**Aircraft Dynamic Stability & Design of Stability Augmentation System**
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**Module 6**
**Lecture No 31**
**Introduction to Stability Augmentation**

Good morning friends. Last so many lectures, we were busy in writing big big equations, approximations and then tried to get some meaning out of it.

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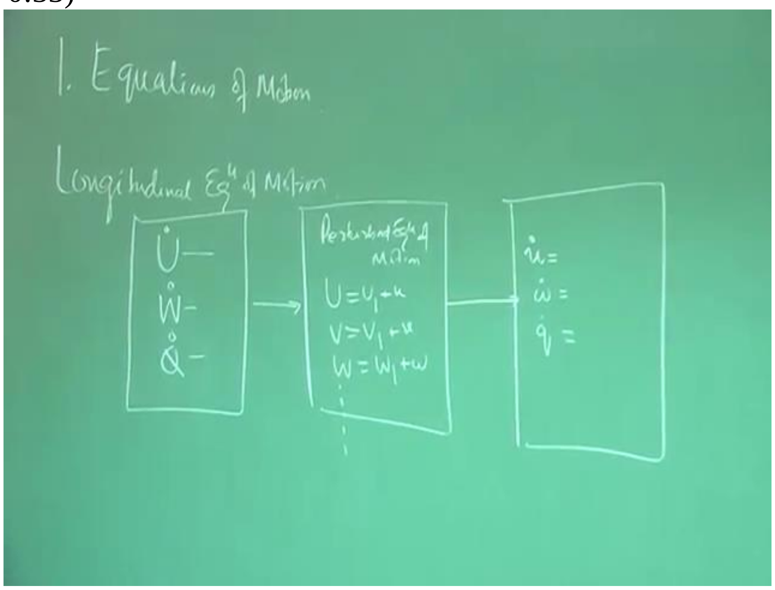


What we did was first we developed equations of motion. Then we segregated longitudinal equations of motion. Primarily, that involved U dot, W dot and Q dot.motion along X direction, motion along Z direction and pitching motion about Y axis. From there, we went into development of perturbed equations of motion and there we did a strategy, small perturbation of motion.

So we changed U to this VS, V1 + small V, W as the W1 + small W and so on with understanding U, V, W small letters, they are all perturbed quantities. From there we developed perturbed equation of motion which was typically in the form, U dot equal to, is W dot equal to and Q dot equal to the forces, aerodynamic forces, propulsive forces and gravity.

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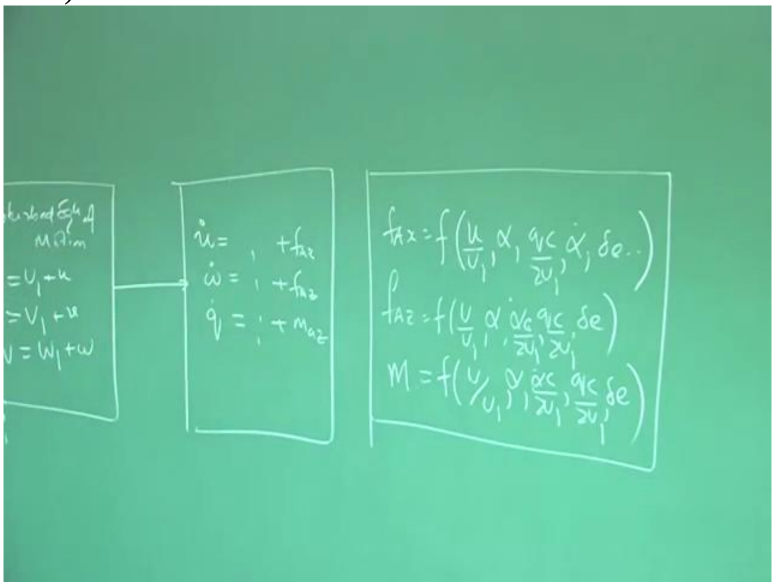

And there was a term like FAX aerodynamic, FAZ aerodynamic, MAZ aerodynamic apart from gravity term, etc, etc. Then what we did? We said okay, let us be clear that FAX is function of motion variable U, U by U1, we non-dimensionalised this, alpha, QC by 2U1. Then alpha dot, then Delta E, etc. And for FAX, we said alpha dot contribution will be negligible.

Similar thing we did for FAZ. So it is function of U by U1, alpha, alpha dot, I will fought out C by 2U1, QC by 2U1, then Delta E. And similarly for M. We said, this is the function of same variable. These are longitudinal case.

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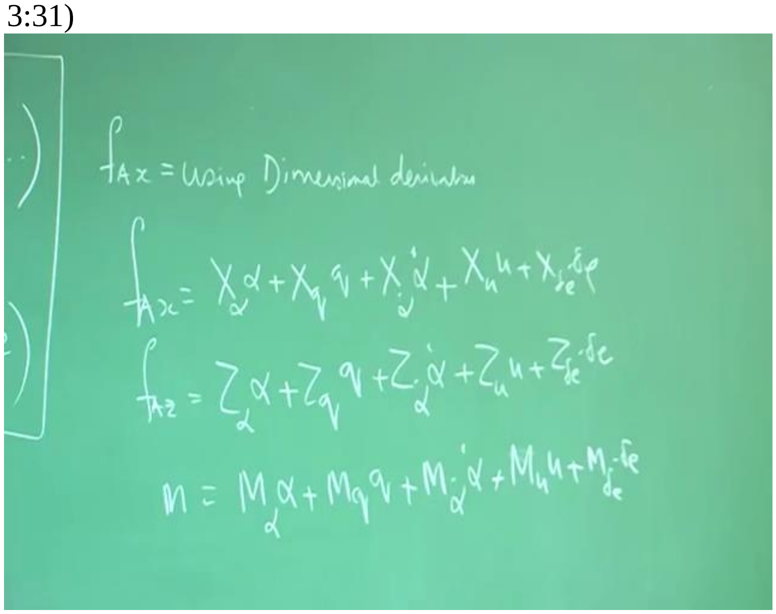

Then we expanded FAX using dementia derivatives and we wrote, FAX is equal to X alpha into alpha, XQ into Q, X alpha dot into alpha dot although we neglected alpha dot contribution. XU into U + X Delta E into Delta E. Similarly for FAZ, we again wrote Z alpha into alpha, ZQ into Q, Z alpha dot into alpha dot + Z Delta E into Delta E.

And in continuation, you know M, it is M alpha into alpha + MQ into Q + M alpha dot into alpha dot + MU into U + M Delta E into Delta E. Then, what was the next step? We substituted this expression in these equations and then found a stability matrix and for solution, to find the roots, we put the determinant of that matrix equal to 0.

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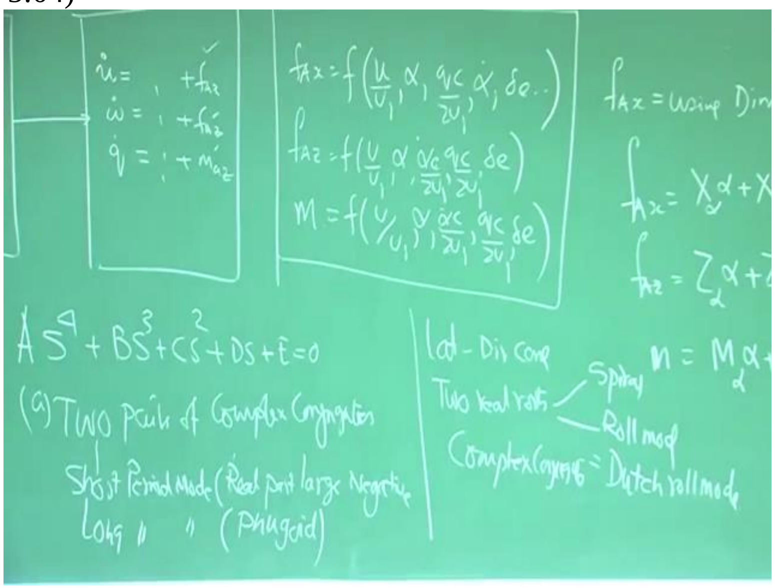


And we got equation of the form AS4 + BS cube + CS square + DS + E equal to 0
where A, B, C, D, E can be evaluated using these dimensional derivatives as well as flight conditions. And for longitudinal case, we have seen, generally, for most of the airplane, you get 2 pairs of complex conjugates and one pair is short period mode. That is where the real part large negative.

And the other one which again is a complex conjugate, long period which is Phugoid mode. All these things we have done. Similar thing we have done for lateral directional case. There also, we got equation of this form. And there, when you do for lateral directional case, we found, there are typically there are 2 real roots. One was spiral mode, one belongs to roll mode and another one which is a complex conjugate and that was Dutch roll mode.

And we have tried to understand how to use approximation to get an idea about the expression of natural frequency and damping ratio. For all these cases, finally we came down to the expression for natural frequency and damping ratio. Why we are doing all these things? Because from those values, we will be able to understand or from the roots of this equation, we will be understanding whether the aircraft is dynamically stable in that mode or not.

Further, the values of Zeta, the damping ratio and Omega capital and will also tell us about the handing qualities of the airplane. That will be dealt in the subsequent lectures. And during that only we will find the requirement of having stability augmentation system. But you could understand one thing, if you are a designer, since your configuration is not completely ready, how do I get these equations?

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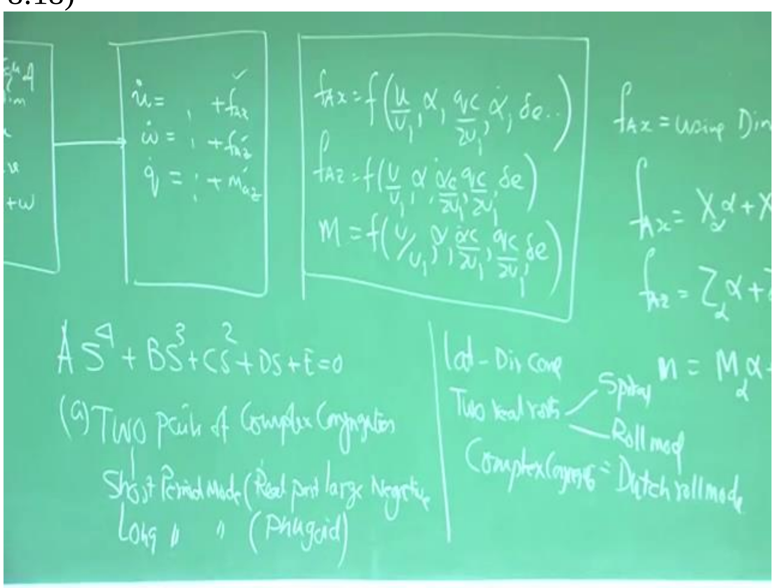


Because A needs complete details of aerodynamic coefficients of the airplane. But the airplane is not fully ready. It is in conceptual stage maybe + 1 conceptual stage. So you need to look for some approximate expression. That is why those approximate expressions were evolved. But more than that, if I am a designer, I will first start with one-dimensional dynamics and try to get feel for some important numbers. And then I will go for the approximate method. And then finally, I will go for the exact method. Okay?

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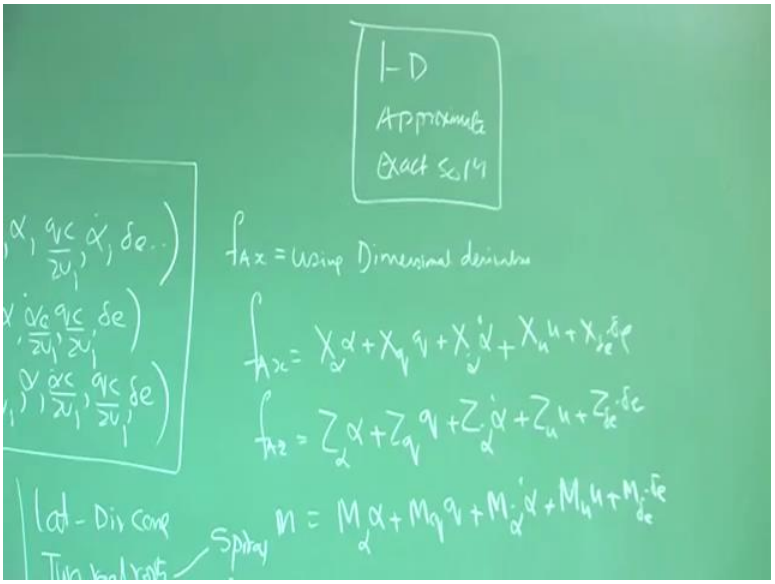


So the approach would be for a designer, he first tries with one-dimensional method. Then approximate, as you know, the approximate methods are not very accurate but it is useful. And then for exact solution. Designer will be always happy following that path. So let us revisit, when I talk about one-dimensional approach for analysing one-dimensional dynamics which is also you are familiar.

And I thought since so much of expressions we have written, by now many of you might have even stopped seeing the lecture because so many questions are coming. And my knowledge to tell you that these are to be done only once. As long as you know the final result, you know how to handle this.

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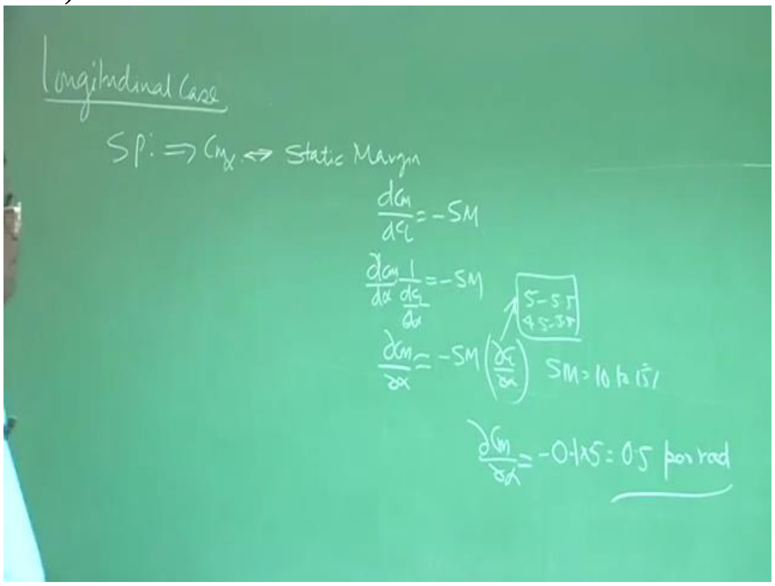


Let us see, if I take a longitudinal case, what happens if a body is disturbed in longitudinal and you know, there is a short period mode and there is a Phugoid mode. Short period mode will be governed by the static stability that is CM alpha. Because alpha CM contoured. CM alpha will decide. And CM alpha, short period mode is decided by CM alpha and CM alpha has directly with static margin because you know starting margin is roughly DCM by DCL equal 2 - static margin and DCM by DCL I can write as DCM by D alpha into 1 by DCL by D alpha.

So, with - static margin, so DCM by D alpha will be approximately equal to - static margin into DCL by D alpha. And DCL by D alpha if you see for an airplane, it will be between 5 to 5.5 or at the most 4.5 to 5.5. This value is saturated around that. So as a designer, I can start picking on the average number and take static margin as 10%, static margin as 10% to 15%.

So your initial value of DCM by D alpha to start with for this combination, static margin is 10% means 0.1 and CL alpha limit is 5. So this is 0.5 per radian. And typical value of CMR aircraft, small small aircraft, lies within that order, - 0.5 to 0.8 per radian. Its order of magnitude 1 per radian. So this is the importance of CM alpha.

And we have seen, the short period dynamics is decided more influenced by CM alpha. Similarly we will find CMQ also, pitch damping also we will decide the transition, the dynamics of longitudinal. We do an approximation.

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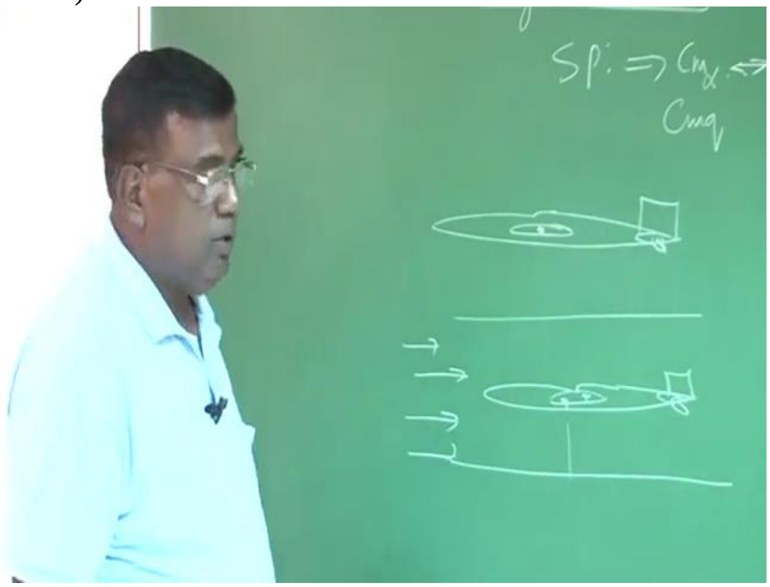

We say, although I know that if this plane is disturbed in longitudinal plane by an elevator deflection, it will not only oscillate like this but also have a plunging motion. Because force is there, so there will be acceleration, deceleration like this. But we neglect this. We say, the time is so short, we will neglect this motion, plunging motion.

We will only say, it is pitching like this which is equivalent to saying which we have already done that if I mount this air plane in a tunnel, and this is V and this is the point where it is hinged and only pitch degree of freedom and you deflect it, the elevator, you give a disturbance. And then it will go on oscillating like this. In such case, this is one-dimensional case because only pitching motion we are considering. So in that case, how do I model it? That already you know. Because I want to demonstrate you the advantage of dimensional derivative.

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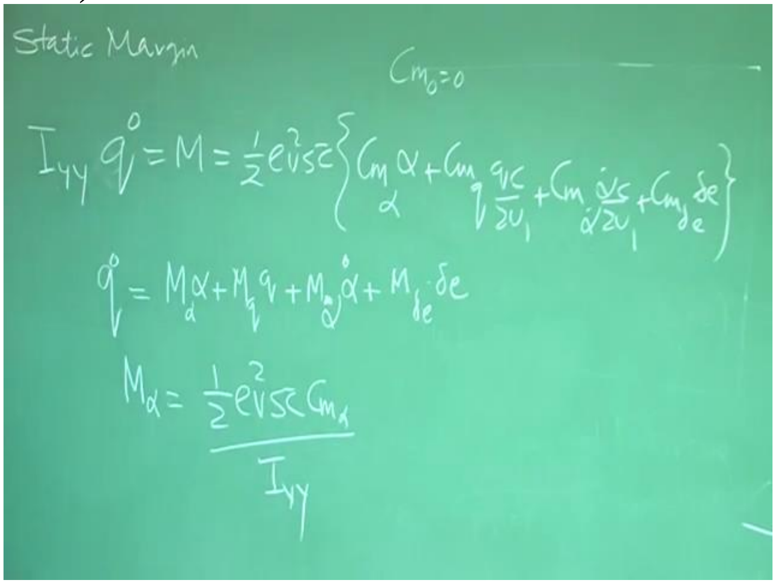


So you know ideally IYY Q dot or small Q dot let us say will be equal to moment which will be equal to half Rho V Square SC bar into CM alpha into alpha CM Q into QC by 2U1 + CM alpha dot into alpha dot C by 2U1 + CM Delta E into Delta E. No problem. I have taken CM not equal to 0 to just check the dynamics part of it which by now you know can be written as Q dot equal to M alpha into alpha MQ into Q + M alpha dot into alpha dot + M Delta E into Delta E.

We have been using this and you can see what is M alpha? M alpha is nothing but half Rho V Square SC bar into CM alpha by IYY. Similarly, you can find out MQ, M alpha dot. All these things we have done when we were dealing with bigger bigger equations. Now see, once I know this then I put enough, because there is only pitching motion, no plunging, so whatever Theta is there, that is alpha.

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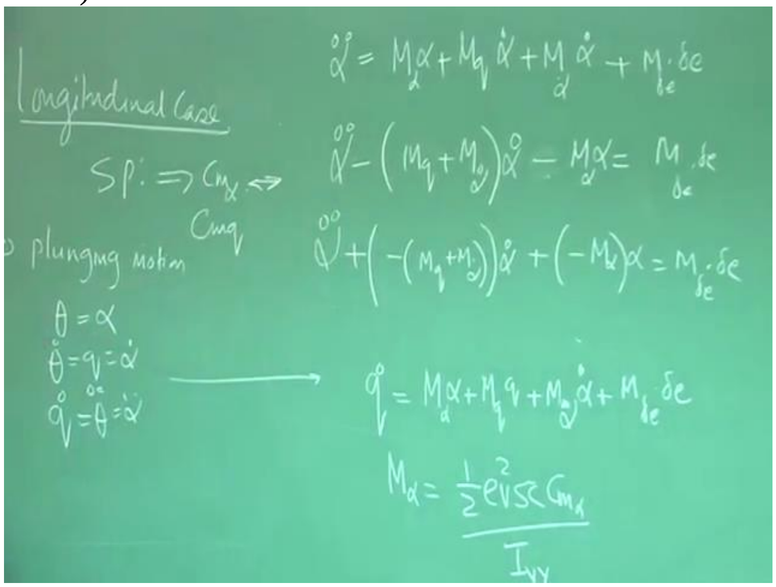


No plunging motion. So whatever Theta, that is alpha. Whatever Theta dot equal to Q is equal to alpha dot. Whatever Q dot equal to Theta double dot equal to alpha double dot. So I can now substitute in this equation. Then I get alpha double dot equal to M alpha into alpha + MQ into alpha dot, because Q is alpha dot + M alpha dot into alpha dot + M Delta E into Delta E.

So I can write this as alpha double dot - capital MQ + M alpha dot into alpha dot - M alpha into alpha is equal to M Delta E into Delta E. Standard form I can write alpha dot + - MQ + M alpha dot into alpha dot + - M alpha into alpha equal to M Delta E into Delta E. No problem. So what does this remind you? This is typically like mass spring damper system.

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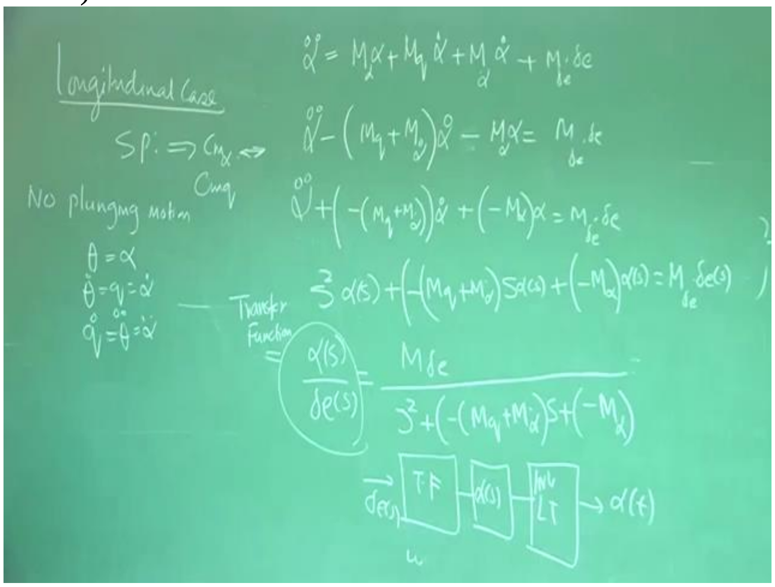


So if I take Laplace here, I get S Square alpha of S + - MQ + M alpha dot S alpha of S + - M alpha into alpha of S equal to M Delta E into Delta E of S. So if I write alpha of S by Delta E of S, that will be M Delta E divided by S Square + - MQ + M alpha dot S + - M alpha. That is all. So what is this alpha of S by Delta E of S?

This is called the transfer function. We have all learned this Transfer function. What is the advantage of transfer function? That if I know these values into force MQ, M alpha dot which I know, which are aerodynamic coefficient and initial parameters, I can easily find out what will be the alpha S for a given Delta of S. Just multiply it. And if I take the inverse Laplace transform, I will get the time domain response.

So the block diagram if you see, this is the transfer function, this is the input which is Delta E of S, it gives alpha of S and then you go for inverse Laplace transform, you get alpha of T. This is one thing. second thing we try to understand here is that what is the damping ratio and natural frequency for this pitching motion only which is called pure pitch motion.

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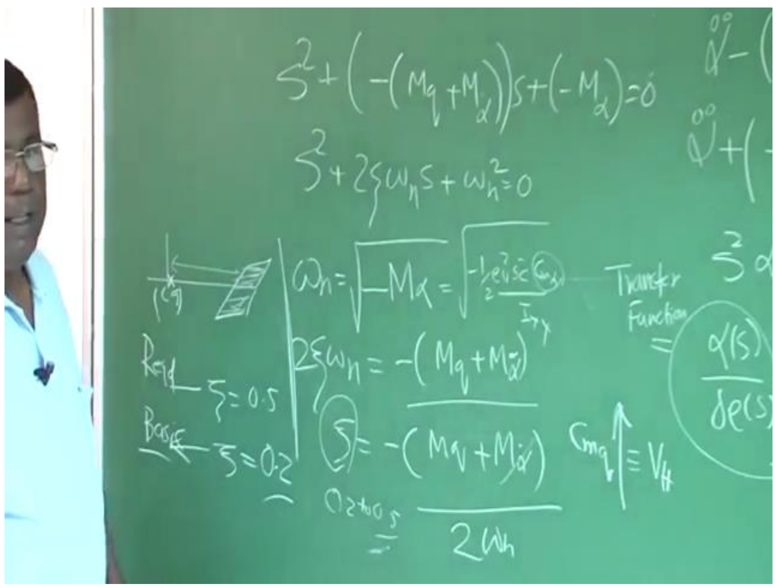


And that you could see, S Square + - MQ + M alpha dot into S + - M alpha equal to 0. This is the characteristic equation and if I compare this with S Square + 2 Zeta Omega NS + Omega N Square equal to 0. So I clearly see, Omega N is under root of - M alpha and 2 Zeta Omega N is equal to - MQ + M alpha dot. So Zeta will be - MQ + M alpha dot divided by 2 Omega N.

Now suppose you are designing an aircraft, you need to have a quick assessment of damping ratio. What you need is approximately, the value of MQ that maybe you have already designed the tail and the CG of the aircraft you have got roughly, you have estimated. So using the lifting coefficient or the lifting parameters of the tail and this distance, roughly you can know the CMQ because of the tail and damping is primarily because of tail and even if you take M alpha dot to be 20% of whatever CM Q value is there then you get an idea of Zeta provided you know Omega N.

But Omega N you know, this is M alpha. This is nothing but - half Rho V Square SC bar CM alpha by IYY. So you know what static margin you are designing. From there you can get an assessment of CM alpha maybe - 0.8. And you know rough value of Omega N, rough value of Zeta. At this stage, you know whether you are close to the design requirement or not.

It is possible that Zeta you want around 0.5. But this is telling the Zeta you are getting around 0.2. This is required and this is basic. Okay? So the question comes, I want to increase the value of Zeta from 0.2 to 0.5. How do I do? Immediately, I do not go to all these big big expressions. No, not at this stage.

I use this pure pitch expression for longitudinal case. I see yes, Zeta we have to increase from 0.2 to 0.5. So how much I have to increase MQ? That is how much I have to increase CM Q? Before that, already you have got rough value of Omega N because you have designed for a particular static margin. The moment you try to increase CMQ you know you have to handle the tail volume ratio.

That is either you have to increase the tail size, this is the horizontal tail, either you have to increase the tail size say for example, CMQ when we are getting 0.2, tail area was like this. Now for 0.5, you may have to either increase the tail size or also you could take it back.

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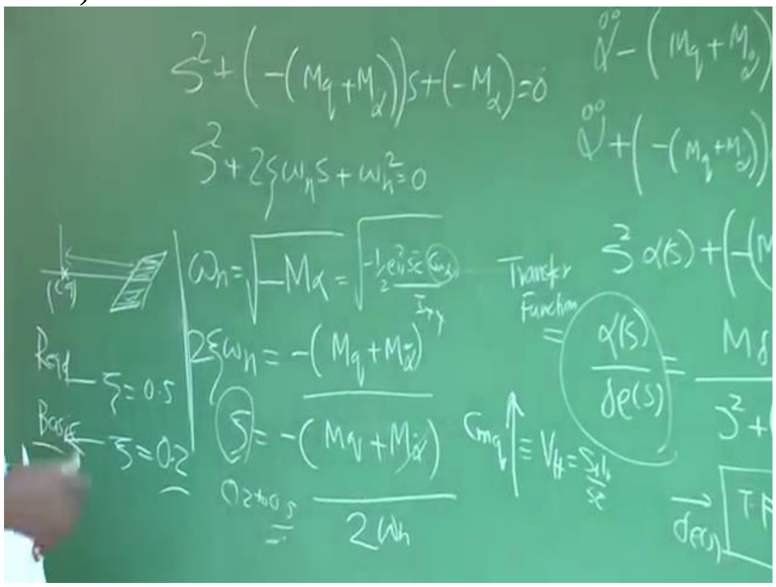

That is ST increase, LT increase and the product, ST LT by SC bar, that you know. That is nothing but tail volume ratio. You have to increase. Then again, you reconfigure the design. Again go for this initial estimates. When you are happy, fine. But if you find situation where this enhancement of Zeta from 0.2 to 0.5 is required occasionally, not all the time or if you want to increase from 0.2 to 0.5, which is fairly large gap.

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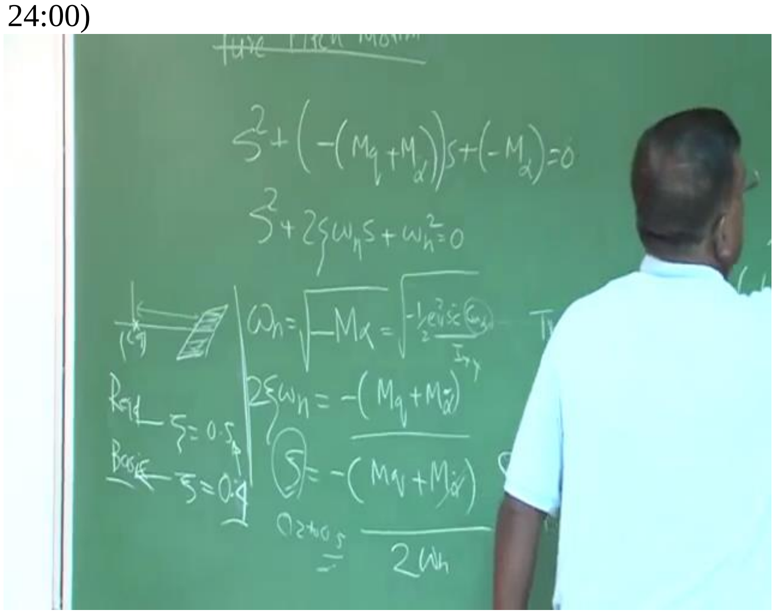

Let us say I was working at 0.4 and I want to go to 0.5. Then if I am trying to change all those tail size, tail length, it is making a lot of problem for my configuration, so then the approach comes, can I do it online? And that is where the question comes, disability augmentation system.

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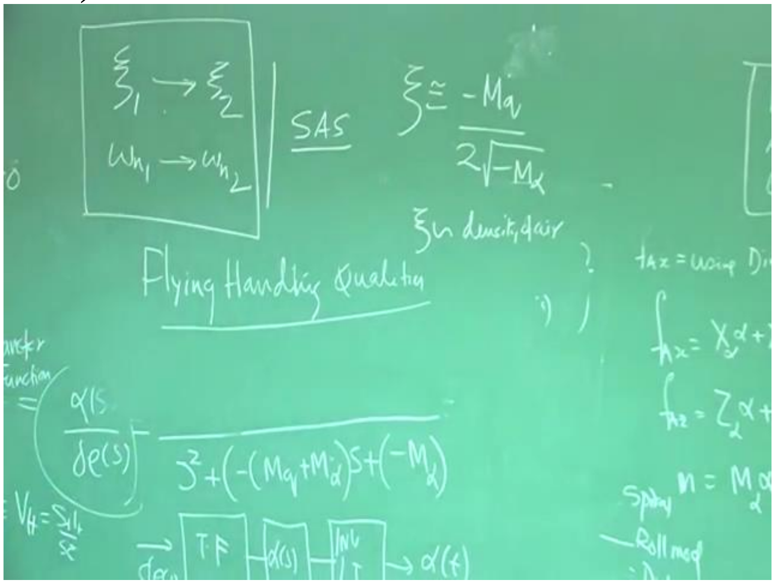


That is I need to increase Zeta from Zeta 1, I will have to increase it to Zeta 2, Omega N 12 Omega N2 let us say for a longitudinal case. Then what is the procedure we follow? And that is what we are calling stability augmentation system. That is, we will be artificially augmenting CM alpha, CM Q, other derivatives so that these Zeta1 goes to Zeta2, Omega N goes to Omega N2.

But do not forget, this requirement is there because to maintain the flying, handling qualities we will appreciate the Zeta value. Let us take Zeta value. As a designer, now I am talking like a designer. I am saying, this is approximately equal to - MQ by 2 under root - M alpha so Omega N is - alpha and M alpha dot, I have absorbed here. Let us say M alpha dot, 20% I have already absorbed here.

Now you could see this Zeta is also function of density of air because MQ is half Rho V Square SV bar will be there. And here under root. So it is function of density of air. So whatever Zeta you have got at sea level, if you go 10 km, Zeta is going to change. And that is where you have a question in mind, I cannot design my tail volume based on only one condition.

What we will do? For majority of the time, wherever the aircraft is operational, we will try to get the Zeta around that. And rest, the variations, we get it through SAS. So they are required to operate for different different altitudes, different different conditions. And that is the beauty, that is why we saw online. Whatever altitude is there, automatically the stability will augment it.

That is the need for a SAS.

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