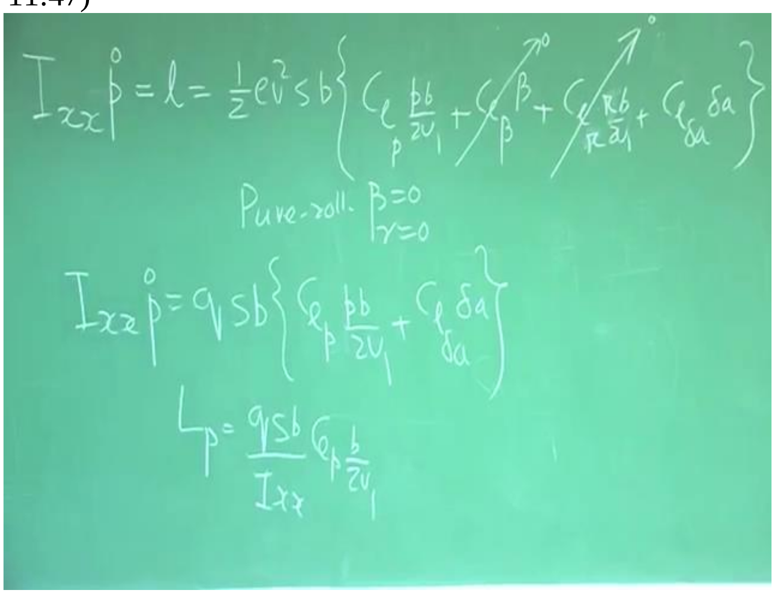


LP will be equal to QSB by IXX CLP by 2U1. Similarly CL Delta you can find out.

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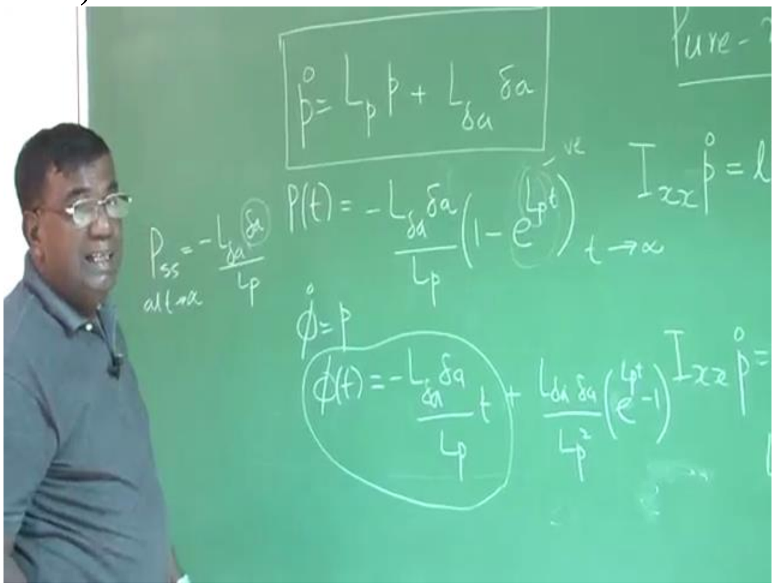


And you could see this, this is a first order differential equation and you can easily find out the expression of P and this P will be given by - L Delta A into Delta A divide by capital LP into 1 - A to the power LP into T. And phi dot is equal to P. We are doing only one-dimensional roll and phi is the bank angle.

So from here, you can find out phi T is equal to - L Delta A into Delta A buy LP into T + L Delta A into Delta A by capital LP square E to the power LPT - 1. So what is the message if I try to read PT, P of T? That is as time goes to infinity, large time, since LP is negative, this gentleman will approach to 0. So, P of steady-state that is as T approaching infinity, this will be - L Delta A into Delta A by LP.

So depending upon what type of Delta aileron you have given, whether step input, pulse, impulse, whatever it is, so you know what will be the steady-state value. And for phi also, you have seen this. LPT is negative. So as your time increases, this LPT E to the power 0, this whole will become E to the power - T, T tends to infinity. So this goes to 1.

And then phi also approaches the value which constantly increases with time T as time increased. So that if I were to plot it, how will it look like?

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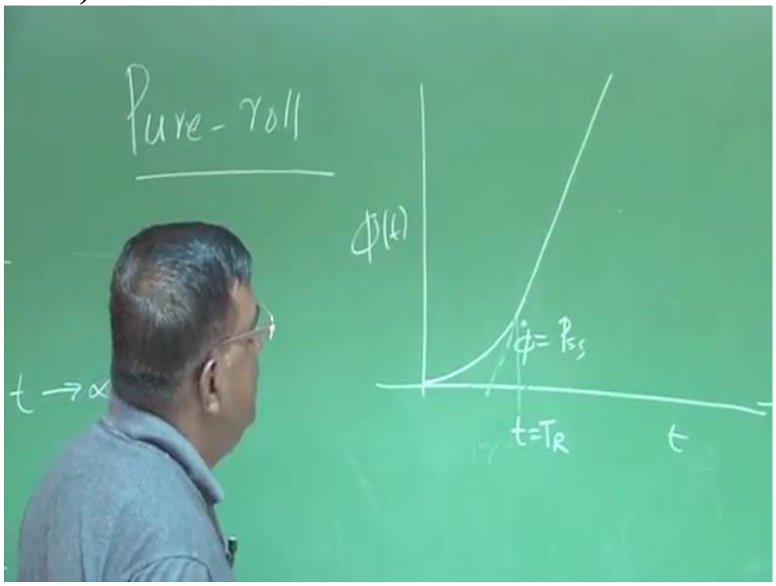


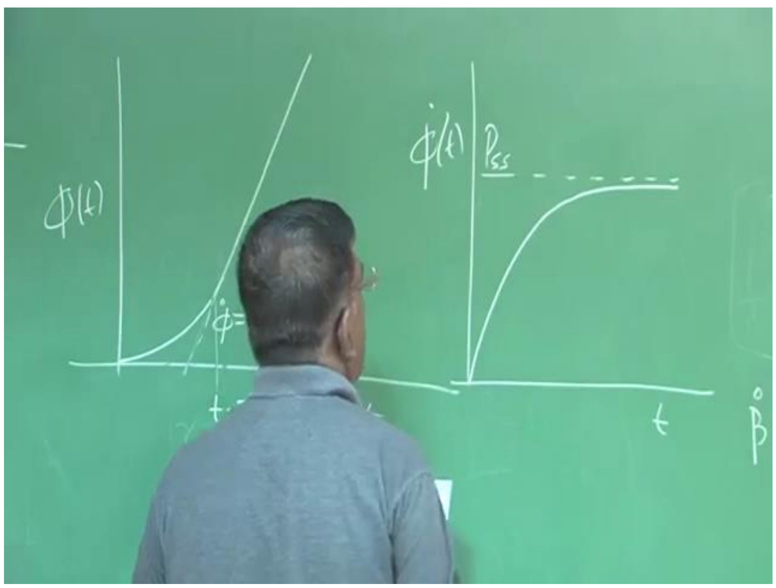


Let us plot phi T versus T. You could see that the variation is like this. So phi goes on increasing with time. But interestingly, you could see if I take a slope at some point, at phi dot, at this point, it is basically P steady-state and that slope, you can draw at some point which will correspond to time equal to, P equal to TR.

And if I try to see what is phi dot doing, phi dot vs time, you will find that it is, so this is PSS or steady-state P.

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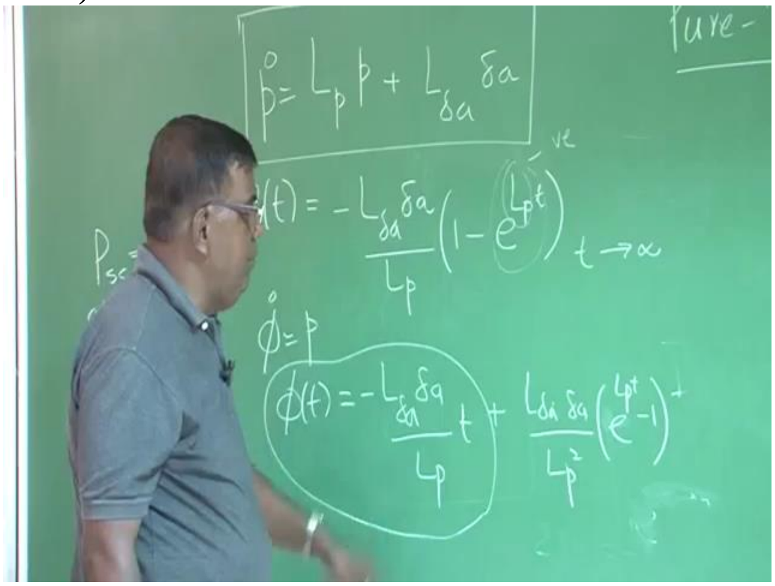

You could see phi of T is L Delta A into Delta A by LP and L Delta A into Delta A by LP square into E to the power LPT - 1 and we know LPT is negative.

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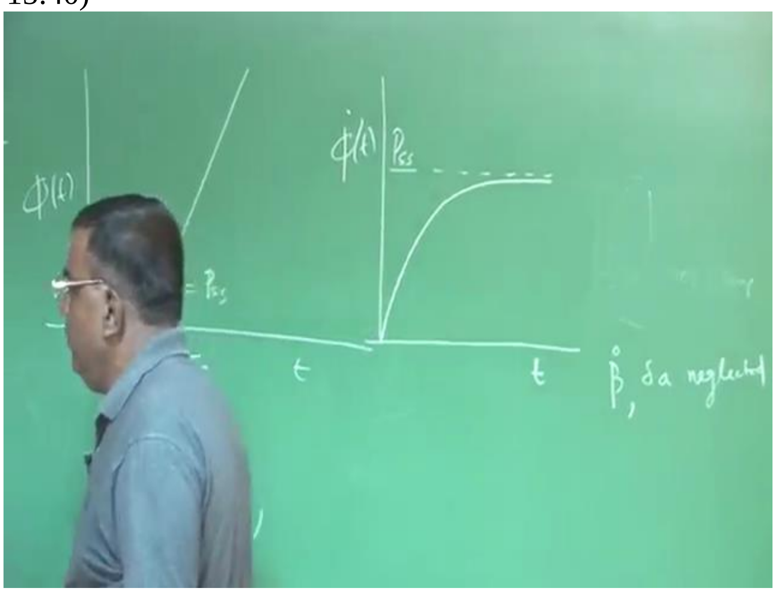

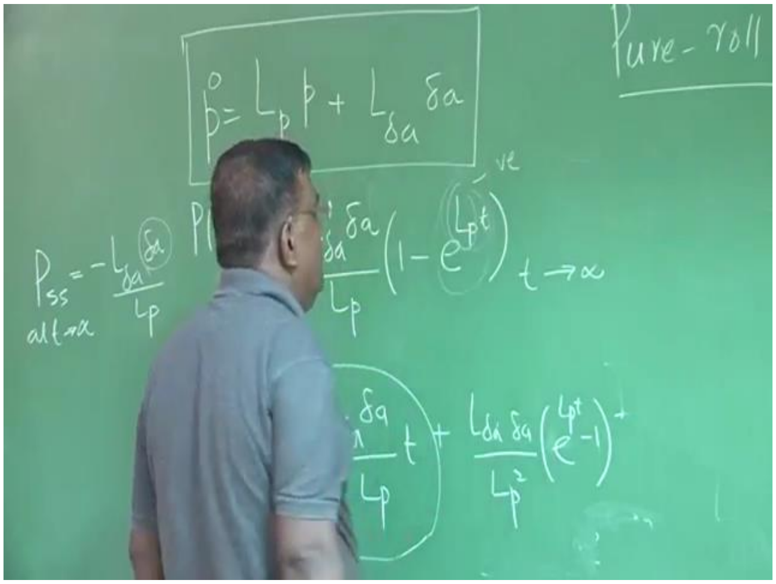


So if I plot it like this, it will reach P steady-state as time goes to infinity. This is here. As time goes to infinity, this gentleman goes to 0 and so you have P steady-state as L Delta A into Delta A by LP. This is which is given here. That is why I am repeating. But the question what we are looking for is not on understanding the variations.

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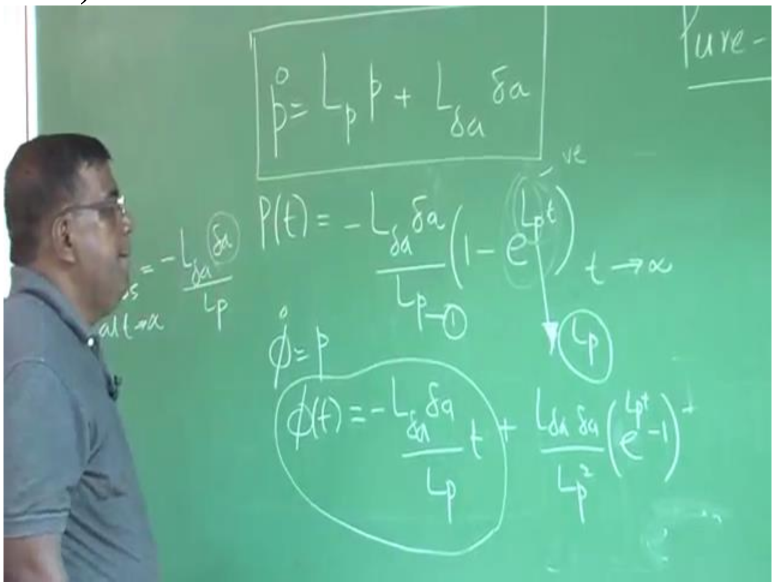

The question is, if I try to alter the rate at which P of T is increasing or going (())(16:25) to a steady-state value, I have LPT, LP here. I have to do something with LP, roll damping which is lying here. Because L Delta is active. It comes through aileron.

And LP which is the roll damping, that decides the stability. So in stability augmentation system also we know if we want to tweak the roll response, I need to play around with LP. That is very important.

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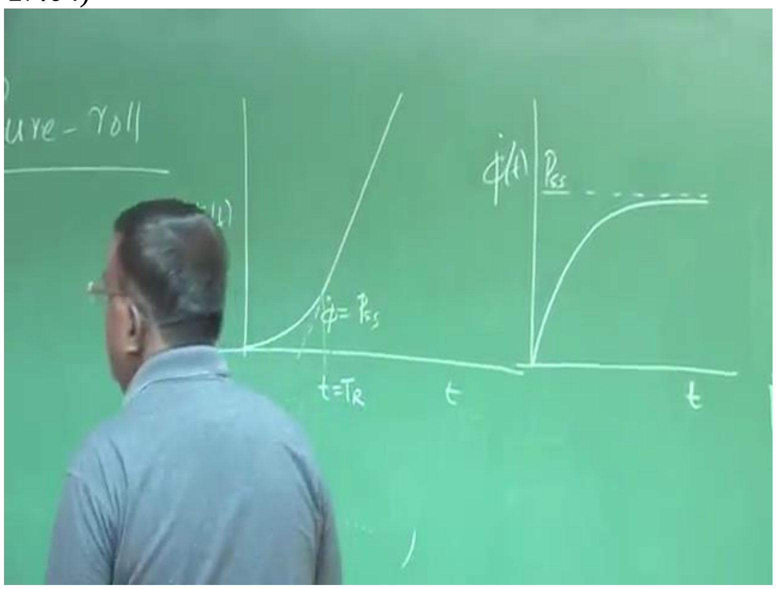


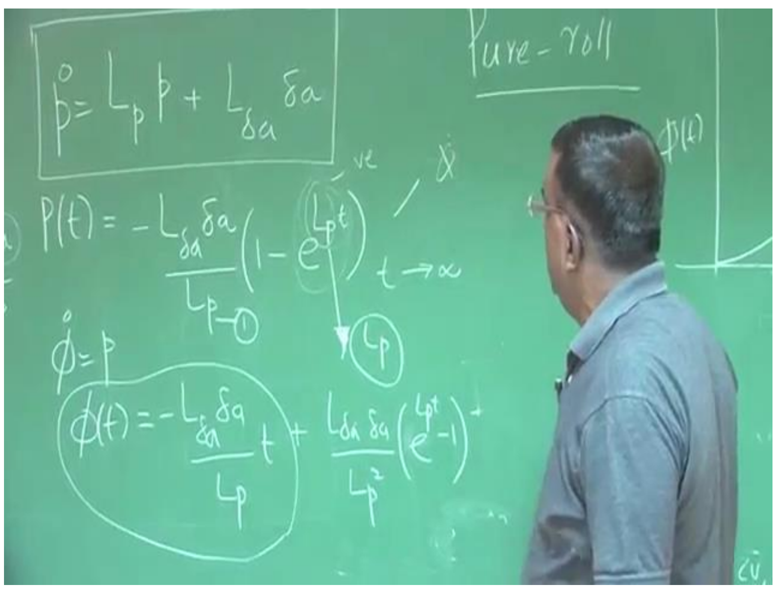


If I again come back here, this is the phi of T variation, this is the phi dot T variation, which are coming from, this is for phi dot T and this is for phi T and you can see that the variation goes like this and we also realise that if I want to do something with the roll response, the LP which is the roll damping derivative I have to play around with that.

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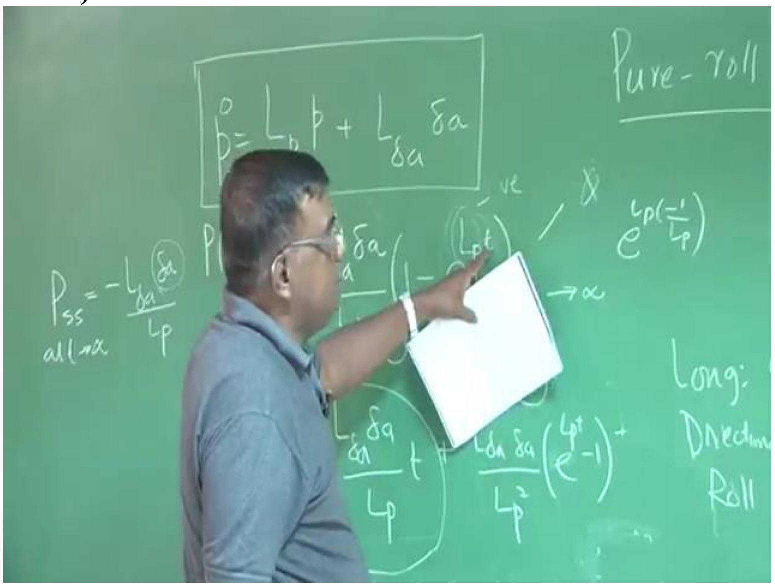


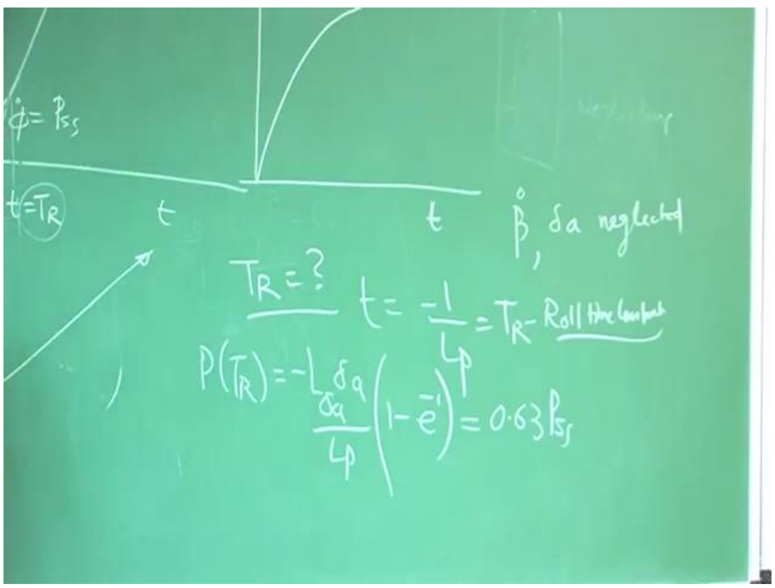


See this one-dimensional analysis for longitudinal, we identified CM alpha is the key parameter, CMQ is the key parameter. For directional, we found CN beta and CNR. And for roll, we found CLP. So we have seen that for longitudinal case, it is CM alpha, CMQ and for directional case, it is CN beta, CNR and for roll, it is CLP that plays an important role.

When I am coming to roll, I also get notice of this term, TR. I am sure you would like to know what is TR? TR, otherwise it will not be complete. So say at T equal to - 1 by LP which I called TR, time constant at roll, if T equal to 1 by LP TR so one by LP TR means E to the power LP into - 1 by LP. Okay. This T is - 1 by LP.

So LP, LP goes, gets cancelled and then you have P at TR equal to, you can check, it is - L Delta A into Delta A by LP into 1 - E to the power - 1 and which is close to 0.63 of T steady-state. So you can understand that roll time constant TR. This is roll time constant. Very important information it gives for designers.

Roll time constant TR is that time at which the value of P goes to 63% of the steady-state value. Okay, most of the designs, you will find this 63% of steady-state value plays an important role in freezing the initial design parameters. So I thought I must complete this.

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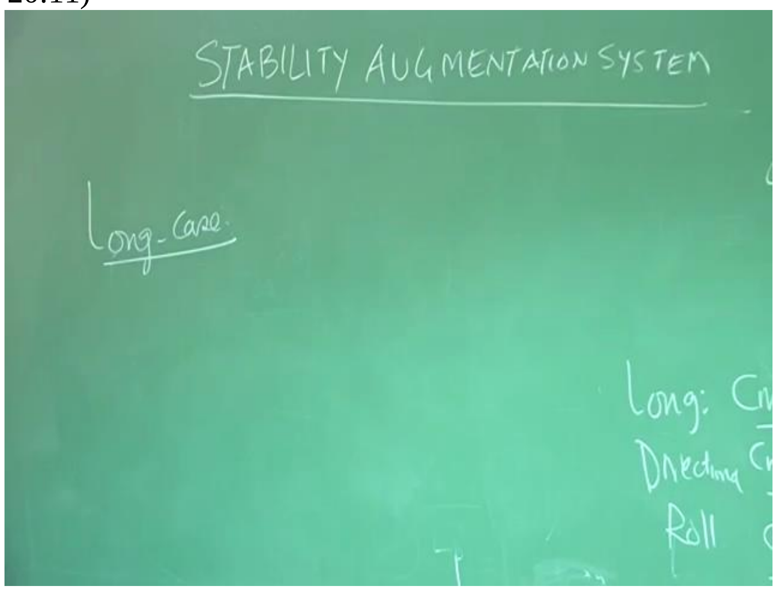


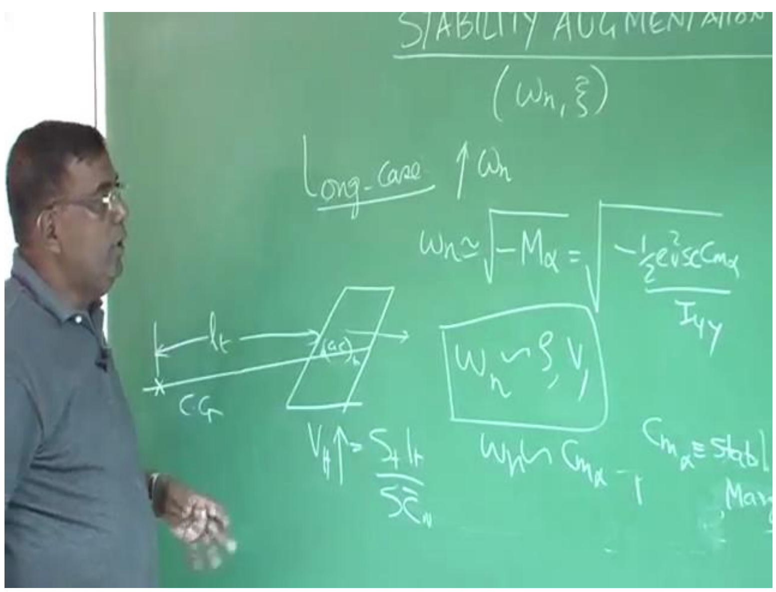

Now I think we are ready for venturing into stability augmentation system. What does that actually mean? Let us say, I take a longitudinal case. We will be mostly focusing on Omega M and Zeta through handling qualities of airplane which I will talk in the next lecture. But just to warm up you, for longitudinal case, if there is a requirement of increasing Omega N, that means you know Omega N we know from approximate analysis that it is proportional to - M alpha which is nothing but - half Rho V square SCM alpha by IYY and C will come here.

If we want to increase Omega N, natural frequency, longitudinal pitching motion, short period, you could see, how can I increase this? Before i talk about increasing this, I see that Omega N will vary with Rho, with V. This is very important that as I go higher and higher, the Omega N will go on reducing. Similarly, as I go on increasing the speed, Omega N will go on increasing. This is very very important to observe.

Also see I can tweak Omega N by changing CM alpha and problem is, what is CM alpha? CM alpha has direct link with stability margin. That you know. So we cannot go on changing the stability margin in-flight. But if at all at the beginning we have to change Omega N and if we decided to change through CM alpha, that means if this is your horizontal tail and suppose somewhere here there is the CG so you have to increase tail volume ratio.

Either you increase the size or droid backwards because VH is ST LT by SC bar wing. So if we want to increase CMR alpha, we have to increase tell volume ratio. That means, increased tail area or this length from AC of the tail to CG of the airplane. So the initial stage once you have done it, now you do not have option to change it. That is not a very good design.

However, as we see Omega N changes with a density, it changes with V. So you may not be very happy with the initial design whatever it is. You may online change the value of Omega N depending upon what is the density of air, at what altitude you are flying, at what speed you are flying, so we want something which I can do it online, you are flying and if required, we could automatically change it.

To the requirement, we satisfy the pilot’s flying quality requirements. So that is where we need to have a stability augmentation system and this is true for directional as well as roll. From next class, our next lecture, I will be focusing on stability augmentation system. I hope, up to this point, after fighting through huge huge expressions, finally you see, I have tried to narrow down to a very simplistic expression from the designer's perspective.

Once you freeze the parameter with this simplistic approach, you can go back and go up to the exactly question and find out the final characteristics of the airplane. That was the aim. Thank you very much.