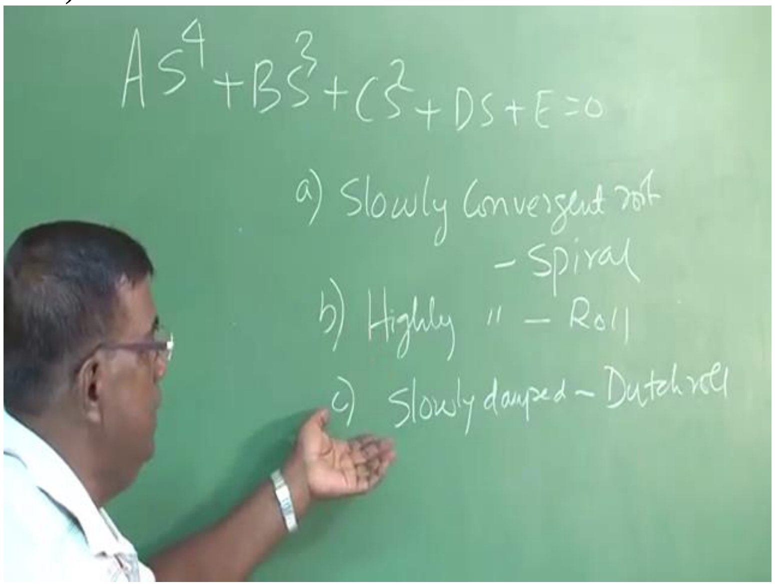
Aircraft Dynamic Stability & Design of Stability Augmentation System
Professor A.K. Ghosh
Department of Aerospace Engineering
Indian Institute of Technology Kanpur
Module 5
Lecture No 29
Spiral and Dutch Roll Modes Approximation

Good morning friends. We will be continuing the lateral directional case.

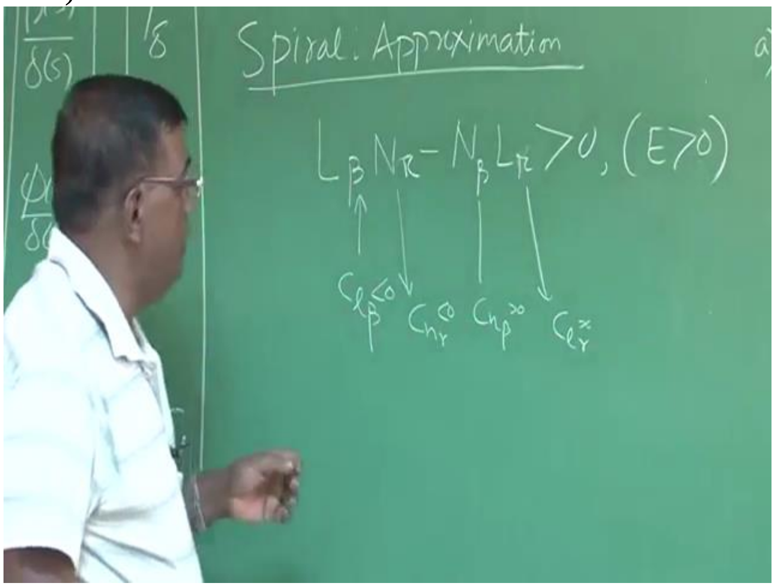
(Refer Slide Time: 0:27)



We are trying to get interpretation where are the roots of this characteristic equation? And you know all this A, B, C, D, E, they can be easily completed once I know lateral directional divisional derivatives. And we have seen generally, for most of the aircrafts, you get roots like this, one is slowly convergent and real root, highly convergent and real root with negative sign and slowly damped Dutch roll, it is like oscillatory roots.

We will now try to use this equation, this matrix equation and take the determinant 0 to find the characteristic equation, AS4 + BS cube + CS square + DS + E equal to 0. We will now simplify this for spiral, roll, as well as Dutch roll mode. And these are approximations. Mostly, they are not very good approximations. But still it has got some relevance to the designers in an approximate manner. It helps a lot.

(Refer Slide Time: 1:34)



First we will try to, first we will do spiral approximation. Before we go for spiral approximation, please remember, we found out the condition, capital L beta NR - N beta LR greater than 0. Remember? This was from the condition, E greater than 0. We said, if E is greater than 0, then it will not suffer from any spiral mode divergence. first of all, let us understand what is this spiral mode?

This is the airplane. If there is a banked disturbance, the airplane goes to the side slip. As it side slips, if it is a stable airplane directionally it turns like this. As it turns, there is a R, yaw rate that will also give moment, a rolling moment. Because as it does like this, you could see from the vertical tail, it is pushing the air towards my left-hand side. So it gets a force on the right-hand side.

So that also gives a rolling moment which further banks the airplane. And then further there is a beta, further there will be turning. So this sort of a combination will go on unless and until you have got CL beta, that is as it banks and side slips and CL beta tries to recover it. So you see that L beta where it is actually non-dimension is CL beta. And CL is less than 0.

What is CL beta? CL beta is again bank, side slip and it tries to come like this. So CL beta negative. NR, you understand what is NR. In NR you have got CNR. It is the yaw damping. It is negative. Then N beta, this CN beta for a Dresher stable airplane, CL beta has to be greater than

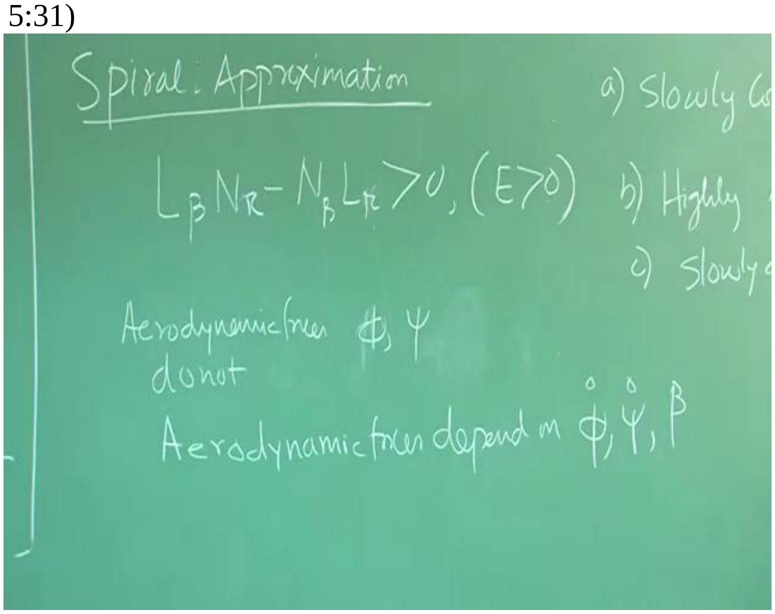
0. And this is CLR. And CLR you understand, if the airplane is having yaw rate like this, let us say when I try to explain you, if this is the vertical tail, this is the airplane.

It is having a positive yaw rate like this. As it goes positive like this, this is pushing the air in this direction. So it will experience a force in this direction. And this force into the distance from the fuselage centreline, that will give me a rolling moment. So vertical tail contribution to CLR is also positive. So now, it is not sufficient to know the signs. But we need to design the airplane in such a way that this L beta into NR - N beta into LR, this should be greater than 0.

That is important. So we understand who are the non-dimensional derivatives. They played the role. We also understand how do I change the value of CL beta. And if I increase vertical tail height, CL beta will increase. If I make it a hi wing, CL beta will increase in negative sense. Like that, as a designer I should know if I want to tweak this parameter, which are the design parameters I should change? Which are the geometric parameters I should change?

So this should be very clear before we go for spiral approximation. And I again caution you that these approximations are generally not very good approximations. Still, they are good enough for a designer to get some feel for basic design parameters. So now we will be talking about spiral approximation.

(Refer Slide Time: 5:31)

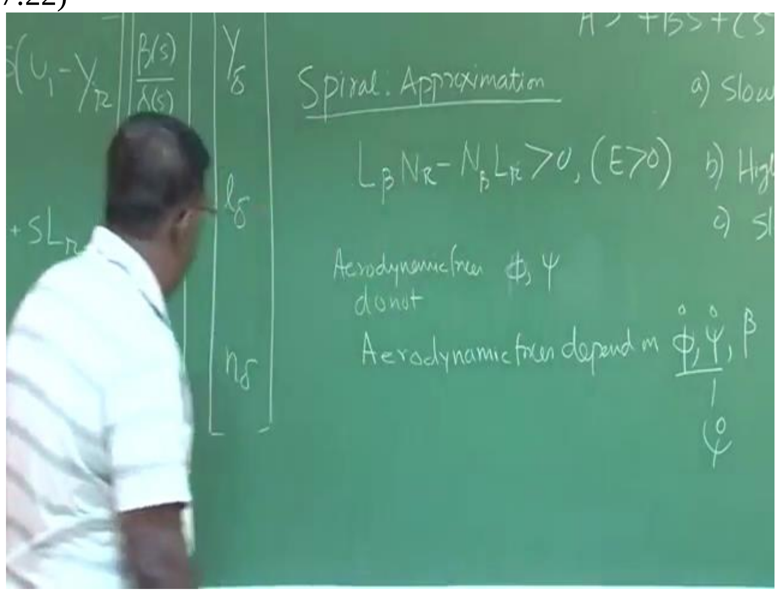


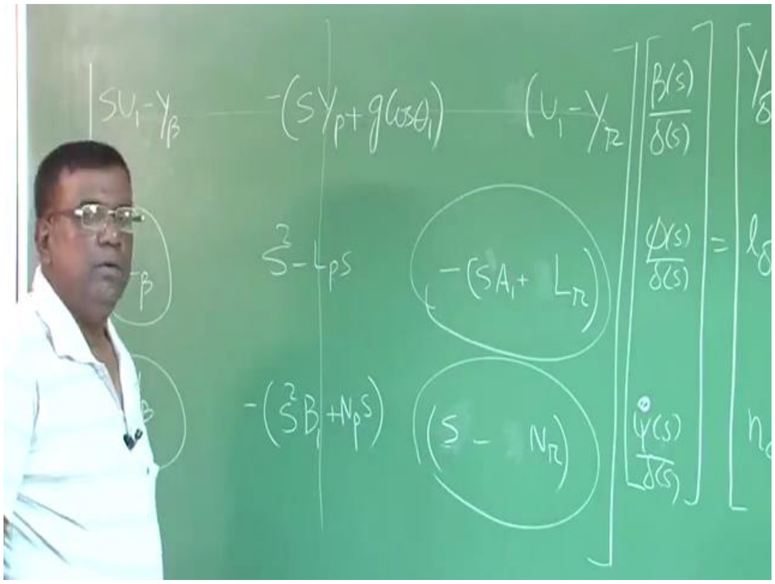
Before we go for spiral approximation, please also understand that the aerodynamic forces do not largely depend upon Phi or Psi. Please understand, we are aware for normal case, the aerodynamic forces will not depend upon Phi or Psi. Please understand, the aerodynamic forces do not depend upon Phi and Psi for normal case. It is a fair assumption to believe that aerodynamic force depends on Phi dot, Psi dot and beta.

Okay. You can understand why Phi dot? If it is Phi, the dynamic force has nothing to do because it is relative air speed. Okay, that matters. Whether it is going like this, going like this or going like this, relative air is same. However, Phi dot is this. As there is a Phi dot, there is a relative air speed vertically up. That gives the aerodynamic forces. Similarly, Psi dot. That is the rate.

So that will also give an induced beta which will give you forces, aerodynamic forces. That is why, it is important to first identify this and then if we now try to simplify which I am again cautioning you, it is not a very good approximation. We have to incorporate this understanding into this matrix and remember, instead of Psi, we are telling the variable is Psi dot.

Refer Slide Time: 7:22)



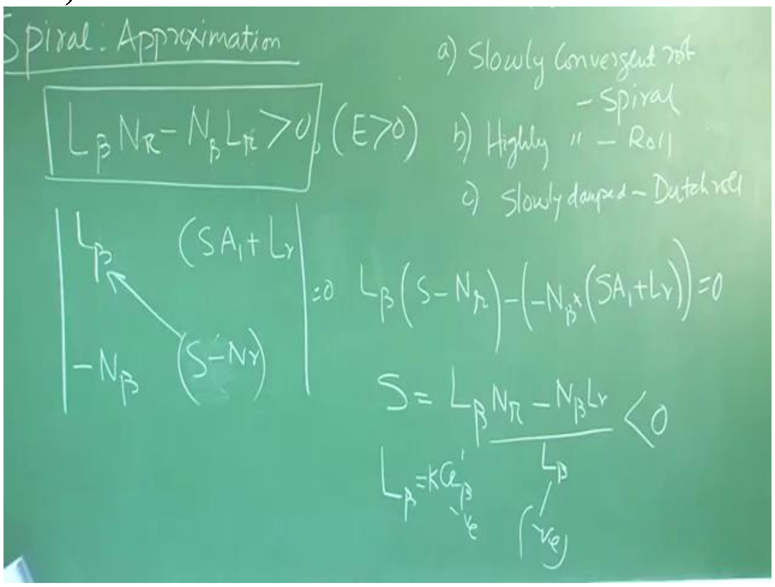


If the variable is Psi dot, then naturally if I write it here, Psi dot then one S will go from here which I have done in the last class. And here, this S will go and here also one S will go. Correct? Because this into this, this into this, this into this. And also you know what we do? Say Phi, it does not matter with Phi. So this term into Phi, this term into Phi, this term into Phi, I drop this.

Because Phi is not relevant for us. What we are left with is this one, L beta into beta, this into Psi dot. Then N beta into beta, S - NR into Psi dot by DS. I am dropping this, I am dropping this. Because I am saying, in spiral mode, what I am focusing is bank, side slip, this, further bank, this, like this. So I am neglecting the effect of any force acting on the Y direction.

Again, this is the approximation. I am more talking about the angular motion. So this is also dropped as if it is taken care or not so important. This is an approximation because when I talk about spiral mode, I am talking about bank, side slip, this, further bank, like this. And though there will be a slight force in the Y direction because it is introduced transient manner but we are neglecting that.

(Refer Slide Time: 9:13)

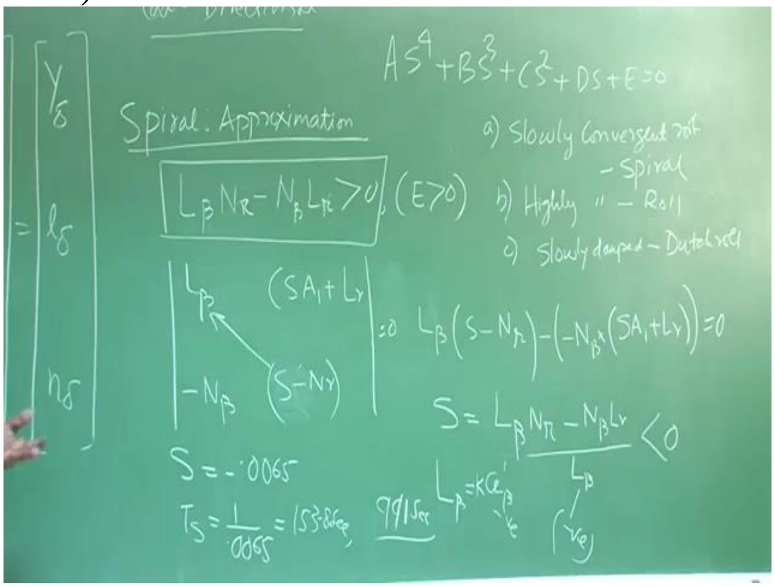


So we are having a smaller determinant which will be having a structure like this, L beta, SA1 + LR, - N beta, S - LR is equal to 0. That becomes my determinant. That because my characteristic equation. And if I do this, if I now expand this, I will find, L beta into S - NR, you could see this one - N beta into SA1 + LR. This equal to 0 which gives me value of S as N beta NR - NB LR by L beta.

And now we see, to make sure that the spiral mode is convergent, not divergent, so this man should be less than 0. And what is the sign of L beta? L beta you know as CL beta. It is first-come into CL beta. And what is the sign of CL beta? CL beta is negative. So for S less than 0, L beta into NR - N beta into LR has to be greater than 0.

Please understand, this condition to be less than 0 and since this is negative, so this gives me a condition which already we have seen. L beta NR - N beta LR greater than 0. So this also we are getting from first approximation. But the question comes, okay it will tell you whether spiral mode is divergent or convergent. But how accurate it will be in terms of numbers?

(Refer Slide Time: 11:34)



So if we do an approximation with the data which we have used, last example for this airplane, then you get the value of S as - 0.0065 which means TS 1 by S. So that will be 0.0065. And this is equal to 153.8 seconds. But exact solution was 991 seconds. Do you see? How much difference? So once we try to talk about quantitative manner, this approximation does not give you very good results. It has very far off results.

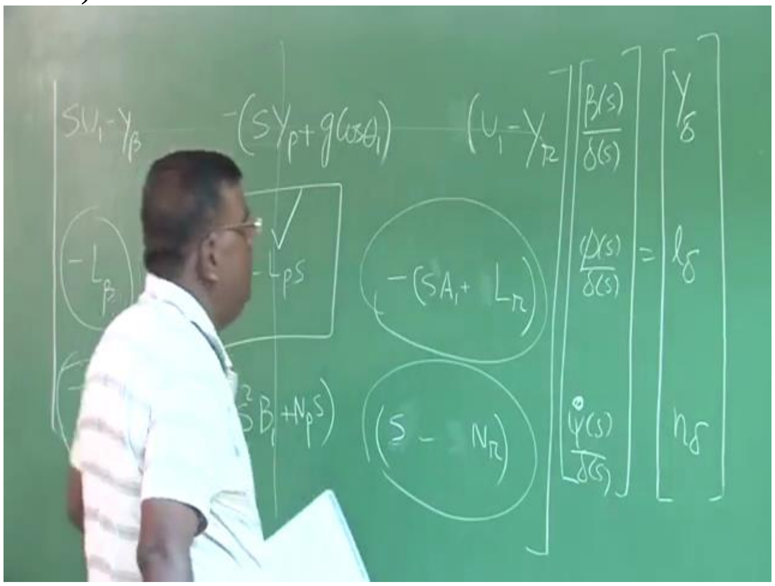
But fundamentally, as a designer, it yields what we got from exact solution that E greater than 0, L beta NR - N beta LR greater than 0. Similar by this approximation also, we got same results. So that is the beauty of this approximation. And that is the reason even if the numbers are not close, we still continue to understand that to get feel for the designer. So this is the spiral approximation. Now we will come to roll approximation.

(Refer Slide Time: 12:52)

Now we will talk about roll mode approximation. And what is this roll mode approximation? If the airplane, remember for lateral directional case, we have identified, there are 3 ways, the airplane will get generally excited, one is spiral mode, like this. Another is roll mode. Another is Dutch roll. These 3 modes.

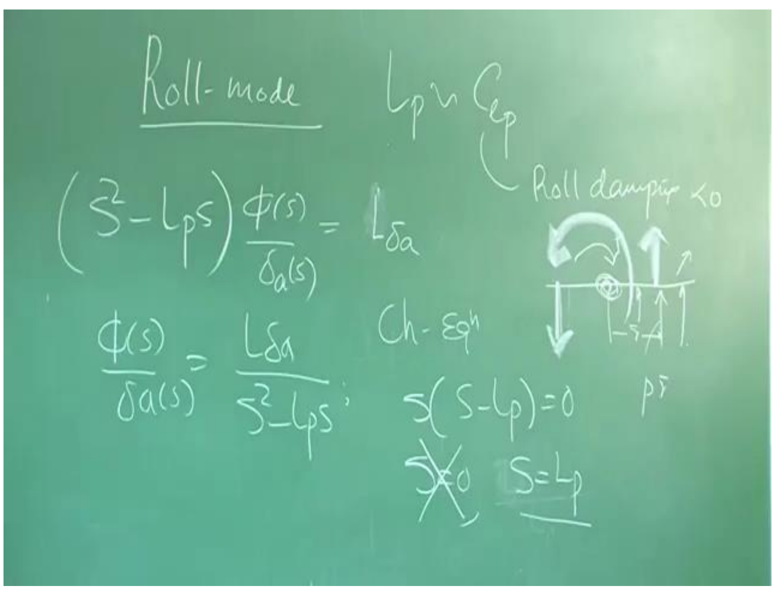
Now we are talking about roll mode. That is if I give a disturbance in the aileron, how this aircraft is going to respond to this rolling moment disturbance? What is the approximation done here is, the assumption is, if it is rolling like this, it is not affecting any motion in the Y direction. No beta is introduced and no Psi dot is introduced. Only Phi, one-dimensional. Okay? Equivalent thinking, in a tunnel, you have given a roll disturbance and it is trying to build or decay, whatever it is.

(Refer Slide Time: 14:01)



If you are doing only one-dimensional roll mode approximation, then you could easily see only we are keeping this. So S square - LPS into Phi of S by Delta of S equal to L Delta A. Or, no yaw motion, no side slip motion. So only this, one-dimensional roll.

(Refer Slide Time: 14:19)



So I can write S square - LPS into Phi of S by Delta, now Delta A, it is aileron input is equal to L Delta A. Right? Now let me write it. L Delta A. I can write capital for clarity. Now if you see carefully, this I can easily write Phi of S by Delta A of S is equal to L Delta A by S square - LPS. So you are experts now.

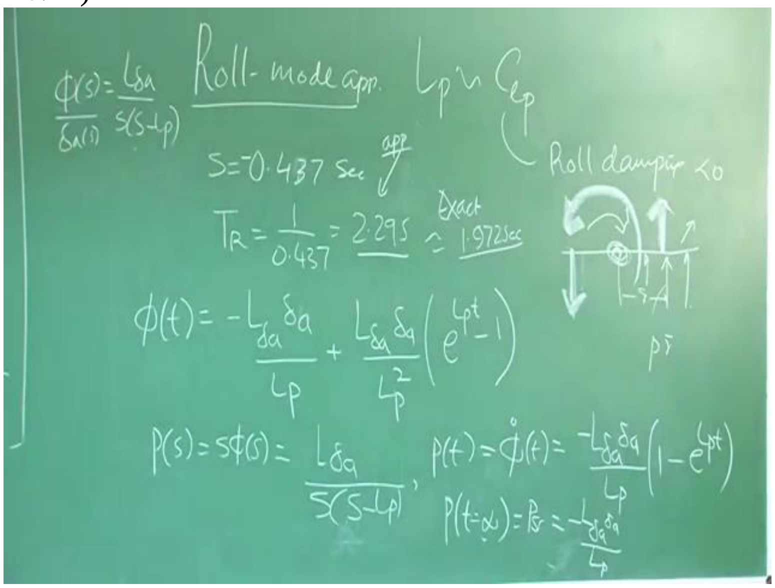
The characteristic equation is S into S - LP equal to 0. This I have got S equal to 0 and S equal to LP. S equal to 0, neutral stability as far as Phi is concerned, I told you again and again, whether the airplane is going Phi 0, Phi 10, Phi 30, aerodynamically they are neutrally stable. There is no change. So you forget that S equal to LP.

It only suggests you, if LP is negative, it will have a first order response and it will be a convergent response. And what is LP? LP you know is having CLP in its term and CLP is roll damping which is less than 0. Just to recall. If this is the airplane and if it is having a roll, say P is like this. Then at every station, there will be relative air speed which will induce angle of attack as it is moving forward.

Similarly, reverse will happen here. This will give you a rolling moment like this. So this is a roll damping because this is a rolling moment. This is proportional to P. For simple reason, this induced velocity is proposed to PR bar. This is the distance R bar from here. This moment is proportional to rate. So we say this is roll damping.

So this is the important truth I have got. As long as this is negative, it is convergent. And now I need to solve this to find out time history of Phi. How do I do that?

(Refer Slide Time: 16:47)



Let us do some numerical search on this expression. With the roll mode approximation, this is roll mode approximation and when we solve that characteristic equation and put the values, we get S equal to - 0.437 seconds and the TR is one of, 1 by this. So this is 2.29 seconds which is coming from approximation. That is one way approximation. But exact value was 1.972 seconds.

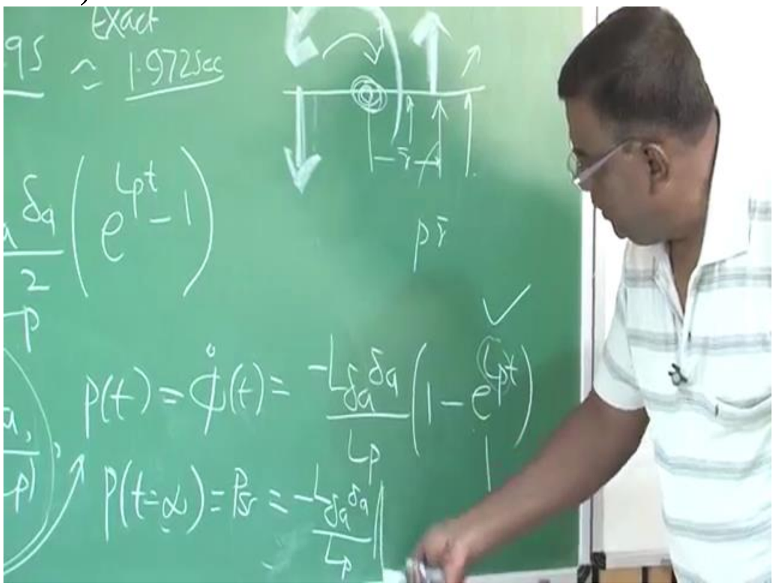
We see that they are not that bad as spiral. Now the question is, if I want to know how this bank angle is developing, then if I, this is my transfer function, Phi of S by Delta of S. So I can always find out the time history by using a partial fraction method. And you will find that from here, I can write Phi of T equal to - L Delta A into Delta A by LP + L Delta A into Delta A by LP square. It requires LP T - 1.

I am sure you know how to find out from this two-time domain which we call inverse Laplace transform which is straightforward. Once you get this, we also know what is the roll rate in frequency domain. Roll rate, we have made an approximation, P equal to Phi dot. So P of S equal to S Phi of S. And Phi of S expression is this.

Phi of S is L Delta A by S S - LP into Delta of S. So this is straight, S Phi of S. That will be L Delta A by S S - LP. And this is, of course Delta A will be there. So once I know P of S is S Phi of S and Phi of S expression already I know. So this is my P of S. So I can easily again from frequency domain, I get into time domain by doing inverse Laplace transform.

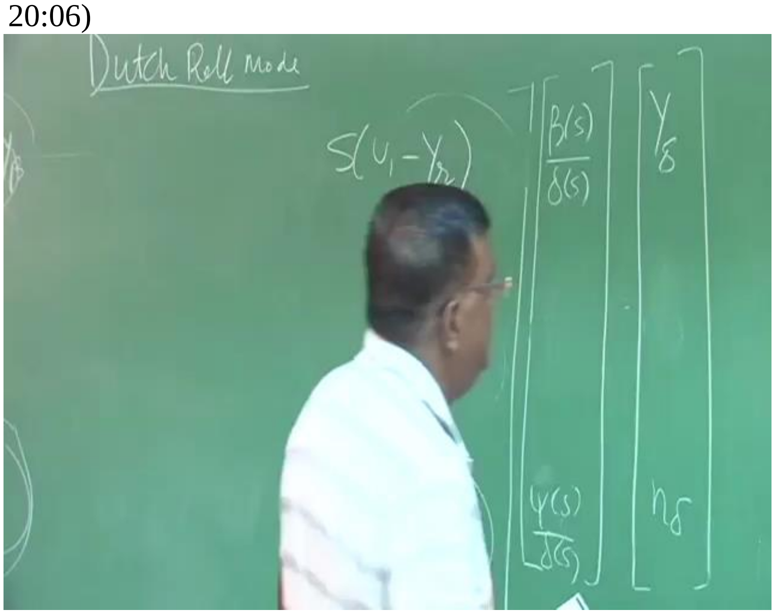
So this will come out to be this. So what are we seeing in this expression? E to the power LPT. LP is what? Positive or negative? LP is negative.

(Refer Slide Time: 19:27)



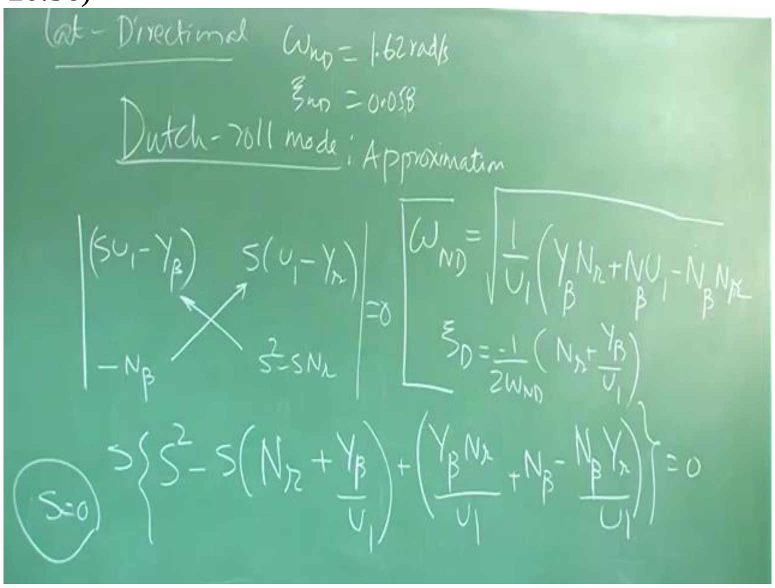
So as time increases, this man will vanish. So you will get a steady-state value which is - L Delta A into Delta A by LP. So once we have given an aileron input, it takes some time to go to a steady-state value, which is here. So we can easily use this to find out the steady-state values, how much the air plane will respond for different types of aileron inputs. This I am completing just to give you a little feel how to estimate the approximate response.

(Refer Slide Time: 20:06)



Now we will complete the last part which is Dutch roll mode. Physically what is it? As we have agreed that airplane generally in lateral directional case can get excited into a spiral, roll and Dutch roll. Dutch roll is primarily like this. This one is going on, see, yawing like this. And also, this motion. Although there will be some banked disturbances, but primarily this and this. This is a Dutch roll. So you go like this. And this has been approximated using, keeping only these 4 terms, I will use beta and Psi. No Psi dot. Please understand.

(Refer Slide Time: 20:56)



And the determinant is typically we have SU1 - Y beta, SU1 - YR and here it is - and beta, S square - SNR. This is equal to 0. Okay? Because we have assumed that there are no Phi disturbances. So you could see, any term with Phi, they anyway vanish and also second part which was, primarily Phi equation is also taken out. These are approximations. Good or bad, we will understand now.

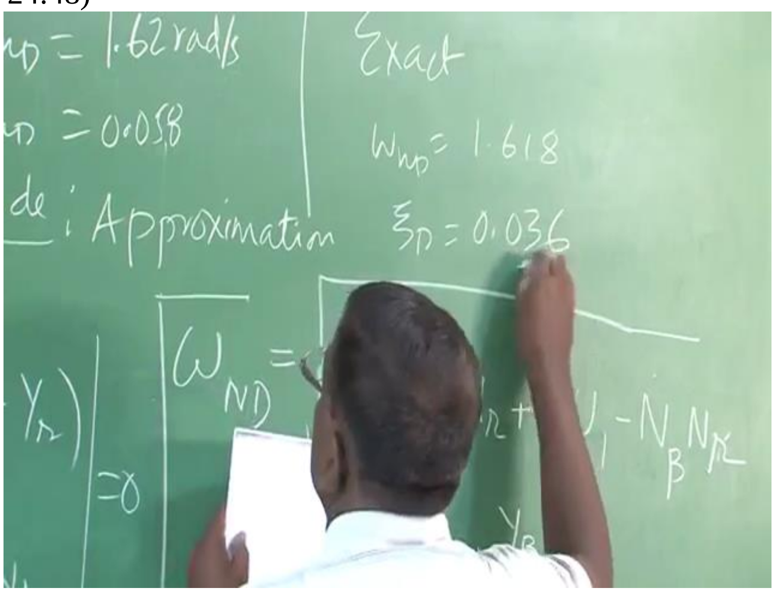
And if I do like this, I get a characteristic equation as S S square - S into NR + Y beta by U1 + Y beta NR by U1 + N beta - N beta YR by U1. Let me check. It is S square - S into NR + Y beta by U1 + Y beta NR by U1 + N beta. So there will be a bracket here. Okay, that you can easily find out by doing this. This and this.

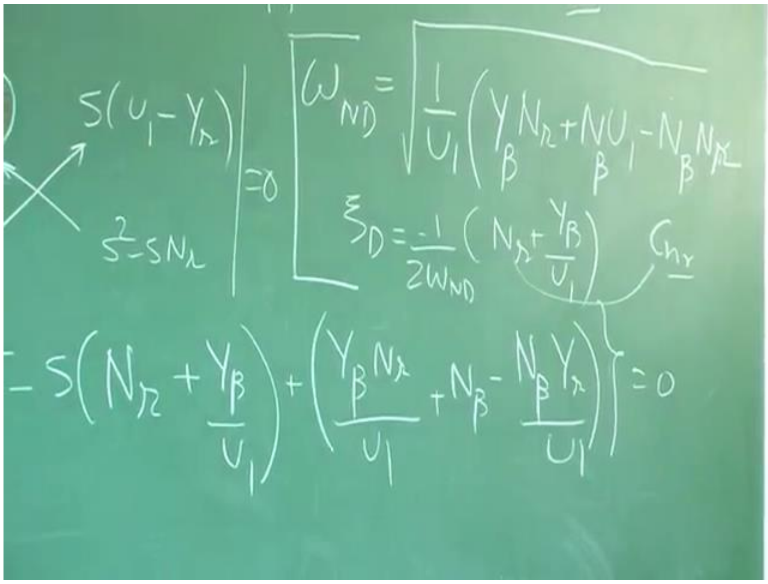
If this is again the characteristic equation for the Dutch roll mode approximation, let me write here, approximation, then you could see that S equal to 0, then it belongs to neutral stability. Because the yawing motion against Psi, this way, this way or that way, it is all aerodynamically similar. So we forget that. So we get again second order equation of the form mass spring damper system.

So we can easily use this and write Omega in Dutch roll as 1 by U1 into Y beta NR + N beta U1 - N beta NR. And Zeta Dutch roll as - 1 by 2 Omega N D into NR + Y beta by U1. This you can directly find out from here because this is nothing but your Omega N square. Now we will try to see how much we get in terms of numerical value of Omega ND with approximation and Zeta ND with approximation and we will see how they compare with the exact solutions.

If you see Omega ND, we will get around 1.62 radian per seconds if we put the values. And Zeta we will get 0.058. This is the Zeta value. One thing is very clear from this approximate value also, this is a low-frequency and Zeta ND is 0.058 means weakly damped. But what was the exact value? Let us check it.

(Refer Slide Time: 24:48)





The exact value was Omega N D was 1.618 and Zeta has 0.036. If you see, as far as Omega is concerned, not that different. But Zeta is also close, not that bad. But the problem what happens you know? Nowadays with all airplane, there will be a lot of large value of IXZ and all, cross moment of inertia. In that case, this approximation is likely to fail very drastically.

This I thought I will complete it because finally we are going for stability augmentation system. So few things you should be clear that if I want to increase the Dutch roll damping, as a designer what do I do? This is Y beta by U1. So I will not focus here. I will try to see how can I increase NR or how can I increase CNR?

And how CNR can be changed by either changing the distance from the vertical tail, aerodynamic centre to CG or (())(26:02) of the vertical tail or tail volume ratio of the vertical tail. To have those ideas, I thought we must complete this before we jump into stability augmentation system. Thank you very much.