Aircraft Dynamic Stability & Design of Stability Augmentation System

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Module 5
Lecture No 28

Modes of Lateral, Directional Dynamics

Good morning friends. We were developing perturbed equation of motion for lateral directional case

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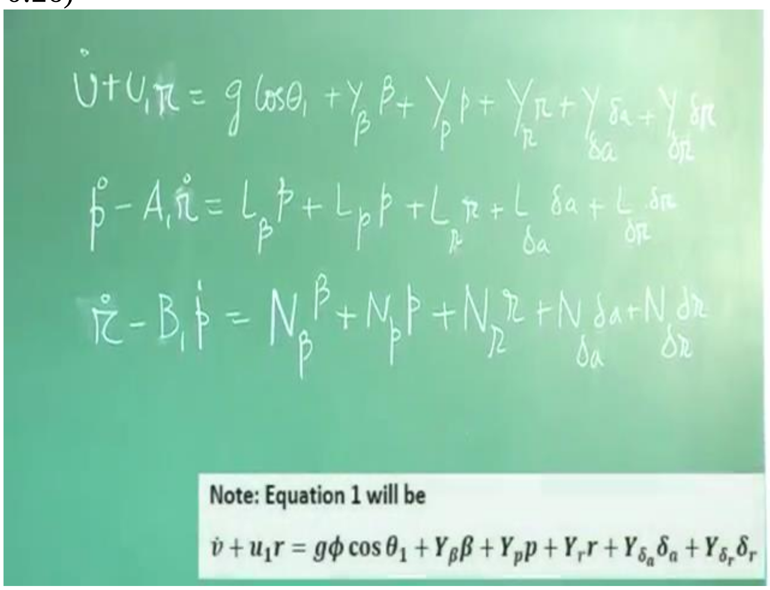

And the first equation we already derived in an explicit manner. This is the second equation. Just to make you comfortable, I will again go through the second equation quickly because these are all mechanical things. And once you have understood how to handle one, you know how to handle other.

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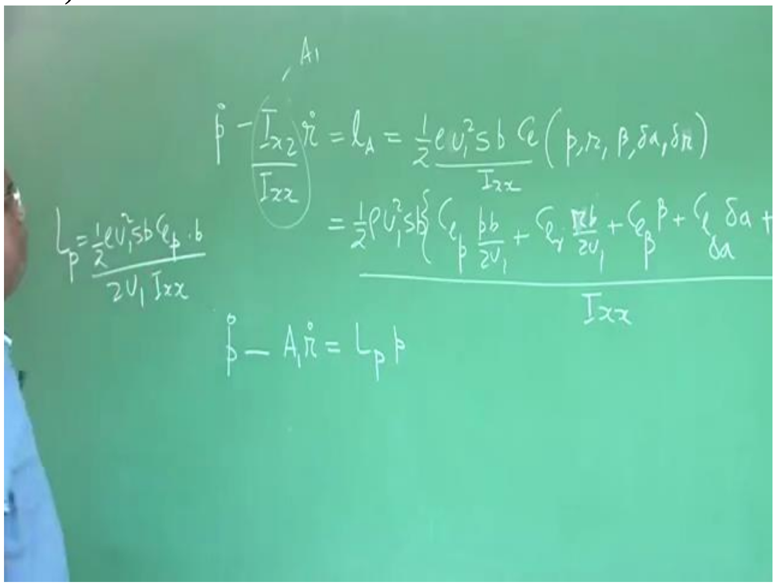

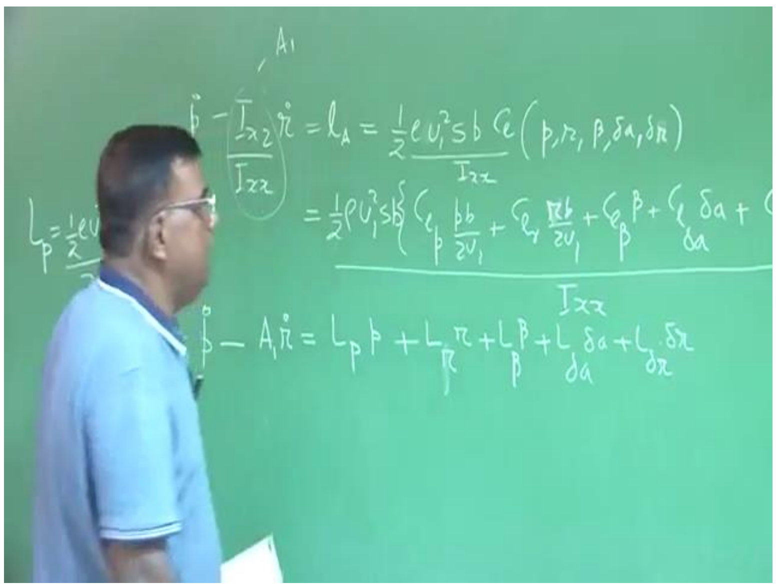

If I try to derive the second equation, we start with the equation IXXP dot. These are perturbed equations. - IXZ S dot equal to LA. and what are these P and R? They are the perturbed rates? LA is the rolling moment. Rolling moment means about the X axis, body X axis. Now, we are also clear that since we have written these equations in stability axis system, we know how to give a correction for IXX and IXZ so that appropriately we have taken the moment of inertia about the stability axis system.

Now we started with this and we know that LA I can write as half Rho V Square or let us say U1 square SB into CL. And CL is function of P, R, beta, Delta A, Delta R. And then we wrote it like half Rho U1 square S into CLP into PB by 2U1 + CLR into RB by 2U1. You are expert now. Why we are multiplying by B by 2U1? 2 non-dimensionalise P and R which are the rates. I can put R like this.

Then CL beta into beta + CL Delta A into Delta A + CL Delta R into Delta R. Let me write the rotation consistently. Okay. Now if I write this, P dot equal to, this will be equal to what? So I have to multiply left-hand side and right-hand side by IXS. So I remove this IXS from here and then there will be IXS here and then there will be IXS here which in turn means there is IXS with the right-hand side.

And now I can write P dot - IXZ by IXS, this we are denoting as A1. So, - A1R dot equal to, half Rho U1 square S B divided by 2U1 into CLP by IXS, this I am writing as LP. So what is LP? From here you can easily see, LP is nothing but half Rho U1 square S B , here B will come. B is missing. This B is here.

So this B should be here. Half Rho U1 square is B. Into CLP into B by 2U1 IXS. So this B into B becomes B square. That is the way you can write in a simplified version. So L beta or LR let us say, I maintain the order. LR into R this L beta into beta + L Delta A into Delta A + L Delta R into Delta R. As simple as that.

That is what exactly is your second equation. Similarly you can find the third equation. Please, as a flight dynamics man, equations are fine. You need to know what is N beta? Who decides N beta? You know in N beta, there is a CN beta. What is CN beta? CN beta talks about directional stability. CN beta has to be greater than 0.

You also know, if I make CN beta very large positive, then it will become very sensitive to wend. Because CN beta is greater than 0, directional stability, if there is a wind like this, it will try to neutralise beta. It will turn like this. So, very very sensitive this frequency, yaw frequency will go on increasing. Right? For a longitudinal case, if CMR alpha is more and more, longitudinal natural frequency goes on increasing.

So these are the understandings you should have. NP means damping, roll damping. That is, the moment it tries to bank like this, there is a damping which will try to discourage this roll or it will come faster to equilibrium once the disturbance is withdrawn. Once I know this and you know that if I am happy with this situation, I want to work in frequency domain, I apply Laplace transform and then once I apply Laplace transform, then the equation will take up the form which is as it was done for longitudinal case the equation will look like, let me write that.

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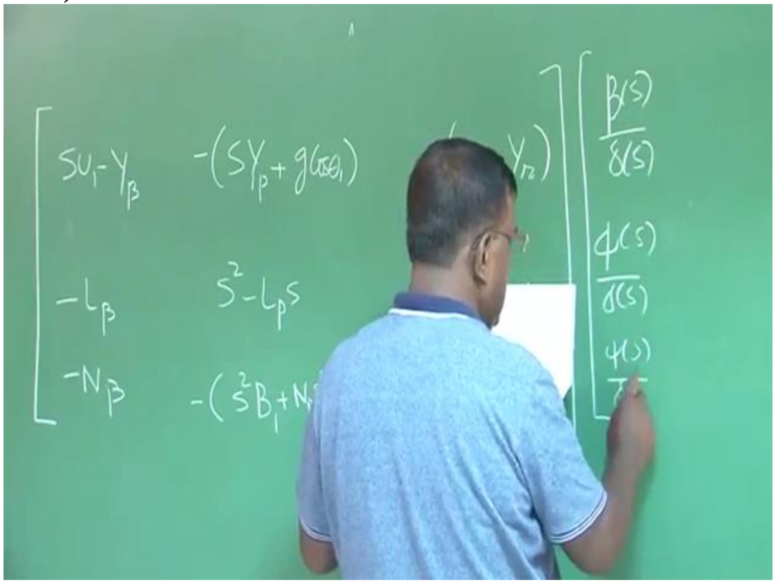

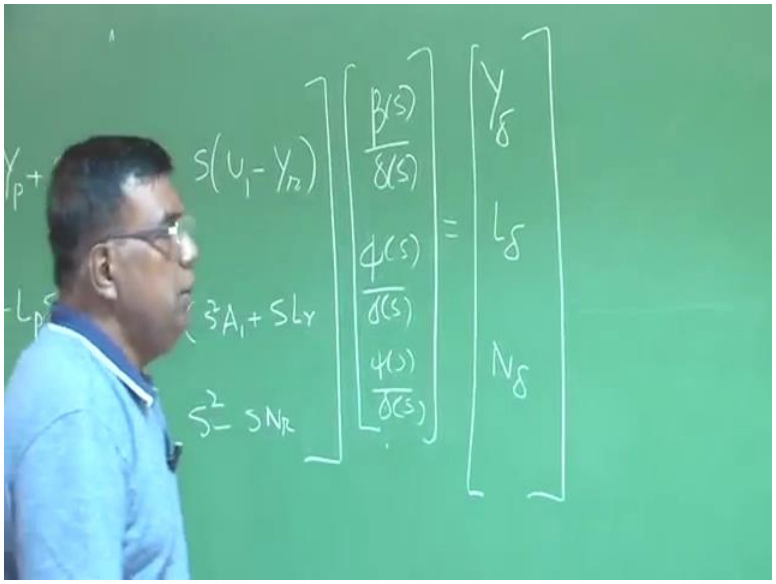

SU1 - Y beta, - L beta -, - N beta. Here it will be - SYP + G Cos Theta 1, S square - LPS, - S square B1 + NPS. And here it will be SU1 - YR, - S square A1 + SLR X square - SNR. This into beta of S by Delta of S. This is Phi of S by Delta of S and Psi of S by Delta of S. This is equal to Y Delta. Then L Delta, rolling moment and N Delta.

I have put Delta here because you could see here, there are 2 control services, Delta A and Delta R. So if I am taking only Delta A, then this Delta becomes Delta A. If only Delta R, this Delta is Delta R and you can take both of them together. How this equation has come? I took Laplace transform of this. So first equation will become what?

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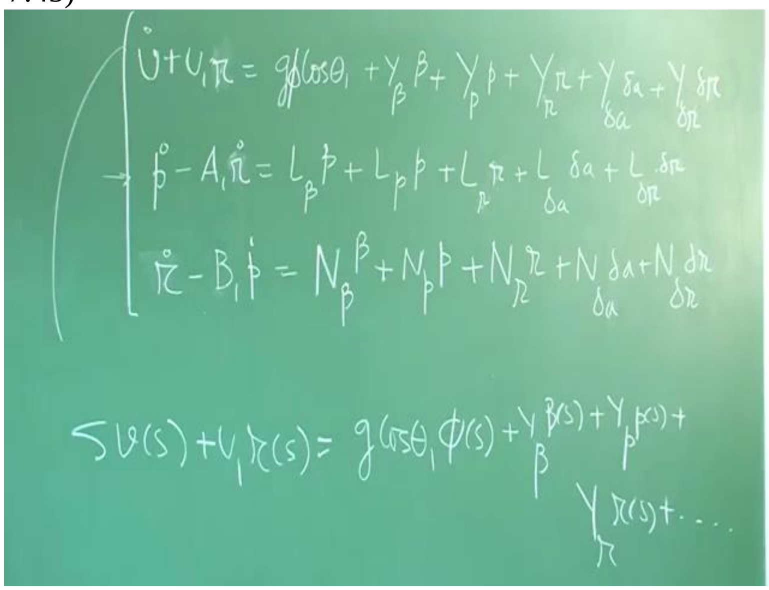

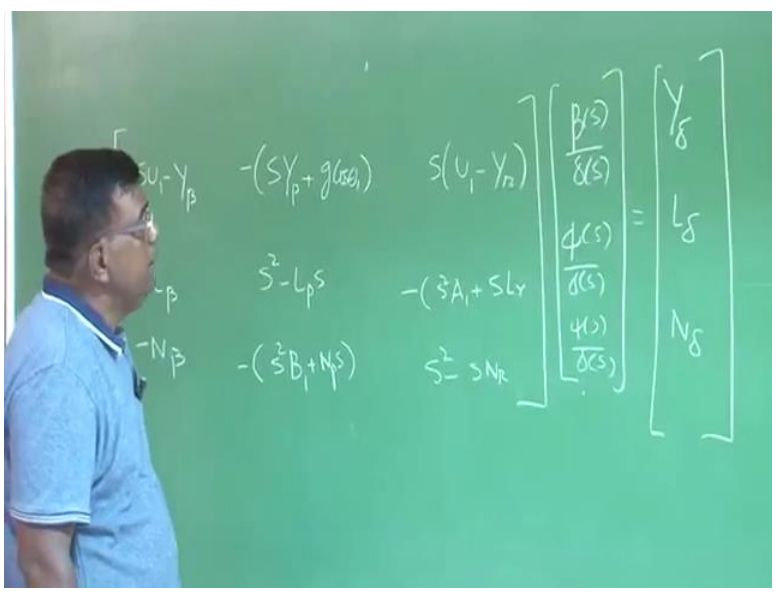

SV of S + U1R of S is equal to G Cos Theta 1 into Phi of S + Y beta into beta of S. Similarly YP into P of S + YR into R of S. And Y Delta into Delta of S and Y Delta R into Delta R of S.

Similarly, I can do it here, I can do it here and then if I combine them, I can write this in this equation form, matrix form. And you know now that if you try to find the characteristic equation for this lateral directional case, that corresponds to the determinant of this equal to 0.

Right? We find the roots and then comment. That is the principle we followed, procedure we followed in longitudinal case also. Before we go to that, just one more attention I want. Whole of this matrix, here it is beta, Phi of S and Theta of S.

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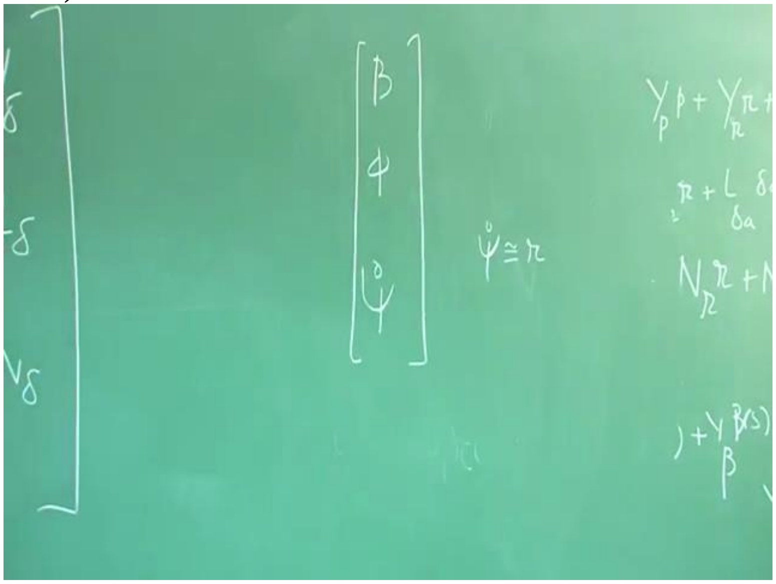

If we want to write it in terms of beta, Phi and Psi dot, in this form if we want to write, and you know for our case, this is a steady-state and disturbance is like this about steady-state. For us, Psi dot is equal to R. Yaw rate and Psi dot are same. But if there is a bank and then there is a Psi dot they are not same. But it is like this. So Psi dot and R are same.

So what we can do? If we want to write the same a question using Psi dot, what will be the change? You could see, for R, we have used Psi dot. Now, Psi dot is a motion variable.

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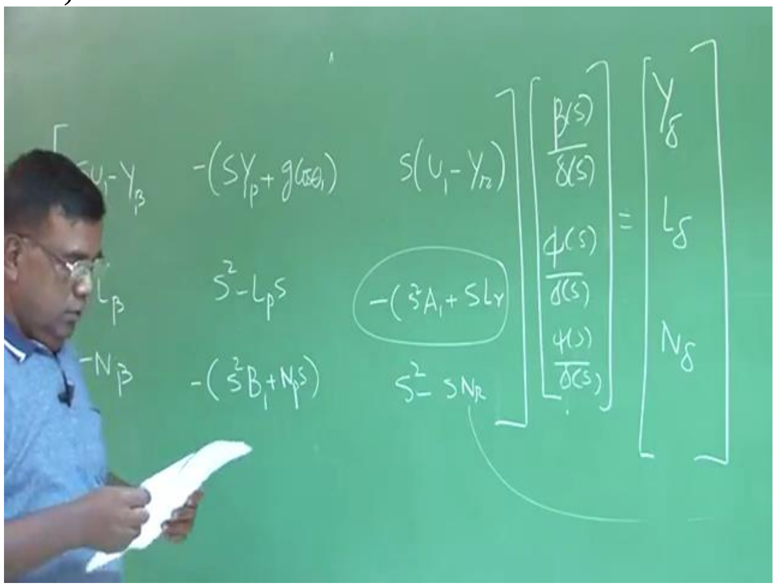

Then the only change that you will get is, wherever this is coming here, if this is Psi dot, then this term, one S will be less. So this term will be - S A1 + LR and here again, one S will be less. So that will be S - NR. Clear? Here it was Psi dot which is equal to R. Now if I am putting Psi dot as a variable, then that one S will be less from the expression which is getting multiplied with Psi.

So this is one is here. So if you see this, this into this, this into this, first with Psi this one. This into this and this into this, if it becomes Psi dot, then this S also will go. So it will become U1 - YR. So here, one S will go. So this will be this. And here, one S will go. So it will be like this. So the whole matrix if I am writing using Psi dot, then one S will be less in this column. This concept we will use later.

So I thought now I will explain you. And now if I see, if I try to write the determinant of this matrix equal to 0 to have the characteristic equation, then what I get is as simple as…

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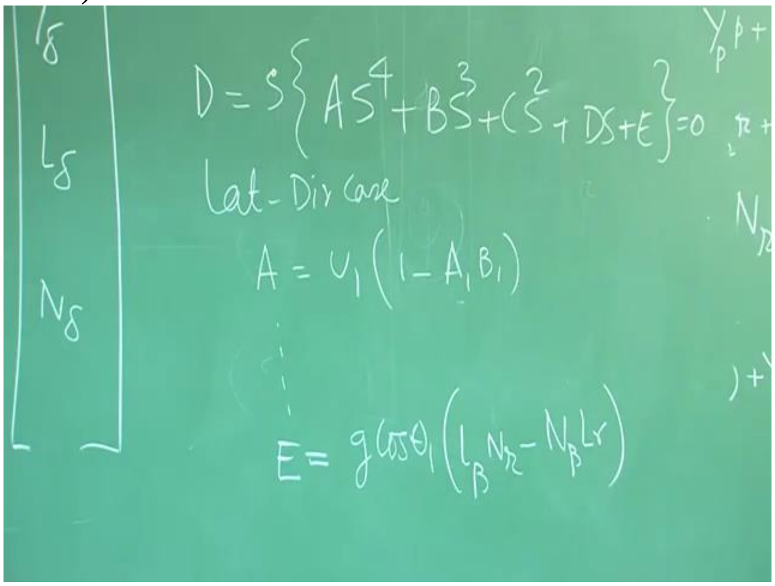

… the determinant I get as S into again AS4 + BS cube + CS square + DS + E equal to 0. Although now the expression for A, B, C, D, E will have the stability derivative, inertial derivative or dimensional stability derivative which belongs to this stability matrix. And you know that we will be giving you that expression. Just for completion, one expression I will give you.

This is lateral directional case. So here, A is given as U1 1 - A1B1. Similarly the important is E is G Cos Theta 1 into L beta NR - N beta LR when there is no thrust. All these detail expressions, I will be posting so that you do not try to remember this, try to understand this. In one page, I will be putting all those expressions for longitudinal, lateral, etc, etc.

Let us understand what is more important for us? If this is agreed, understood, then now you could see that if I want to find the roots of this equation, determinant equal to 0, then what do I get?

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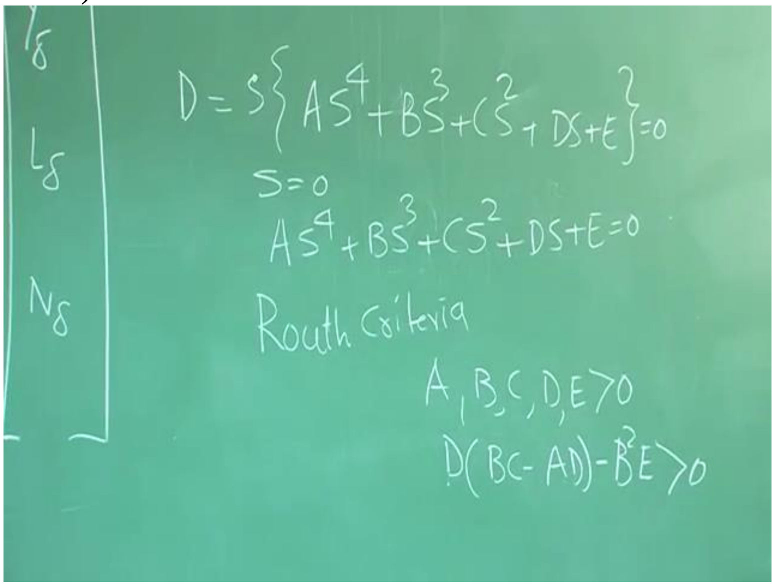

I get S equal to 0 and AS4 + BS cube + CS square + DS + E equal to 0. S equal to 0 is the case. It is a lateral directional. It is like a neutral stability when it is referred to Psi. See, whether the airplane goes like this, it goes like this, it goes like this aerodynamically they are neutrally stable. Like no change. No resistance because for a airplane, it is a relative air speed.

So Psi 2 degree it is flying, Psi 3 degree it is flying , Psi 4 degree flying. It is the same thing. So we call S equal to 0 suggests that. So we neglect that. We understand. Now from this again, we have to apply Routh's criteria and you know Routh's criteria, A, B, C, D, E greater than 0 is to be satisfied. For Routh's criteria, A, B, C, D, E should be greater than 0.

And D into BC - AD - B Square E greater than 0. We also understand, for example if E is less than 0, that means one root has come, real part has come in the positive side. So all those details we know. Now what next? In longitudinal case also, we have seen, we have given some attention to the expression for E.

Before we come to that, we know this, solution for this equation, AS4 + BS cube + CS square + DS + E for lateral directional case, this is lateral directional case we are talking about.

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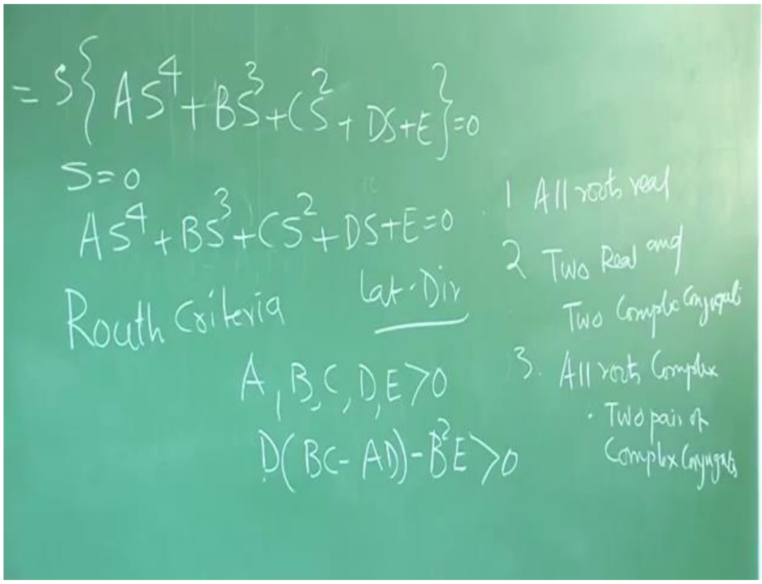

Generally we expect-

1. All roots real. Or it could be
2. 2 real and 2 complex conjugate.
3. Third possibility is all roots complex. That is 2 pair of complex conjugates.

Please see here, 2 conjugates, complex conjugates but here, 2 pair of complex conjugates. Already in second case, 2 real roots are there. Mostly what happens, this case 2 is the common type of root we get for most of the airplane and that belongs to mostly we find for lateral directional case, we get this case 2- 2 real and 2 complex conjugates.

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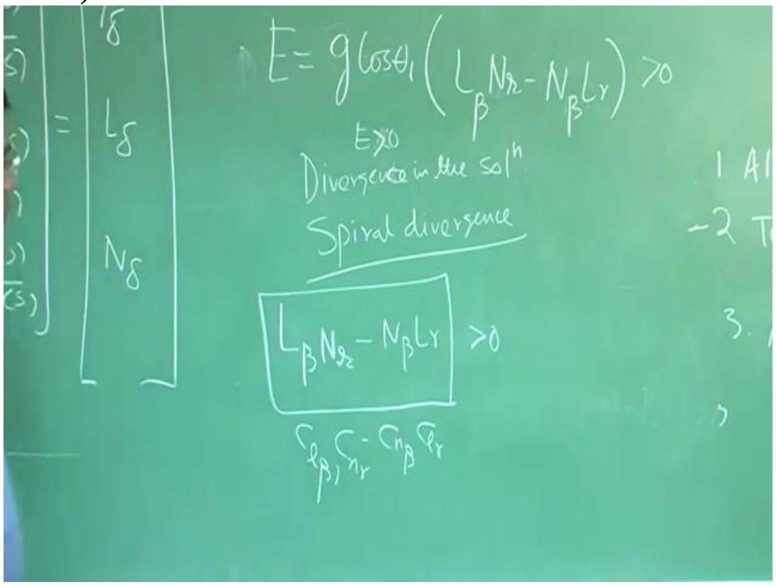

And if you see that if I give a closer look to E where E is G Cos Theta 1 into L beta NR - N beta LR this has to be greater than 0 for satisfying dynamic stability criteria. What is seen that if it is not greater than 0 then in time domain, there is one divergent root. What we found that, if E is not greater than 0, suppose say E is less than 0, then we find, there is a divergence in the solution.

We suggest dynamic instability. And that is typically the spiral divergence. Typically what happens in a spiral divergence? If this is the airplane, because of banked disturbance, the airplane starts side slipping. As it side slips, beta is introduced. It comes like this. As it turns, there is another lift. So it again, left-wing becomes more (())(17:55) cut.

The spiral divergence try to understand, if the airplane is moving like this and there is a banked disturbance, as there is a bank, it starts side slipping. As it is side slips, there is a beta. The moment there is a beta, the airplane tries to turns like this. And as it tries to turn like this, the left-wing gets more velocity, relative air velocity.

It further banks and again further, turns like this. So it goes like this. Unless and until you ensure that E is greater than 0. If E is written than 0, then you avoid spiral divergence. Most of the airplane is having a very weak root in spiral divergence. So it is manageable by the pilot. Do not worry too much about it.

But fundamentally what we understand? The combination of L beta NR - N beta LR, this should be greater than 0. So CL beta plays an important role, CNR plays an important role, CN beta plays an important role and CNR. You could see, these 4 derivatives are to be created in the airplane through the layout of your tail, wing, etc such that this is satisfied.

So now you could see, although we are talking about CL beta for lateral stability in static sense, you could see that also plays important role in dynamic stability. So that is the connection.

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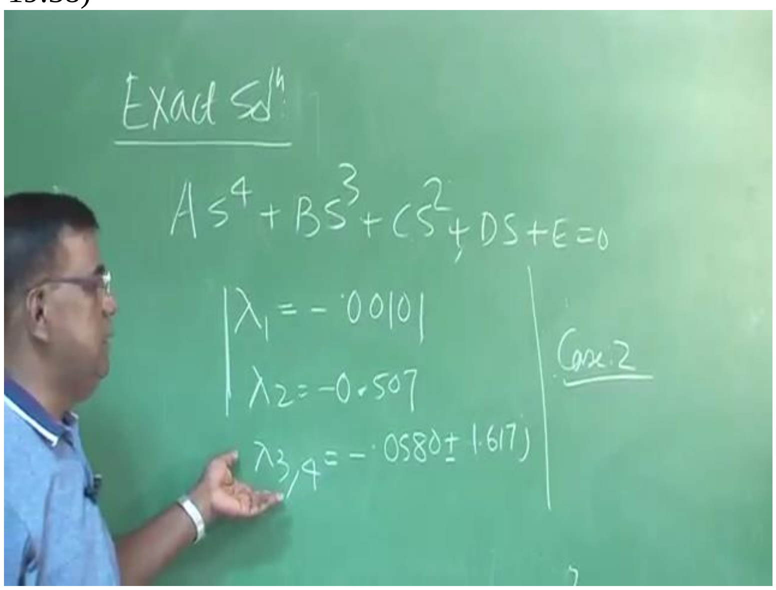

Now become for exact solution. AS4 + BS cube + CS square + DS + E equal to 0. Again this is the characteristic equation. And for a typical airplane, business jet airplane, we found that Lambda 1 was - 0.00101, Lambda 2 is - 0.507 and Lambda 34 complex conjugate that is your case 2, + - 1.6 7J. How did I get these values?

We know this is the characteristic equation. We have an airplane whose aerodynamic coefficients are known, derivatives are known, inertias are known, dimensional derivatives are known. And using the expression of A, B, C, D, E for lateral directional case, we got this equation. Then solve numerically to get this. And you could see that typically most of the airplane the roots are what we mentioned for case 2, 2 real and 1 complex conjugate or 2 complex roots.

One complex conjugate is equivalent to saying 2 complex roots. So, 4 roots. Now I could easily see here that these 2 are real. So there are no oscillations. However, this is, this route, it is a complex conjugate. It has oscillations. And also, you could see, the real part is not as negative as short period real part.

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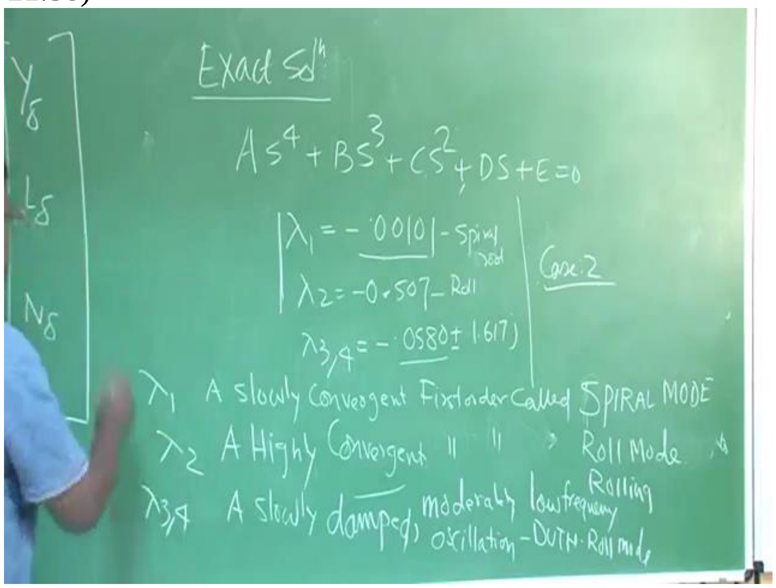

So once I understand that now what type of response or dynamic response I will be expecting, I need to understand one is, first one the weak root and that is spiral root. second one is roll which belongs to the roll damping, roll dynamics that is if I give an aileron input and withdraw it, it will come back. No oscillations. Results time. Now we will try to get more insight about Lambda 1, Lambda 2 and Lambda 34.

We could see Lambda 1, I can clearly write, it is a slowly convergent first-order called spiral mode. That is the Lambda 1, weak. Lambda 2 we say a highly convergent. Why convergent? Because negative. See, this was also convergent. The value was so less. So it is slowly convergent. Here, the value is large.

So this is convergent because of - but magnitude is large. So we say highly convergent. First-order, of course first-order you could say here. And this is called roll mode. Okay? Or rolling mode. Right way, I should use rolling mode. Or role mode whatever you like to use. And that third one, third and 4th, this pair, it is slowly damped. Again, this is slowly damped.

Why slowly damped? first of all, why damped? Because negative. Real part is negative. Slowly because magnitude is less. Then moderately low-frequency oscillation called Dutch roll mode.

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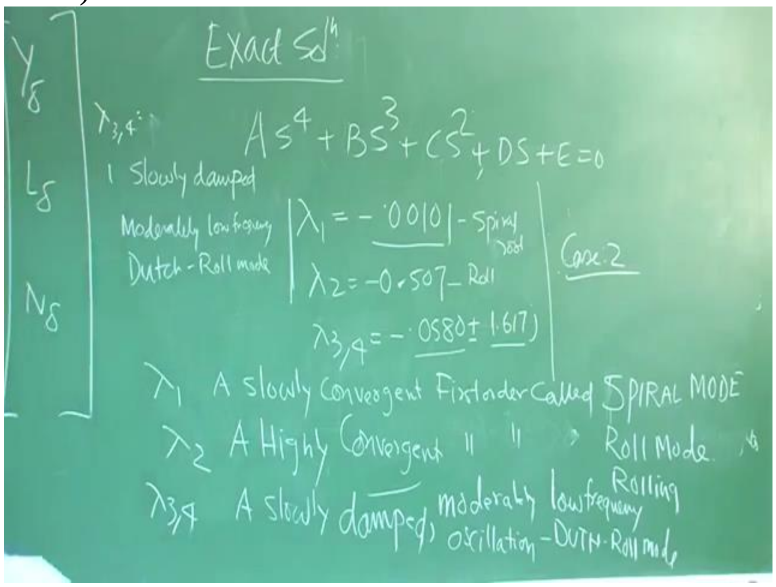

Let me write clearly here, this Lambda 34, what are the keywords? One is slowly damped. second word is moderately low-frequency. That you can see from the value of this. In combination, you know how to find out the frequency. You have to again treat it as a second order system, S square + 2 Zeta Omega N S + Omega N S square equal to 0.

You will find these are moderate low-frequency. And this is also called Dutch roll. This is called Dutch roll mode. Typically, approximately, this will go on doing like this. This oscillation and then like this. This combination. This one this goes here. This sort of a combination. A little bit of bank.

Actually, it is a combination of everything. In approximation, you may neglect one of those but primarily, this is this and one is going like this. They are combination. So you get some sort of a oscillation like this. That is a Dutch roll.

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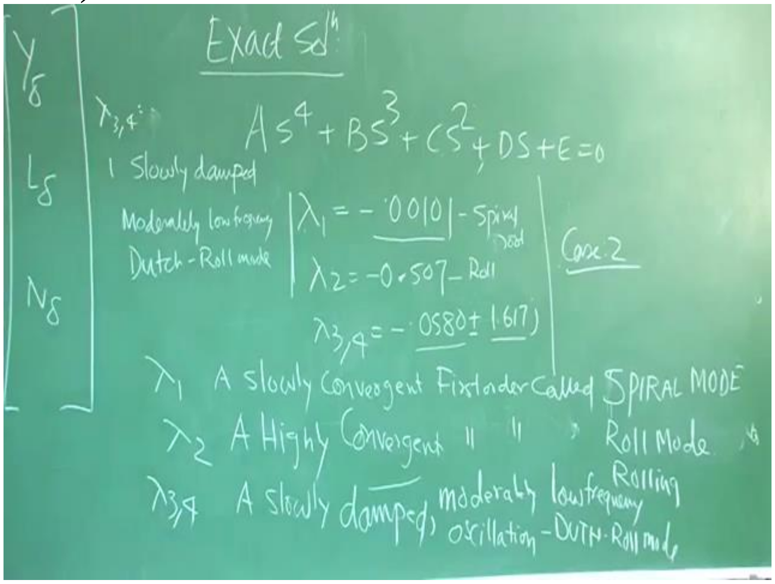

So what did we learn now? We said that for lateral directional case, for most of the airplane, the roots are of this nature. One weak first-order, one strong first-order. And this is a complex. This belongs to roll mode, this is parallel mode and this is the Dutch roll mode. We will go one by one to understand what exactly it means. Now let us play around with this Lambda 1, Lambda 2 and Lambda 34.

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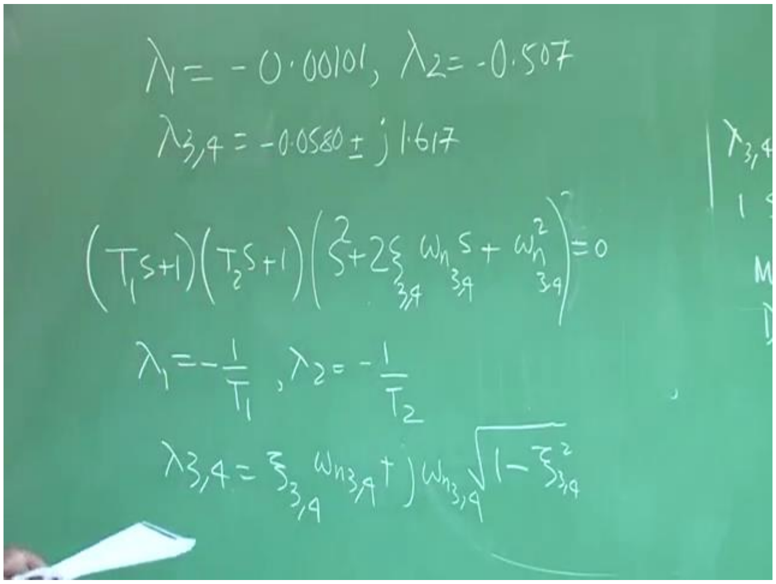

So if I write Lambda 1 is equal to - 0.00101 and Lambda 2 is equal to - 0.507 and Lambda 34 as - 0.0580 + - J1.617. And I know these 2 are first order, this is second order. So I can write the equation in the form, T1 S + 1 T2 S + 1 into S square + 2 Zeta 34 Omega N34 S + Omega N34 square. This equal to 0. This is the characteristic equation. This is second-order. This is first-order.

If I do that, then from this, I get the root Lambda 1 equal to - 1 by T1, Lambda to equal to - 1 by T2. And Lambda 34, I can write like this Zeta 34, Omega N 34 + Zeta Omega N 34 1 - Zeta 34 square. Either you do it like this or 4 second case, you understand, I find the 2 roots and form equation S square - some of the roots into product of the roots equal to 0. So in that way also you can find out. Choice is yours.

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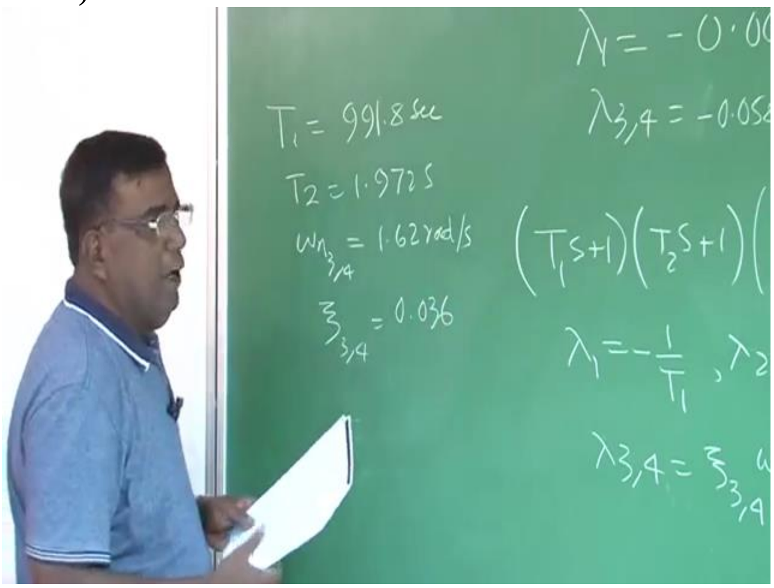

If I do that, then I get the solution as T1 equal to 991.8 seconds, T2 S1 .972 seconds and Omega N 34 I get is 1.62 rad per second and Zeta 34 I get as 0.036. I hope you understand, I can write this we question using this roots in standard form and first root will be T1 S + 1 equal to 0. So from that, one root. From here another root and from there I can get one complex conjugate.

And then you know how to handle these to find out Omega N as well as time constant. Right? That conforms to our point that this is slowly convergent, takes so much of large time. This is fast convergent and this iss having a natural frequency and damping ratio is low. So it is not highly damped. Slowly damped which is a Dutch roll mode.

With this, we have just concluded through an example how to handle lateral directional dynamic case by solving exact characteristic equation which is given by what is written, AS4 + BS cube +

CS square + DS + E equal to 0. We also identified that generally most of the airplane will have roots of this nature, 2 real and 1 complex conjugate.

And we know how to identify which one is spiral mode? Which one is roll mode or rolling mode? Which one is Dutch roll mode? Right? So next, we will go for approximation. Thank you very much.