Aircraft Dynamic Stability & Design of Stability Augmentation System
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Module 5
Lecture No 27
Perturbed Equation of Motion for Lateral, Directional Dynamics

We are discussing about lateral, directional dynamic stability and in particular, we are trying to find out if there is a simplified way of understanding this phenomena which would help designers to initially freeze some design parameters. That is the basic requirement. We have discussed about longitudinal dynamic stability. We have assumed that we are talking about small angle of attack.

We have also assumed that the rates are also small. Why? For a simple reason, if the rates are very high in the longitudinal plane, if angle of attack is very high, then this longitudinal disturbance dynamics will also have an effect on the lateral or directional dynamics. Since we have assumed small angle of attack, small perturbation, we have assumed that longitudinal dynamics is decoupled from lateral, directional case.

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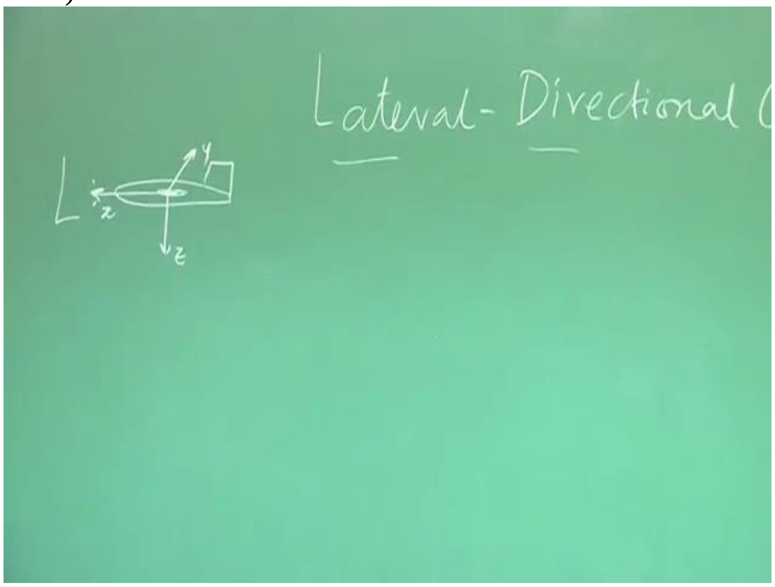

But now when we are talking about lateral, directional case, we understand lateral means if this is the airplane, this is the X-axis, this is Y, this is Z. Lateral means, we are talking about motion about X axis, roll or bank. And directional is motion about Z axis, yaw. But one thing we should understand is that lateral and directional case, they cannot be decoupled generally. That is, you can understand if suppose this is the directional case, that is right-wing going back, what will happen?

As right-wing goes back, this left-wing will come forward. So the relative air speed of the left-wing will increase. Hence the lift will increase. So it will bank also. So there is a coupling between directional and lateral case. So reverse will be there. If there is a lateral disturbance, there will be a directional influence on the whole airplane dynamics. That is why we need to go with lateral and directional case together. We are not decoupling lateral and directional case.

Although we see some approximate studies where we only assume lateral dynamics and directional dynamics separately also yields some useful information specially for the designers. And our approach was, first you write the equations of motion which already we have derived. Now introduce perturbation and then follow the similar steps as we have done for longitudinal case.

We will take one equation and we will try to revisit and they will see that we are very comfortable. Nothing extra we are doing here as far as technique is concerned.

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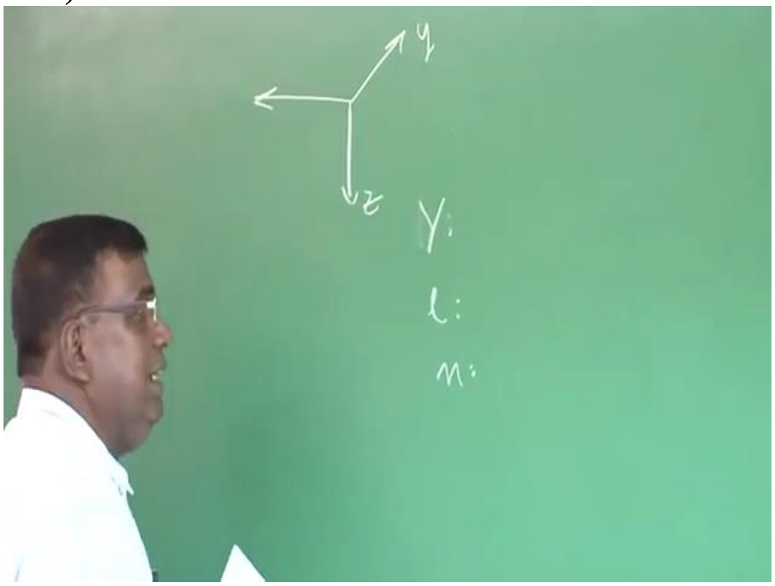

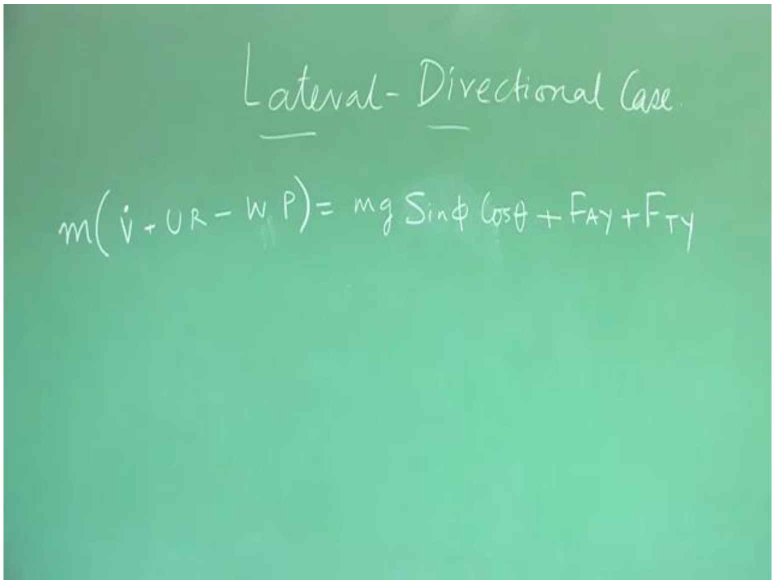

If I take equations along, force equation along Y, then rolling moment about X, yawing moment about Z, these 3 that is this motion, this motion and this motion. That is, if you see like this, this motion, this motion along Y, roll about X and yaw about Z. I will take first Y equation and if you recall, the equation was of the form MV dot + UR minus W1 into P is equal to MG Sin Theta 1. I do not write 1. I am just writing Theta.

Let me check it. MV dot + UR minus WP. I do not put 1 here now at this point. This is a general equation. WP equal to MG Sin Phi Cos Theta. So this is wrong. MG Sin Phi Cos Theta + FAY and we can always write FTY. Let me check again, on right-hand side, it is MG Sin Phi Cos Theta correct, FAY + FTY. This is the equation of motion along Y direction. What was V? V was the component of total velocity along local Y direction.

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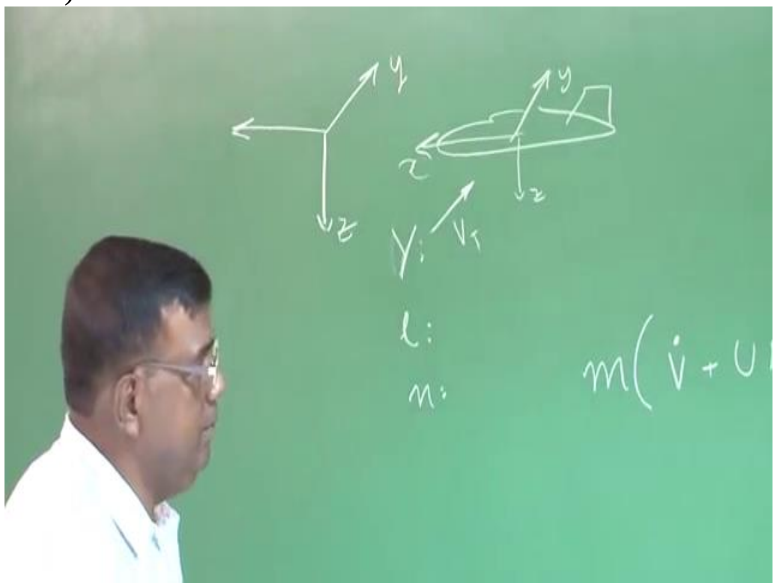

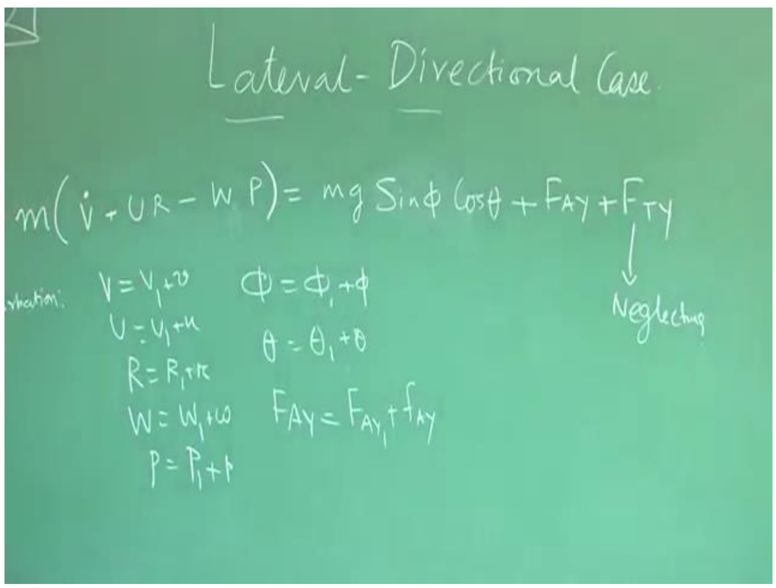

That is, if this is the airplane, this is X, this is Y, this is Z and lets that this is the total velocity, VT or V Star whatever you write, then this V is the component of this velocity resolved along Y direction of the body fixed axis system. Now next step was introduce perturbation. So introduce perturbation, what we have to do? V will be called to V1 + small V. All small quantities are perturbed quantities but they are small perturbation.

Then U is equal to U1 + small U, R equal to R1 + small R. Then W is equal to W1 + small W, P is equal to P1 + small P. Similarly you could see Phi is equal to Phi 1 + small Phi, Theta is equal to Theta 1 + small Theta. Then, FAY is equal to FAY1 + FAY. I am neglecting the thrust component. We are neglecting it with understanding that we know how to handle this if I know how to handle FAY term. What was FAY? FAY is the perturbed aerodynamic force along Y direction.

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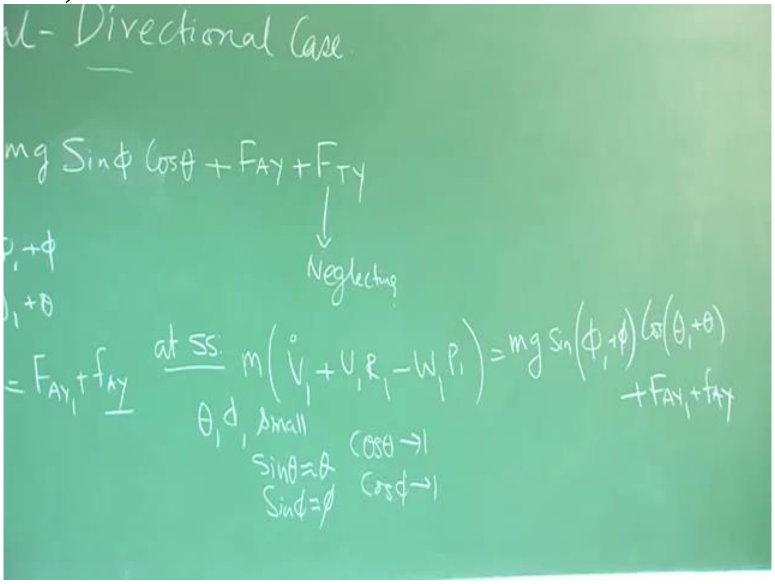

And also we know, at steady state, what is true? It is M V1 dot + U1R1 minus W1P1 is equal to MG Sin of Phi1 + Phi. Then Cos of Theta 1 + Theta + FAY1 + FAY. And also you know that we are assuming Theta, Phi, all are small such that sin of Theta is approximately Theta, sin of Phi is approximately Phi and Cos of Theta goes to 1, Cos of Phi goes to 1. Nothing new we are doing. So what we will do?

We will expand this the way we did it for longitudinal case, we will substitute these conditions and then simplify and if I give you one more step to just recall what we have done, what you have to do?

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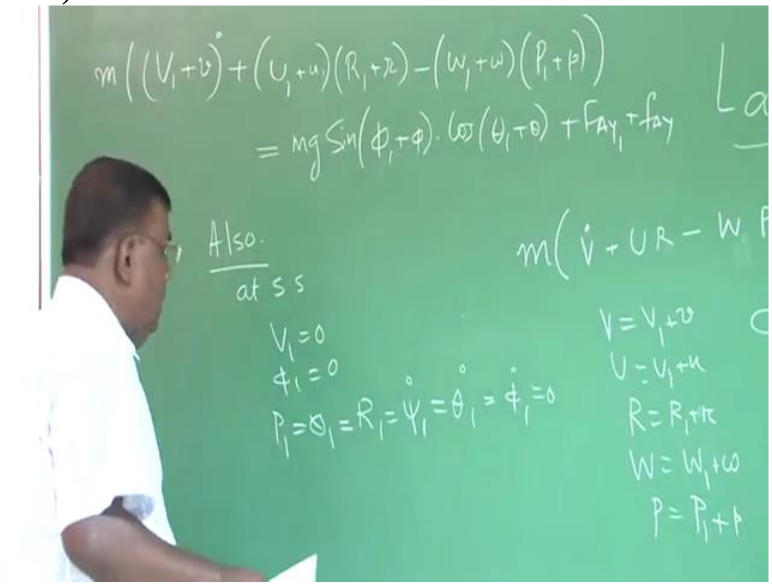

For the first equation, you write MV1 + V dot + U1 + small U into R1 into small R minus W1 + small W into P1 + small p. This is equal to MG Sin of Phi when + small Phi into Cos of Theta 1 + Theta + FAY1 + FAY. You could easily see this which you have seen for longitudinal case also. Since at steady state, this is true, this can be easily taken out from here, MV1 dot, then V 1R1, W1 P1.

Similarly expand this Sin Phi as Sin A + B, Cos A + B. Expand it, put this approximation. And when you do all these things, you will get an equation so you know that as we have done for longitudinal case, we have to expand it, take out the term because they are automatically satisfied, used this approximation and also note, this is nothing new I am doing. Also not, as steady-state, V1 we have taken as 0, cruise, Phi 1 equal to 0, P1 equal to Q1 equal to R1 equal to Psi 1 dot equal to Theta 1 equal to Phi 1 dot, all are equal to 0.

We are talking about cruise, steady, level, unaccelerated flight. So this also goes to 0. And now when you put all these things here, you get a very simplified equation of the form…

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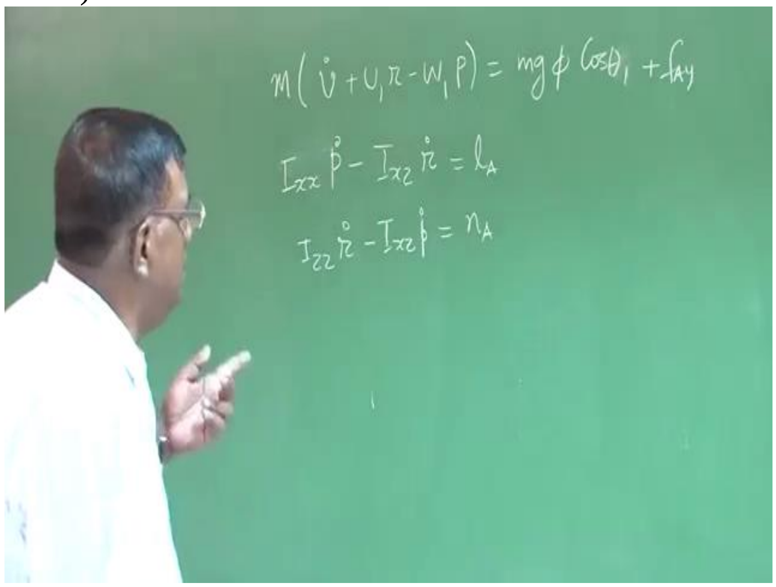

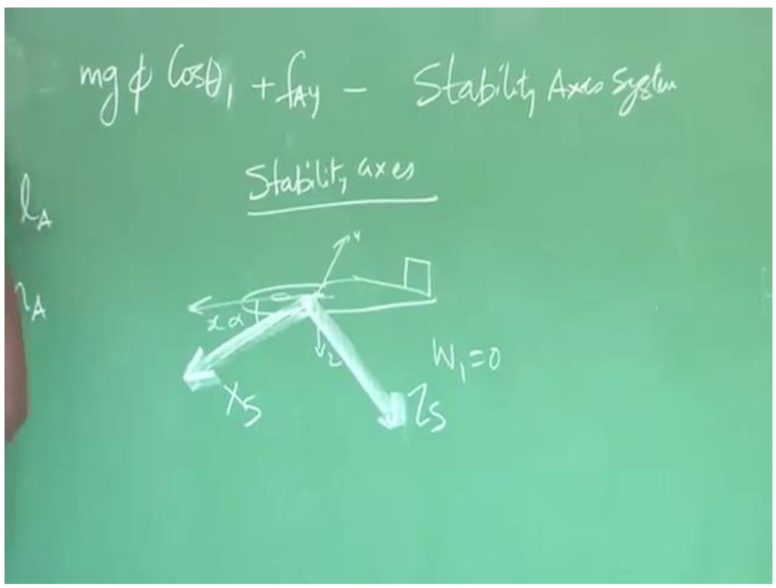

… when I do all those substitution, then I get the equation in the Y direction, is a perturbed equation of motion as capital MV dot + U1R minus W1 P is equal to MG Phi Cos Theta 1 + FAY. Cos Theta 1, what is Theta 1? It is the attitude of the airplane during the cruise. So that need not be 0. That is why we have kept that Theta 1 so far. And also note here, the moment I put another condition that we are using stability axis and recall what is stability axis?

If this is the airplane and this is the normal body X, Y and Z and let us say this is the relative airspeed, and this is the angle of attack, then XS means the X axis is aligned with the velocity vector in the vertical plane and this becomes ZS. The advantage is, if this is XS, this is ZS, then naturally W1 is 0. We further simplify this equation and we get, we remove this term. So we get this equal to this in stability axis system.

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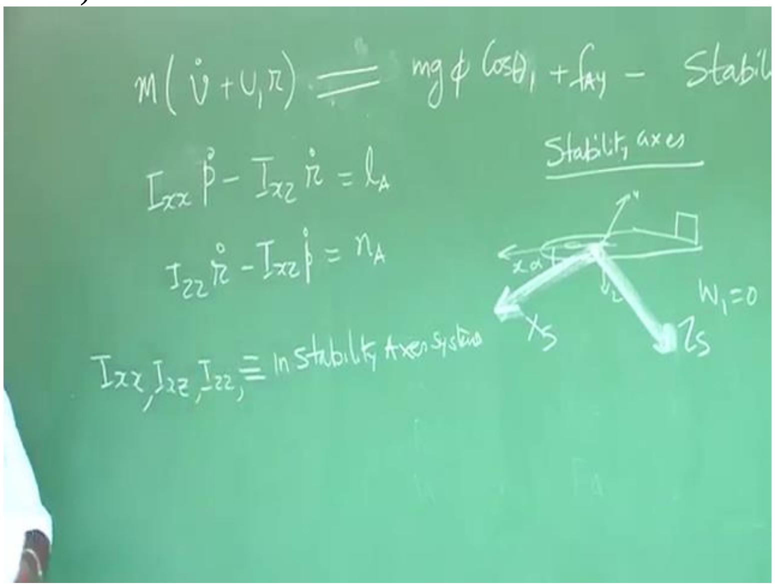

So now the equation becomes much simpler. So I write this equal to this and we should be careful that we are now using stability axis system. The moment I use stability axis system, please understand, earlier this IXX, IXZ were about this XY and this Z axis. Now you are solving everything in stability axis system. So this IXS, IXZ, IZZ, IXZ, they also should be in stability axis system. So IXS, IXZ, IZZ, all this should be in stability axis system.

What mathematically it means that you know the moment of inertia about X, Y, Z axis. This is one axis transformation with alpha. So we have to calculate the new values of moment of inertia about the stability axis system and for that you can directly use transformation which I will just give for completion.

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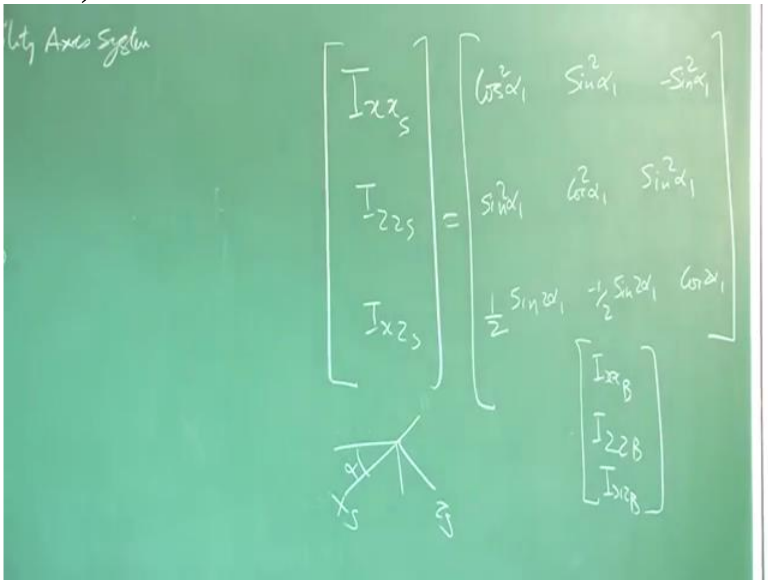

This is IXX stability axis, IZZ stability axis, IXZ stability axis. This will be Cos square alpha 1, sin square alpha1, minus sin square alpha 1. This is sin square alpha 1, Cos square alpha 1, sin square alpha 1 and half sin 2 alpha 1, minus half sin 2 alpha 1 and Cos 2 alpha 1. Into IXX body axis, IZZ body axis, IXZ body axis. This you can check any literature, how do I do body axis transformation that is if this is X, Y and Z, then how the values which we have measured with respect to X, how can we transform them about another axis which is rotated by angle, alpha.

You can please crosscheck this. Okay, the important part which you should understand. When I am using the stability axis system, how much this moment of inertia is going to change? Because it is angular motion, lateral directional, right? 2 motions. One is roll, one is yaw, angular motion. How they are getting affected. So we need to have some numbers with us to get some feel for numbers.

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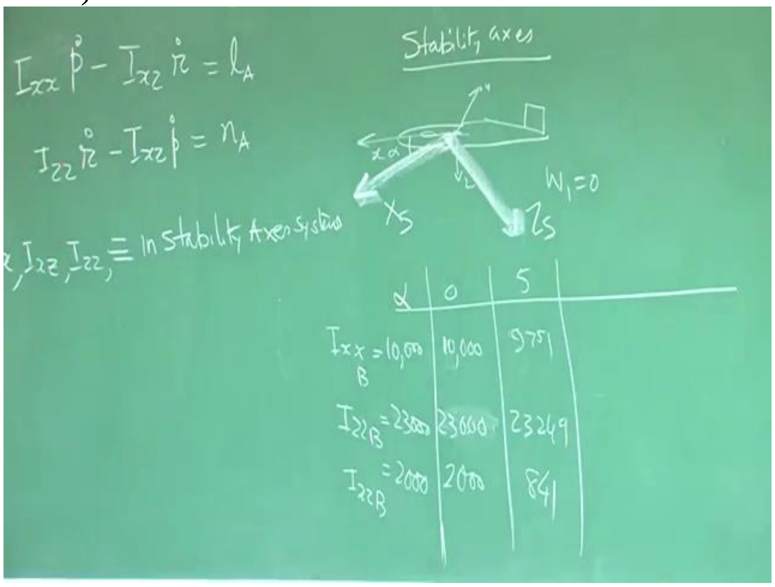

For completion again, I am giving you some sort of influence matrix where you can easily understand how much it will happen. If it is alpha 0 degree and 5 degree, I take 2 cases. If IXXB is 10,000 some unit. Then alpha 0, it is 10,000. For 5 degree, it becomes 9751. Similarly, for IZZB, if it is 23,000, and for 0 degree, it remains same, 23,000. For this, it becomes 23249.

Similarly IXZB, if it is 2000, it remains 2000 of course alpha 0. But here it becomes 841. So it is not really to be ignored because a lot of cross coupling happens through IXZB and then all this are showing a lot of influence. Okay? So that is why, as a designer, you need to have this feel, how much percentage these values will change before you comment on the final result. Okay if this is true and if we have understood up to this point, let me erase this.

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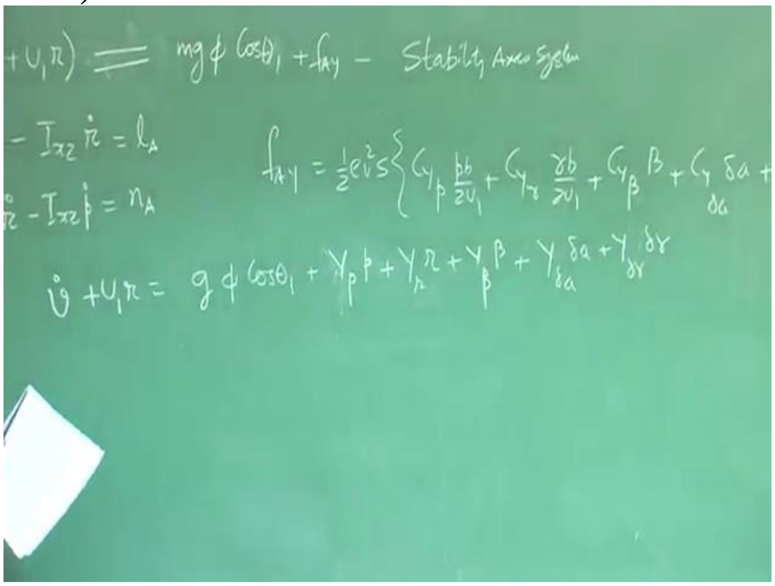

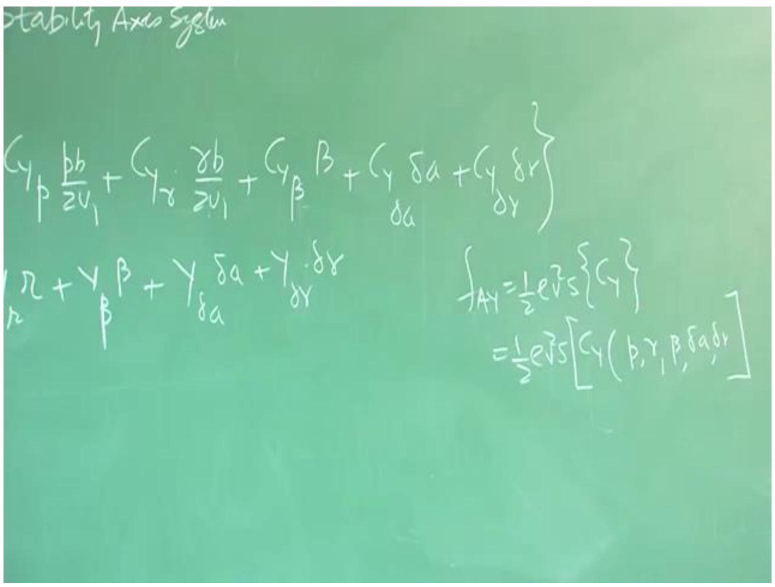

Now what was done? Remember, this is FAY, LA and NA are there, are to be expanded in terms of motion variable or control variable and we have already seen that this can be written as say FAY I can easily write as half row V square S into CYP into PB by 2U1 + CYR into RB by 2U1 + CY beta into beta + CY Delta A into Delta A + CY Delta R into Delta R. We have seen this, how to write this.

Now if I substitute this here and then divide by M, then I can write this equation in a dimensional derivative form which we did for longitudinal case and that would be V dot + U1 into R equal to G. M will get cancelled. G Phi Cos Theta 1 + Y beta into beta or we will not write Y beta, we will write YP into P + YR into R + Y beta into beta + Y Delta A into Delta A + Y Delta R into Delta R. Are you clear what we have done?

We know this expansion we have seen. Because FAY, what is FAY? FAY is half row V square S into CY. And what is CY? CY is CY function of P, R, beta, Delta A, Delta R. And assuming aerodynamics to be linear, we have expanded this as CYP into PB by 2U1. And PB by 2U1, you know B by 2U1, we have multiplied to non-dimensionalise P, all the rates. So this is the form the equations are taking and useful for, you want to see yourself what will be the value of Y beta.

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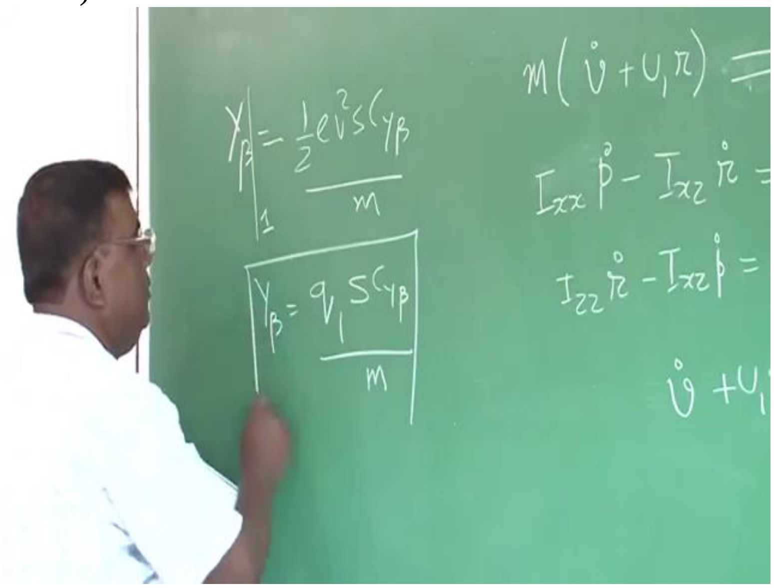

You could see from here, Y beta will be what? Instead of FAY, I will write this expression. Divide by M. Beta is here. That will be half row V square SC by beta divided by M. You could see, this is half row V Square S into CY beta by M. But we know that. We will evaluate this at steady-state. So Y beta will be written as dynamic pressure, Q1. 1 means at steady-state. So Q1SCY beta by M.

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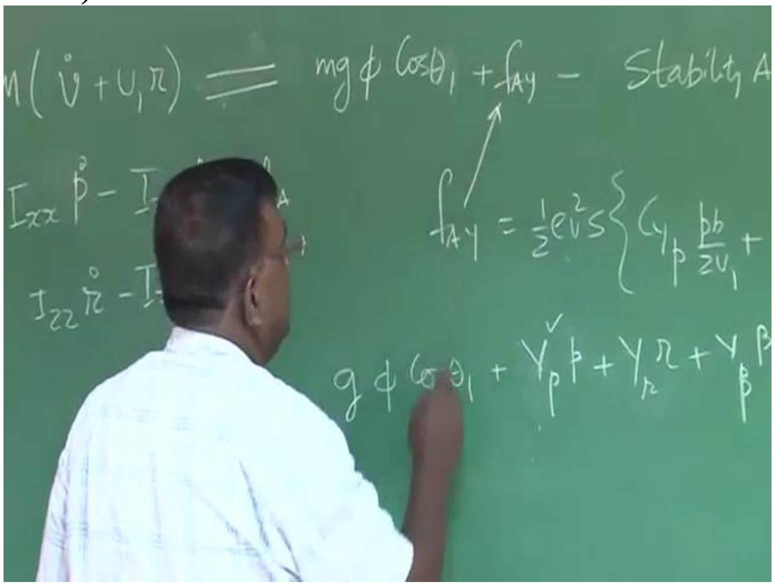

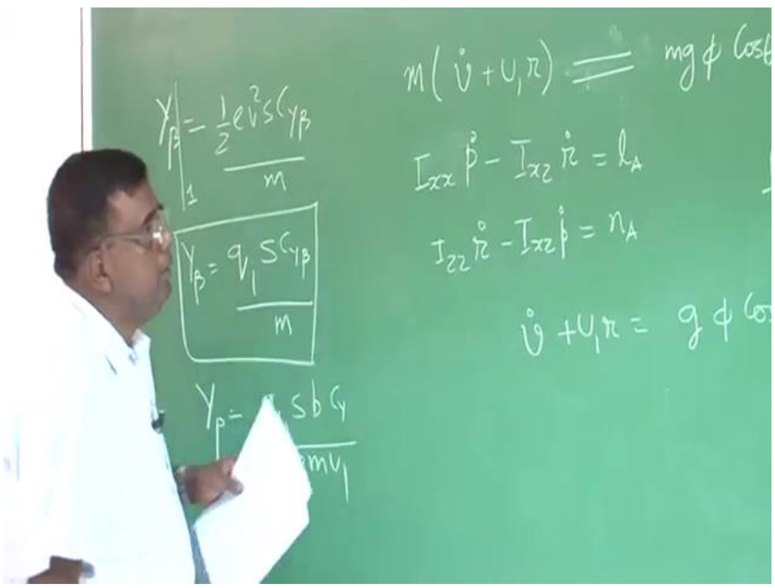

Similarly if you want to check what will be the expression for YP, it is very simple. Come here, we are actually putting this expression here. So P is here. So half row V Square SCYP into B by 2U1 divided by M. So YP will be Q1SBCY by 2 MU 1. Is not it?

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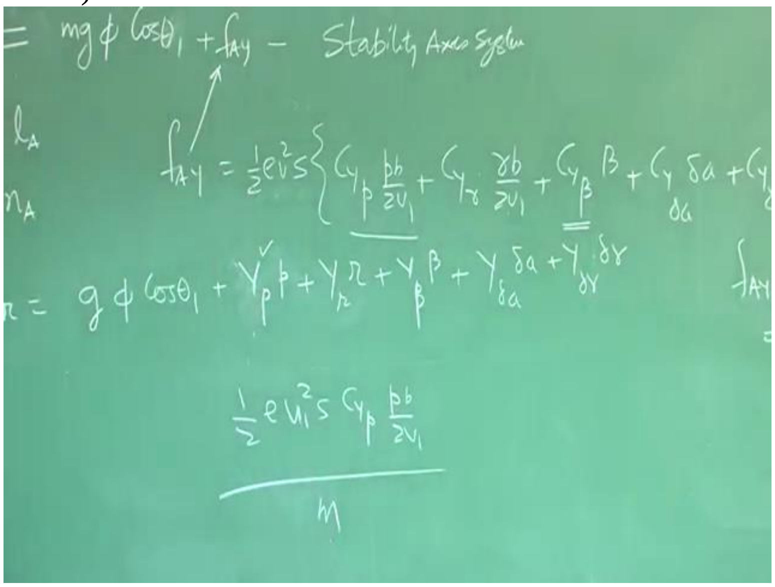

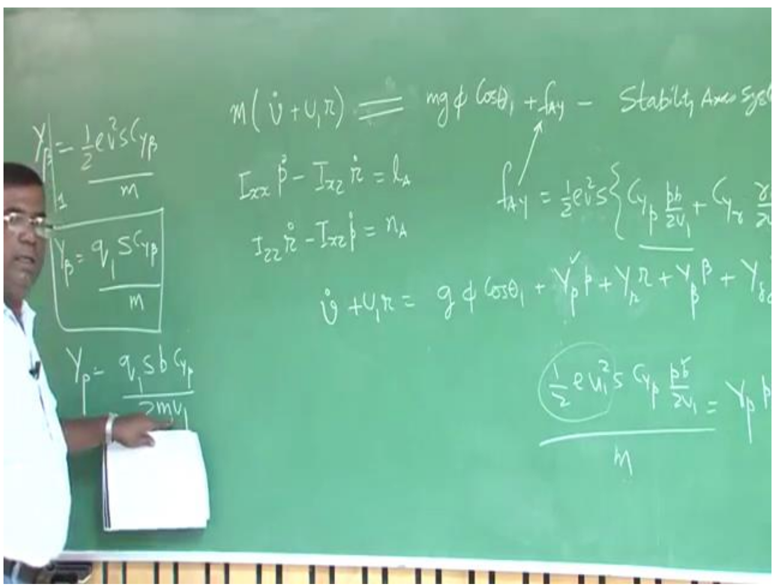

Check yourself here. If I take this term, this is half row U1 square because evaluating at steady-state, into CYP into PB by 2U1. So this will be divided by M. Because I am writing in terms of V dot. This is V dot is here. So M divides on both sides. So what will happen here is I can see this, I can write as YP into P. So what is YP? Half row U1 square which is Q1. Q1 is here, S is here.

This B remains here. BCY is there. This 2 is here, this 2 is here. M is here, U1 is here, U1 is here. Okay, right? So that becomes your YP. Similarly you can check other dimensional derivatives and if you do the similar exercise for lateral and directional case, then you will get equation of the form let me write this complete.

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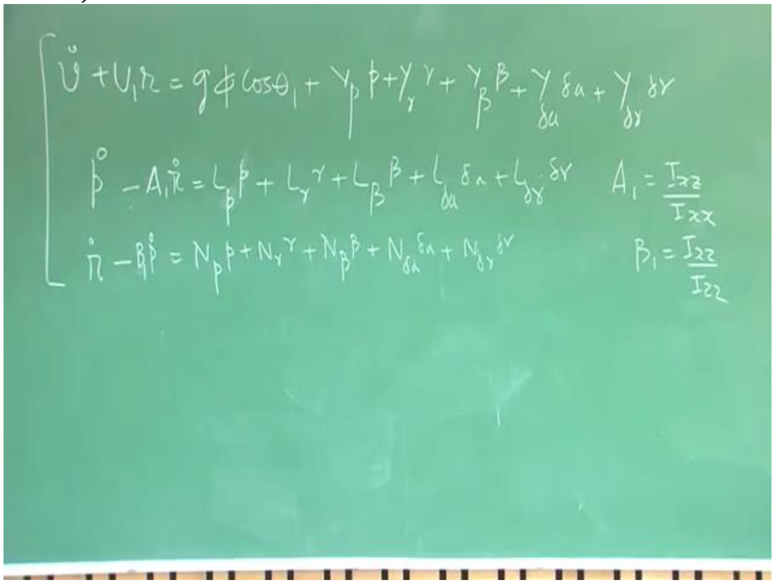

V dot + U1R is equal to G Phi Cos Theta 1 + YP into P + YR and R + Y beta into beta + Y Delta A into Delta A + Y Delta R into Delta R. Similarly from the second equation, you will get P dot minus A1R dot equal to L beta into beta or relate me write LP into P, same sequence. LR into R + L beta into beta + L Delta A into Delta A + L Delta R into Delta R. And capital A1 is nothing but IXZ by IXX. Let me write B1 also here as IXZ by IZZ.

And 3rd equation we will get, R dot minus B1P dot is equal to capital NP into P + NR into R + N beta into beta + N Delta A into Delta A + N Delta R into Delta R. We will get these 3 equations and this will represent short period equation for lateral dynamic case. And we have seen that we have made the aerodynamic model fairly simple. We have not talked about beta dot. There could be Delta R dot also but we have taken a simple case.

I am sure you will be, once you know how to do this, you will be able to do this second and 3rd equations. What is important for you to know? What are the expressions for LP, LR, L beta so that you can crosscheck yourself by doing it?

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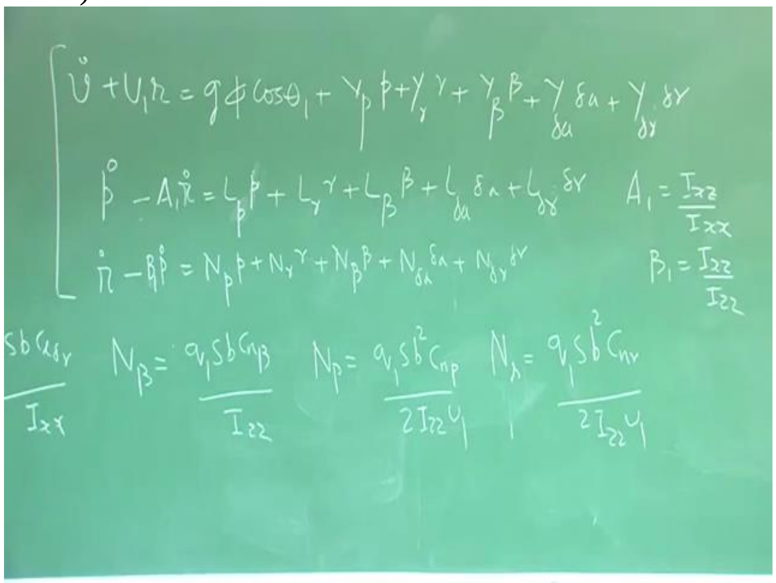

So let me write those, LP is Q1SB Square CLP by 2IXXU1. LR is Q1 SB square CLR by 2 IXXU1. Then your L Delta A will be Q1 let me write this SBCL Delta A by IXX. And then if you see L Delta R will be Q1SBCL Delta R by IXX. Then, N beta will be equal to Q1SB very simple you could see that, by IZZ. Then, NP will be Q1 SB Square CNP by 2 IZZU1. Then NR will be Q1SB Square CNR by 2IZZU1.

You can find out Ñ Delta, etc. what I will do? I will be giving page on whole of this expression. You need not remember this. Even, you need not remember these equations. I am again and again telling you. These are standard. Once you have to do to make clear-cut understanding, what are these things. So now once you have this, what is the next step?

Then next step is, this is an time domain. We understand, if I solve this in time domain, I should be able to track the values of small V, R, P to comment on the dynamic stability of the machine. But we will be following the laplace transform approach, we will work in the frequency domain in the S plane so that this differential equation becomes linear algebraic equation. So that will be our next step.

And then we will find matrix, then we will find the characteristic roots as, similar way as we have done for longitudinal case. So I would like to stop here so that you at least digest this. Please remember one thing, no need to remember, no need to cram anything here. Only understand, if somebody tells you N beta, what should come to your mind? CN beta what is CN beta? CN beta is directional stability.

CN beta is greater than 0. NP. You should think what is CNP? NR. You should think of CNR. All these derivatives, I have explained you the sign and etc in the last lecture. That is more important.

As a designer, you should have that in your mind that you know if you want to increase beta, how to increase CN beta? Whether I increase vertical tail, whether I take the CG backward or I put something with the wing or make the fuselage a little differently. That is what is required from this expression. Okay, thank you very much.