Aircraft Dynamic Stability & Design of Stability Augmentation System
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Module 1
Lecture No 03
Spring-Mass-Damper System: Over and Critically Damped

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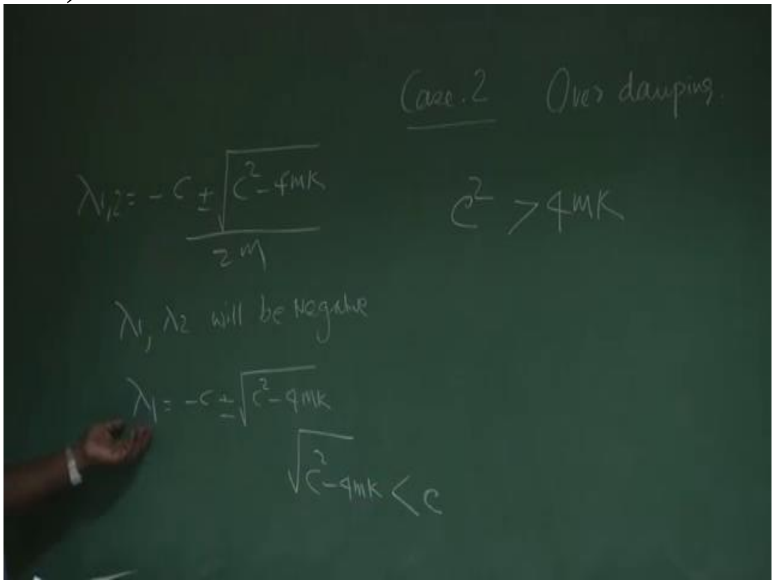

In continuation to our lecture, now we will talk about case 2 which is 'Over damping'. Remember that is the case when C square is greater than 4 MK. And if you see that roots lambda 1 and 2 is - C + - under root C square - 4 MK by 2M. So if C square is greater than for MK, then naturally this is positive. No complex part. Now you could see that both lambda 1 and lambda 2 will be negative. Is it clear or not?

First time we see lambda 1 - C + - under root C square - 4 MK. This C square - 4 MK. MK is positive. So this will be always less than C square. Or this root, C square - 4 MK is always less than C. Obviously because something has been extracted out from C square. If this were 0, they would have been equal. So naturally, lambda 1 will be negative and lambda 2 also will be negative. This is an interesting point. Both lambda 1 and lambda 2 will be negative. If C square is greater than 4 MK, no complex part. It is a very win-win situation. There will be damping fast, it will be over damp.

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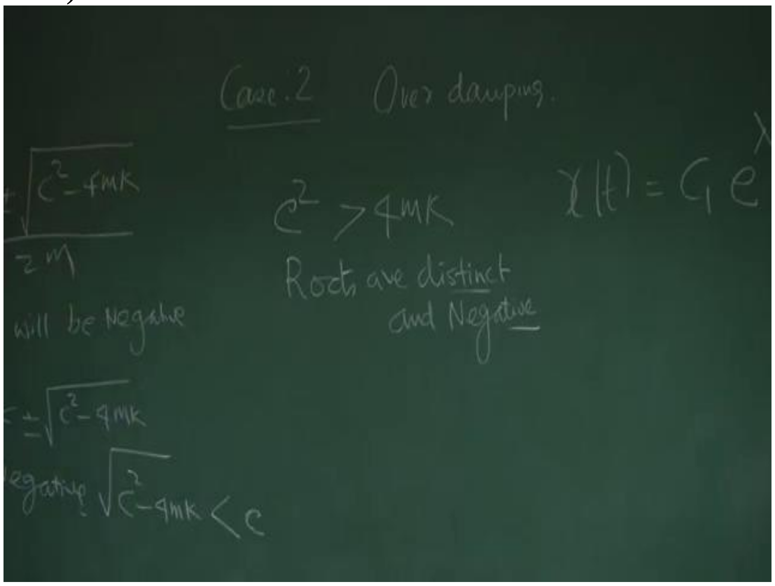

So if that is true, then X of T, we can write as C1 E to the power lambda 1 T + C2 E to the power lambda 2 T where lambda 1 and lambda 2 are negative and they are distinct. The roots are distinct and negative. This is important. What do you see here? What is X of T? X of T was a perturbed quantity.

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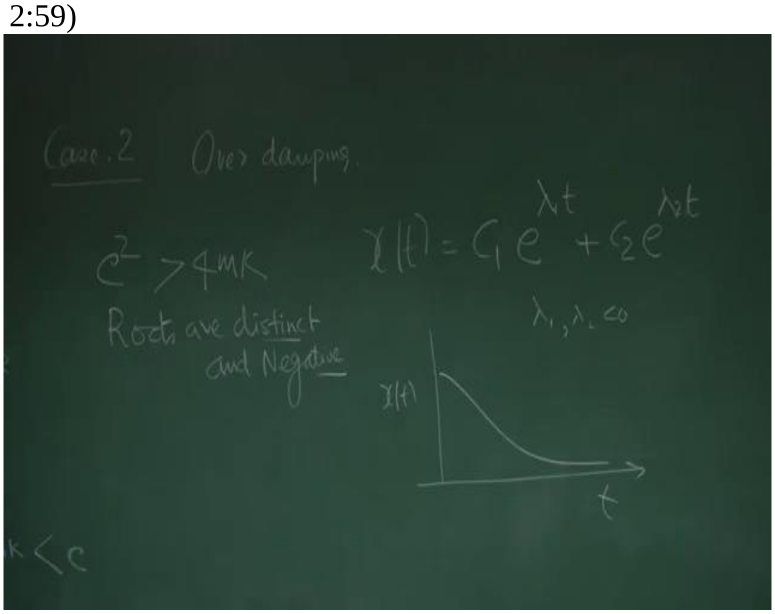

So as time progresses, since lambda 1 and lambda 2 are negative, so naturally there will be decay but when there will be oscillation? No, there will not be any oscillation. So the variations will be typically something like this. If it is X of T and time, it may vary something like this. It will come to equilibrium. Something like this. See here, there will not be any oscillation. Lambda 1 is negative, lambda 2 is negative. So remember this is again a mass spring damper system. But no oscillation.

What is the trick? The message is, if you can select damping in such a way, higher damping in such a way that it is greater than 4 into mass into K, stiffness constant, then there will not be any oscillation. So the designer has the liberty. If you want an over damped system, if you select a high value of C in such a way that the disturbed quantity goes like this, perturbed quantity goes like this. That is very important.

So this also demystifies that mass spring damper system does not necessarily mean oscillator system. Correct? And the designer has so much of liberty. Depending on what sort of response he wants, he can select the value of C, M and K. Now, under damping is over, over damping is just over. Now what comes to your mind? It is obvious, you will be more interested. The boundary between under damping and over damping. And that is our case 3.

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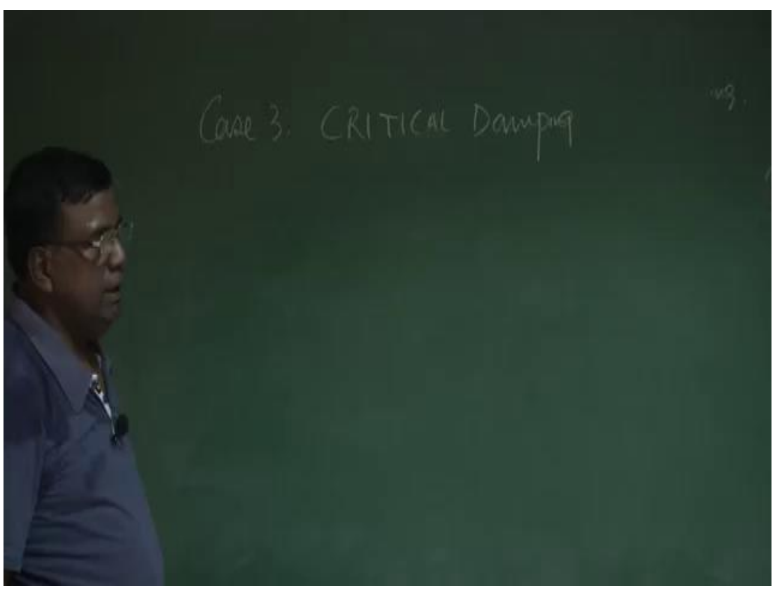

It is called as critical damping. Before I come to this, let me take you back to the physical situation. What did we realise?

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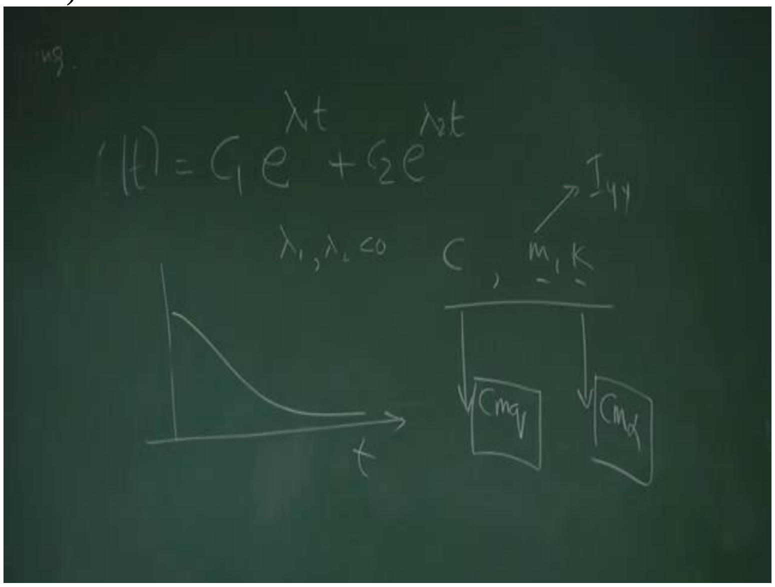

We realise that, to select the value of C, M and K, to ensure or to get whatever you desire as oscillatory response or over damped response you have to select C, select the mass and select the value of stiffness. If I want to translate this to aircraft, C will be translated to CMQ, its damping derivative K will be CM alpha. What I am discussing is, we know that if I want to get a particular type of response, let us say we want and over damped response, then C square should be very higher as compared to 4 MK or C should be very high compared to MK and this product.

So if I translate this to aircraft, C means my pitch damping derivative CMQ and K means CM alpha and M could be IY bar in longitudinal motion. So that is how we are building a relationship between, the understanding through mass spring damper system through aircraft in longitudinal motion. That is why we are spending so much time on this.

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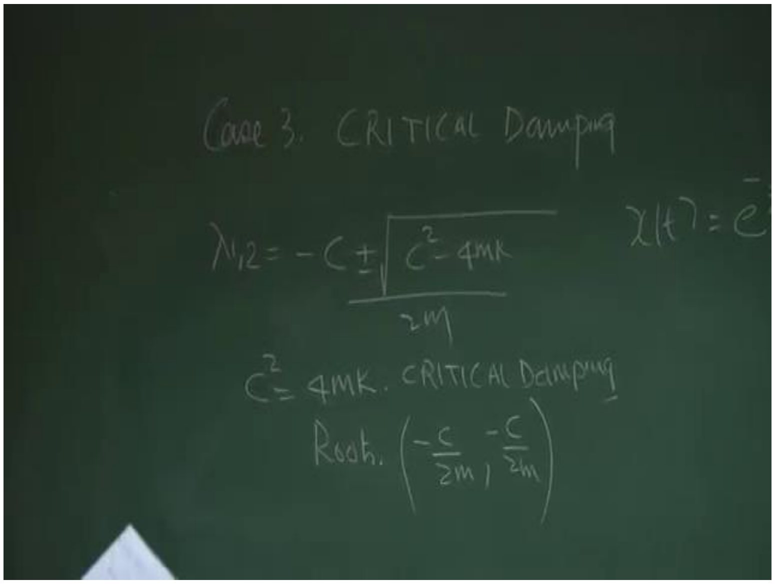

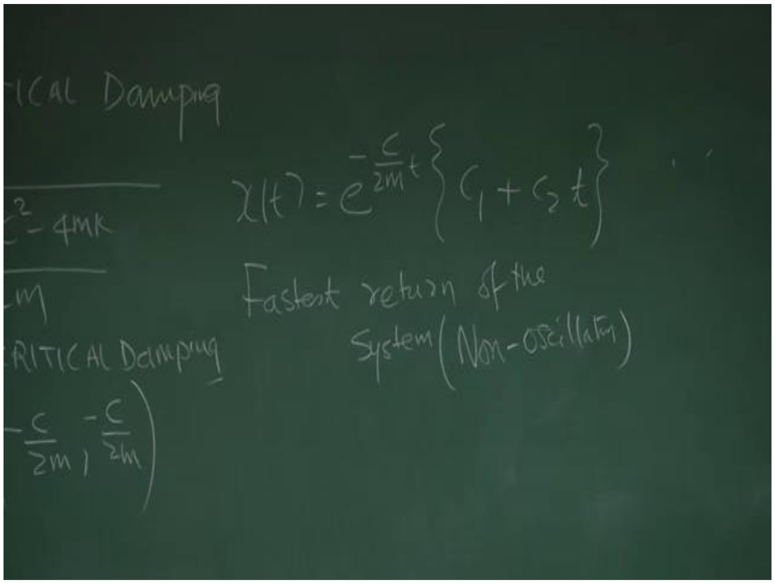

Now let us come to the critical damping case, case 3 where lambda 1 lambda 2 is - C + - under root C square - 4 MK by 2M. For critical damping, you know that C square is equal to 4 MK. This is critical damping. And you will understand, if C square is less than 4 MK, it is under damping. If C square is greater than 4 MK, it is over damping and C square equal to 4 MK is the boundary. It separates under damping and over damping. And this is also termed as critical damping. And you could see here, it has roots which are repetitive in nature and they are - C by 2M, - C by 2M.

And you know solution for this root, for this differential equation because it has repetitive root. You will write X of T as E to the power - C by 2M T into C1 + C2 into T. Carefully observe. One can think of, as time is increasing, this may diverge. But see here, this is exponential. So this decay will be faster as compared to the increase because of time. So this also will take a very fast. In fact, you will see that this response is the fastest response above all the 3. It takes shorter time to come back to equilibrium. We will see that.

And it is also said that it is the fastest return of the system which is non-oscillatory. So what is the message? Message is, if damping C is less than C corresponding to this critical damping, then it will be under damped case. If C is more than this critical damped, it will be an over damped case. And third thing, the fastest return of the system in non-oscillatory mode if the damping C value is equal to C critical. So this demands that we need to know a little bit more on C critical and we will try to understand what this C critical means. We will also try to understand what is natural frequency system.

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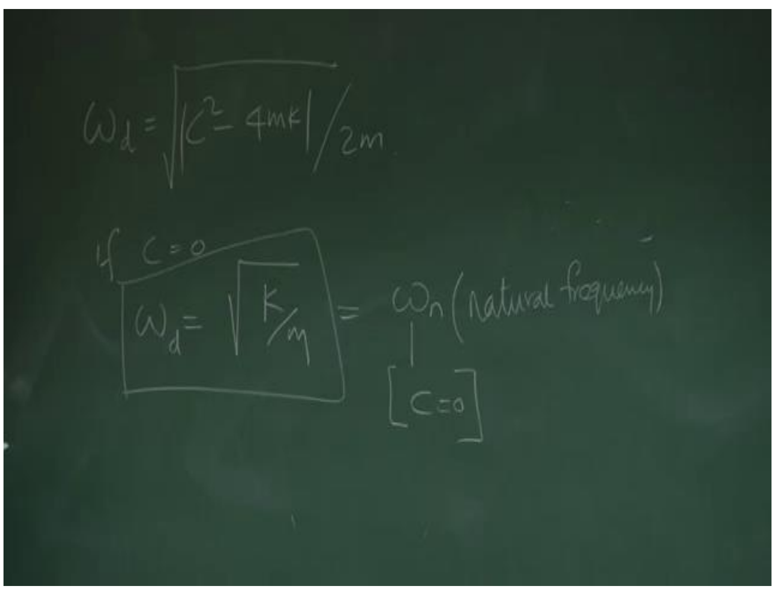

Remember, we defined omega D, damped frequency as under root C square - 4 MK by 2M. And if C is equal to 0, omega D is nothing but under root K by M. If I put C equal to 0, I can show that omega D is equal to this and this is nothing but natural frequency. When I talk about natural frequency, I am talking about the case when there is damping. Now, if I try to give a statement on natural frequency, it is here more natural frequency.

I will define, natural frequency is the frequency with which the system will oscillate if damping is 0. So whenever I think of natural frequency, keep in the back of your mind that C is equal to 0 No damping. Generally, there is a lot of confusion and people will go on talking about damped frequency but actually it is natural frequency. Sometimes natural frequency, you talk about damped frequency. So when you are talking about natural frequency, there is no damped frequency. Okay? So this should be clear to you. And damping ratio is another term.

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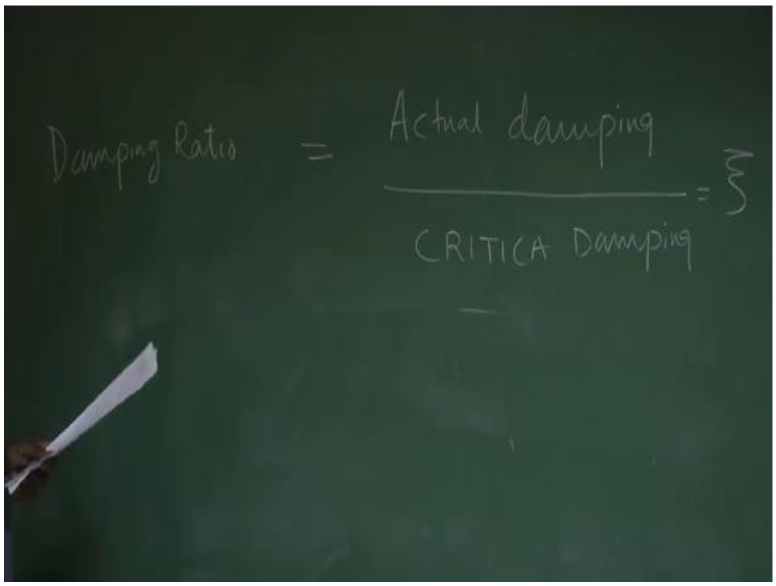

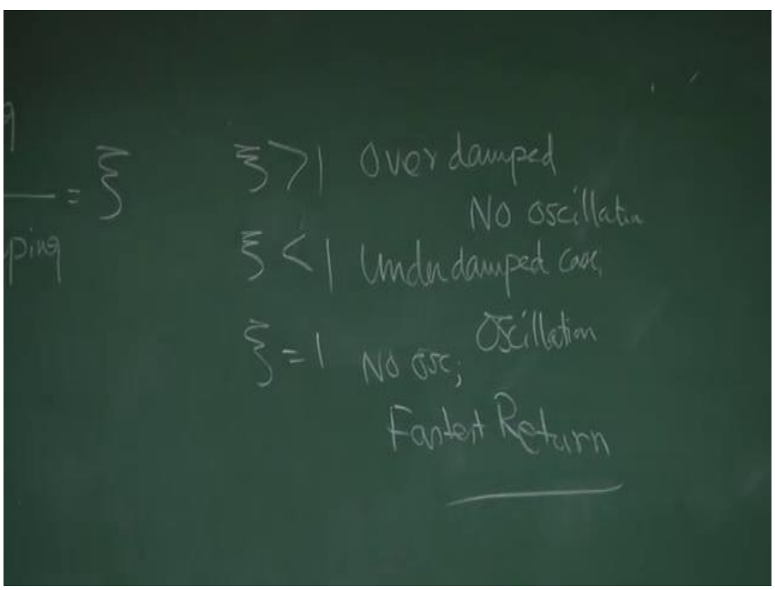

We see, damping ratio. This is also a very important thing to understand. It is actually the ratio of actual damping by critical damping denoted by zeta. Can you see here, in this ratio Zeta is more than one, that means what? That means one the system will behave like an over damped case. Right? This will be over damped and no oscillation. If Zeta is less than 1, that is actual damping is less than critical damping, it is under damped case. So there will be oscillation.

And of course, it goes without saying, if Zeta is equal to 1, again there will be no oscillation and it will have the fastest return. No oscillation and fastest return to equilibrium. So this is important. This ratio, damping ratio has drawn lot of importance and that is why there is a need to understand this ratio, the meaning of this magnitude.

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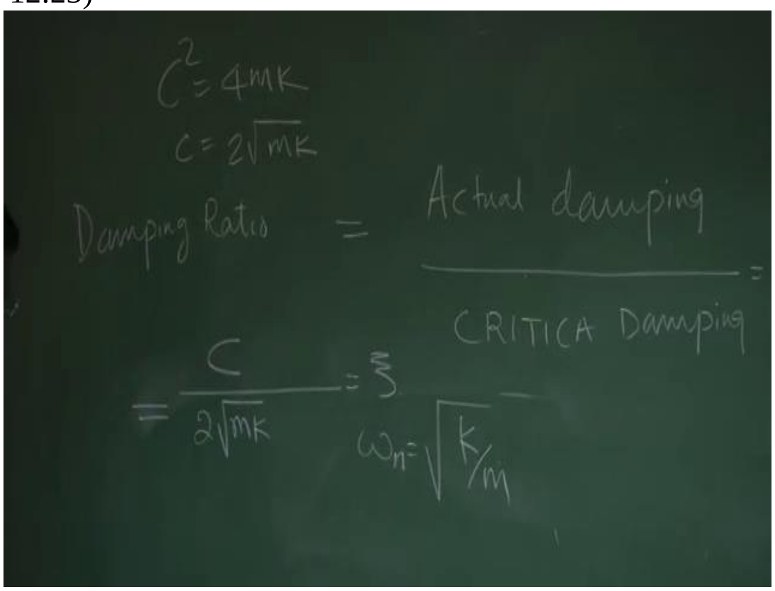

So let us see, if I further go into this, I put actual damping is C and critical damping, you know it is 2 root of MK. Wherefrom it has come? Because C square is equal to 4 MK for critical damping. So if I take C is equal to 2 root MK, the square root and we have assumed that M, K, C, all are positive. So that is the ratio, C actual and the critical damping case. So this is nothing but Zeta. Now let us see. We also know omega N is under root K by M. We will do some trick. You see, this will be useful.

We may use another one lecture of 20 or 25 minutes talking about (())(13:14) system at the most, then we will dash into aircraft. But it is required that you understand this thing. You might have done this thing long back. But try to understand, get the physics out of it. Okay?

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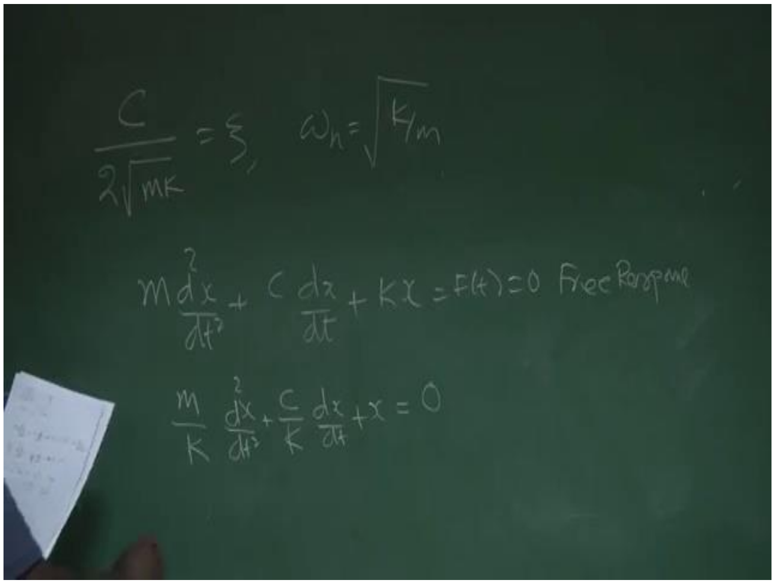

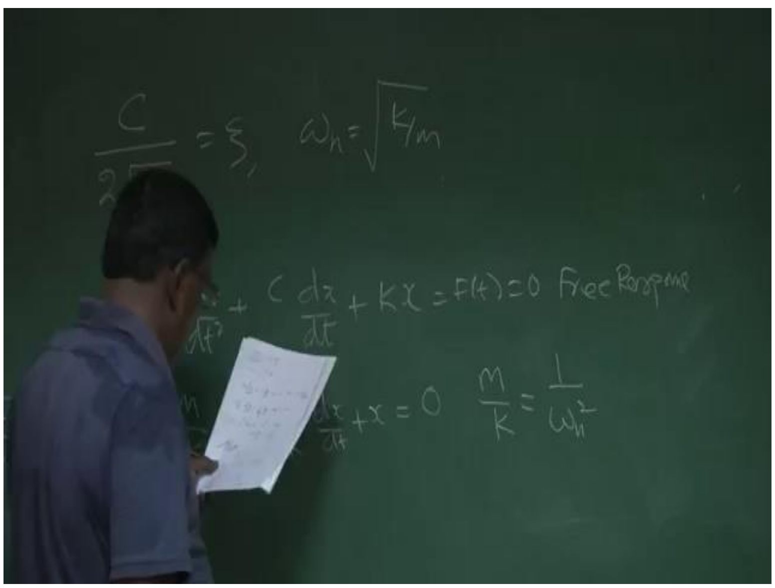

So, C by 2 MK is Zeta and omega N is equal to under root K by M. So if I now see, MD square X by DT square + C DX by DT which is our equation of motion for a mass spring system equal to F of T equal to 0 or free response. So please understand, I have used this term, equation of motion. When I write this, MD square X by DT square + CDX by DT + K of X, it has come from the force balance, Newton's first law. So this will be referred as equation of motion.

If I do that, I can write this as M by K D square X by DT square + C by KDX by DT + X is equal to 0 for free response. Then we also know, M by K is nothing but 1 by omega N square.

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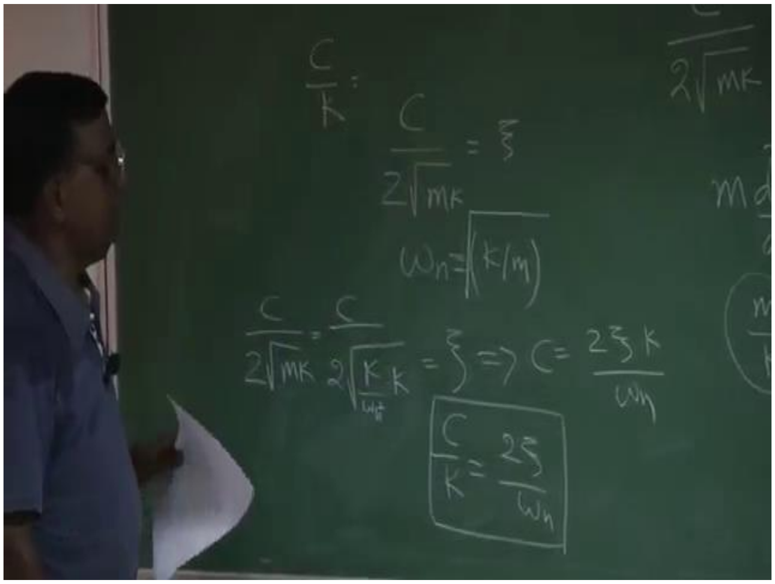

Then question comes, what is C by K? We want to know what is the value of C by K and we will use it, C by 2 root MK is equal to Zeta and we know omega N equal to K by M. Right? So we can easily see that C by 2 root MK is equal to C by 2 root, for M I write K by omega N square into K because you know, omega N is equal to K by M so for M it is, that expression I am using here. Omega N is equal to under root K by M. Never forget that.

This implies C is equal to okay, this is nothing but Zeta by definition. So if I manipulate this, I get C is equal to 2 Zeta K by omega N. Okay? And what is our aim? Our aim is C by K. C by K will be 2 Zeta Omega N. So what we have got? M by K as 1 by omega N square and C by K as 2 Zeta by omega N. Now I substitute it here to get the final equation. What will happen?

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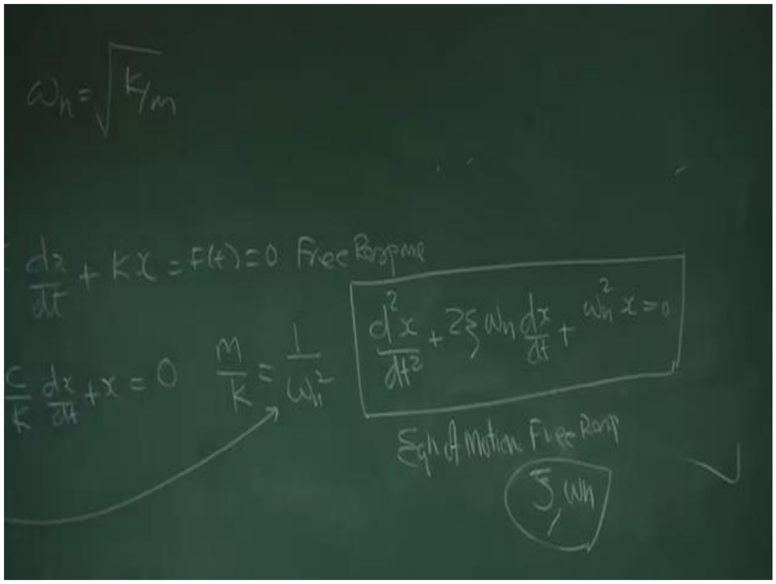

So if I substitute it here, this equation will become D square X by D T square + 2 Zeta Omega N DX by DT + omega N square X equal to 0. This is an equation of motion for free response using Zeta and omega N. So this is equation of motion for what? For free response using Zeta and omega N. And in most of our analysis, we will be using this form.

So what we are doing so far? Please understand. We realised one thing that if I want to understand the dynamics of airplane, we need to understand dynamics of mass spring damper system. Why? Because there is a stiffness parameter or attribute of airplane which comes through CM alpha, static stability. There is a damping type attribute which comes through pitch damping, CMQ. It also comes in the other plain. Like lateral and directional also it comes.

This is one. Second thing we understood is that for a mass spring damper system, depending upon the roost, depending upon the selection of system variable, you can control the, you can guide the response or transient. This is important because after all, finally we will be studying all this dynamic stability to analyse the handling quality of the airplane. So we know that whether I want to design an over damped system, I will design a critical damped system or under damped system. Depending upon that, I can choose those variables, the system variables.

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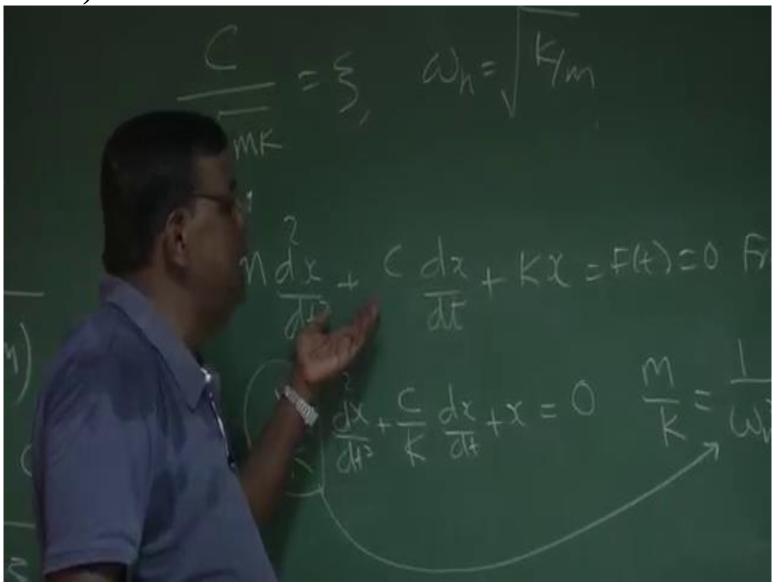

Now to know more about it, in terms of mathematical modeling, we thought of converting this equation of motion into a form where you use damping ratio and natural frequency. These 2 are very important parameters when you try to evaluate handling quality. We define damping ratio as the ratio of actual damping to critical damping and natural frequency as the frequency of oscillation when damping is 0.

The most important part is why from this sort of a form to this? Because we will see that Zeta Omega N and Zeta into Omega N, they will be exhaustively used to define the handling requirements, the handling quality requirements. So whatever you understood here, although through mass spring damper system, we will have one more lecture on this, that is all. Within 20 or 25 minutes, you will find, once you master this, the understanding of dynamic stability becomes simpler.

The ones who go for a shortcut, they do not understand this, they try to understand the question of aircraft, they always complain, oh, it is highly mathematical, very complex It is neither highly mathematical nor complex. It is physics-based as long as you understand mass spring damper system. I am sure we will not commit that mistake of not understanding this. Any problem? Put up a question in the forum. Thank you very much.