

Aircraft Dynamic Stability & Design of Stability Augmentation System
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Module 5
Lecture No 25
Lateral, Directional Stability Derivatives

In the last lecture, we completed longitudinal case. We were trying to see how to assess dynamic stability of an airplane for longitudinal case. And today we will make a beginning for lateral, directional case.

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Long-Case

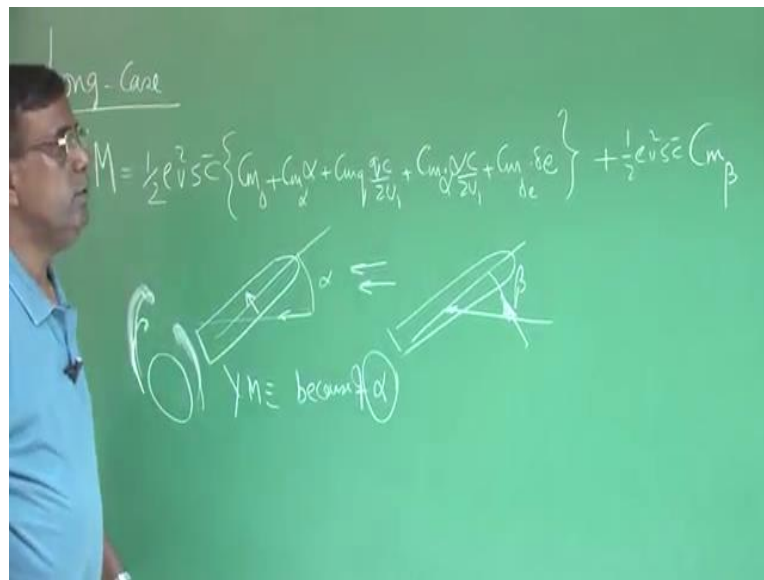
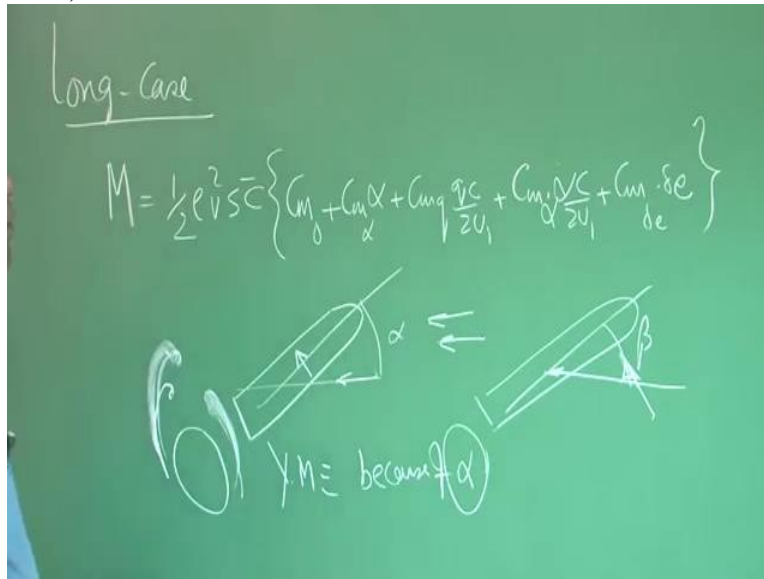
$$M = \frac{1}{2} \rho V^2 S \bar{c} \left\{ C_{m_0} + C_{m_\alpha} \alpha + C_{m_q} \frac{qC}{2U_1} + C_{m_{\dot{\alpha}}} \frac{\dot{\alpha}C}{2U_1} + C_{m_{\delta_e}} \delta_e \right\}$$

Before we go to lateral-directional case, let me give some passing remark on longitudinal case. Please understand that for longitudinal case, we have assumed small angle and also the perturbations are small. And that helped us in decoupling longitudinal from lateral and directional case.

But for high-performance aircraft, the things are little more complicated. For example, for longitudinal case, when I model pitching moment I write it like this, half Rho V square SC bar CM not + CM alpha into alpha + CMQ into QC by 2U1 + CM alpha dot into alpha dot C by 2U1 + CM Delta E into Delta E.

But you will find, if the angle of attack is much higher and which violates our small angle approximation, then there are additional terms that have to be incorporated. Also if the rates are very fast, again we have to incorporate loss of term which are predominant by unsteady effect.

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Just to give you one example before icon put. If I take a cylinder and I go on increasing the angle of attack, let us say flow is coming like this. What we will see is, there will be vertices because air is coming like this, there is one component in this direction and they flow and they will create an asymmetry vertices.

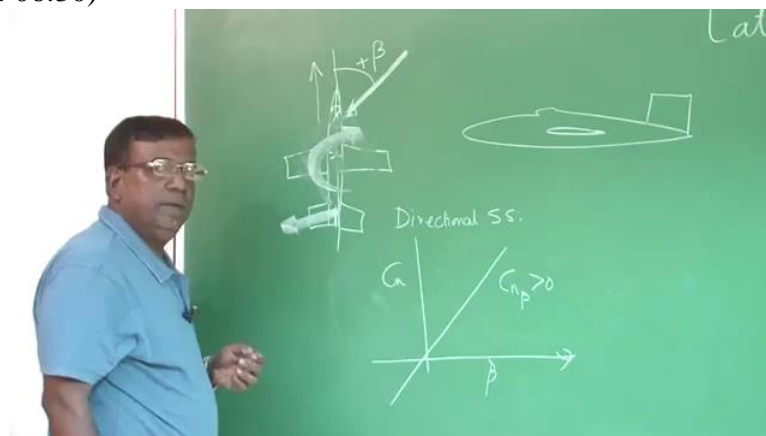
One could be like this, one could be like this and this will create a side force and which will again create a yawing moment. So this yawing moment for high angle of attack, this yawing moment is actually coming because of angle of attack which is pretty high. So in yawing moment when you would be modeling yawing moment, we have to see that for high-performance aircraft, the yawing moment will become function of angle of attack.

Similarly if we take Beta, that is, this is the body and when is coming from here, side wind and let us say this is beta. If this beta is large, then because of beta, there will be pitching moment and in such cases we have to add additional term, I say half Rho V square SC bar into CM due to beta. So there is a coupling. You could see, there is a coupling between directional and longitudinal cases.

Similarly there will be terms which are dependent on the rates, on the history of how the flow has built up because of high oscillations. So, whole of this aerodynamic model becomes highly nonlinear. However, so for whatever we have developed, we have assumed things are linear. So that becomes a challenge for analysing dynamic stability by using this method.

Yes, if you can linearise this judiciously then this can be easily applied. But this is an area which you need to understand and keep in the back of your mind. So we come back to lateral-directional case.

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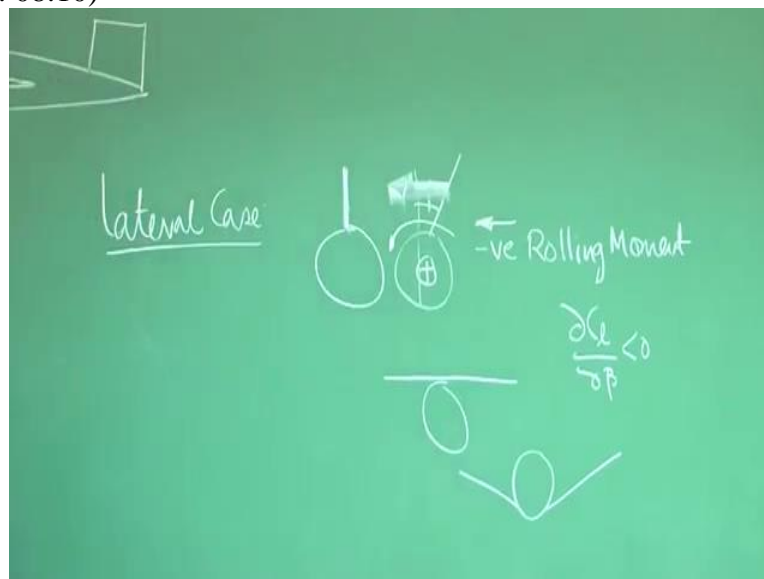
Before we start, we go into dynamic stability, we revise few things. If this is my airplane, and how do we define beta? Beta was, this is the airplane, I am seeing the top view. I am applying in this direction and this relative wind is coming from my right. If I am the pilot sitting here, then this is positive beta.

And what was the condition for directional stability? Directional static stability, condition was if I plot C_N versus β then $C_N \beta$ should be greater than 0. That is, $C_N \beta$ greater than 0. So how do I understand that? If there is a β , the vertical tail will generate a force in this direction which will give moment about CG.

Which moment? Yawing moment. And what is the sign convention for a yawing moment? We know that if I am flying like this, right-wing going back is yawing moment positive and right-wing going down is roll moment positive. In this case we are talking about yawing moment.

So for a positive β , if he wants to counter this β , if he has to have a tendency to make β 0, then he has to turn towards right or generate a positive yawing moment and that is how $C_N \beta$ greater than 0 was a condition for directional static stability. And primary contributor was vertical tail or vertical tail. This we know.

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For lateral case, understanding is this. Suppose I am looking from the back and is my vertical tail and because of some reason it has developed a bank. I am going like this, banked disturbance has come. So if it is statically stable in lateral mode, then it should automatically generate moment which will try to give a negative rolling moment. So for a positive bank, negative rolling moment.

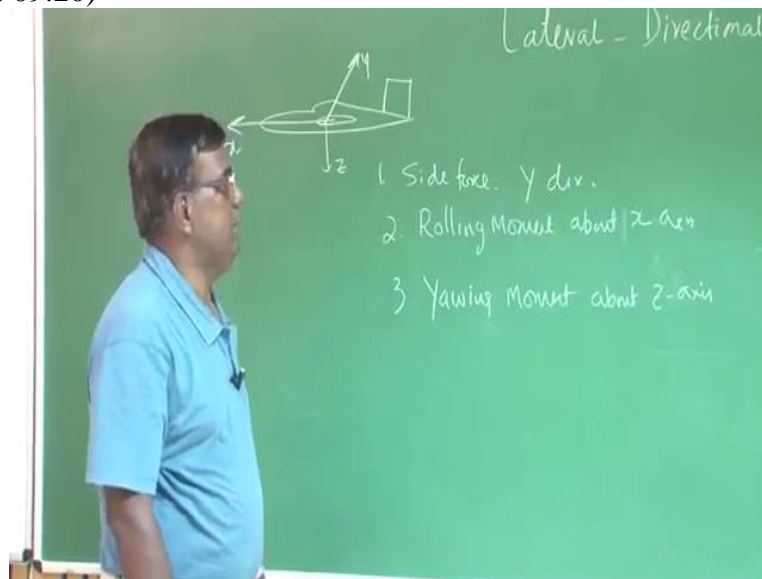
So how do we model this? What is the mechanism? Mechanism is if the airplane is like this, if there is a bank, it will start sidestepping. As it side slips, there is a vertical tail which will give a

force and that will give moment. That is, if this is having a bank, it will start side slipping. So relative air will be like this. This will give a force on the vertical tail.

That will give a moment about the Central line which is in negative moment or negative rolling moment. And in non-dimensional form, we said, DCL by D beta less than 0. We have seen that I can have lateral stability through vertical fin or vertical tail. Also we can have through high wing or by giving a dihedral. All these things we have covered in the earlier course.

I just thought I will revisit this. Now we are talking about dynamic stability. We are talking about the transition.

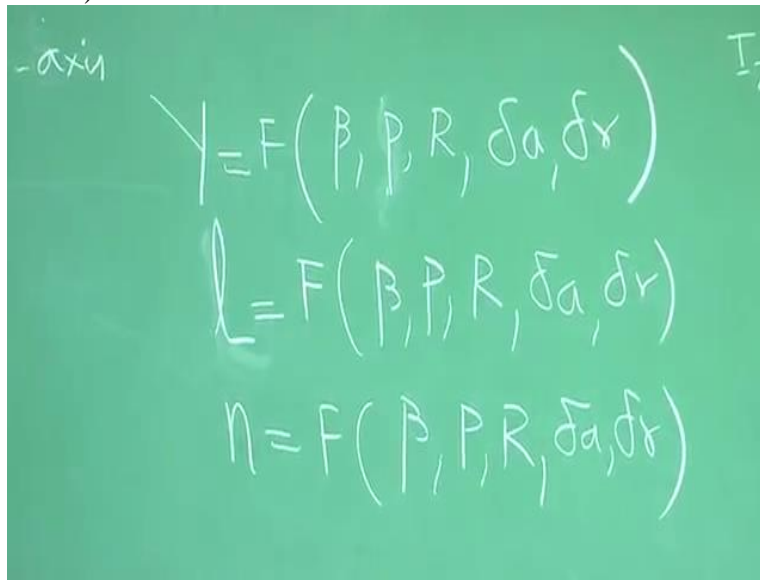
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So it is important that we understand what are the forces and moments we will be primary concerned. If this is X, this is Y, this is Z, one will be side force that is a long Y direction. Second will be rolling moment that is about X axis and third will be yawing moment about Z axis. And we know the sign convention.

As far as rolling moment is concerned, right-wing going down when I am flying like this is positive. Right-wing going back, when I am flying like this is even moment positive. So the next question comes, how do I functionalise this side force, rolling moment and yawing moment, in terms of control variable and motion variable.

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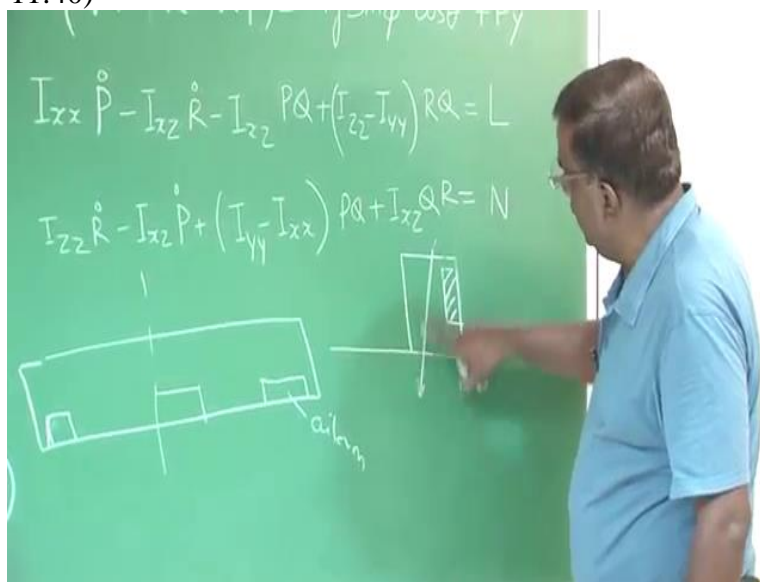
$$Y = F(\beta, P, R, \delta_a, \delta_r)$$
$$L = F(\beta, P, R, \delta_a, \delta_r)$$
$$N = F(\beta, P, R, \delta_a, \delta_r)$$

There are some faint markings on the board: '-axi' in the top left and 'Iz' in the top right.

So if I write Y as side force and it is a function of β , let us say roll rate, yaw rate, aileron deflection and rudder deflection. We are talking about small angle and small perturbation. Similarly, if we assume this is K as a good approximation, rolling moment, I say L . Please do not get confused. L is also used as lift. But we are talking about rolling moment.

So let me write small L . So again I will have, rolling moment is function of β , roll rate, yaw rate, aileron deflection, rudder deflection. And yawing moment N again function of β , P , R , ΔA , ΔR . Do understand what is ΔA ? ΔA was aileron deflection.

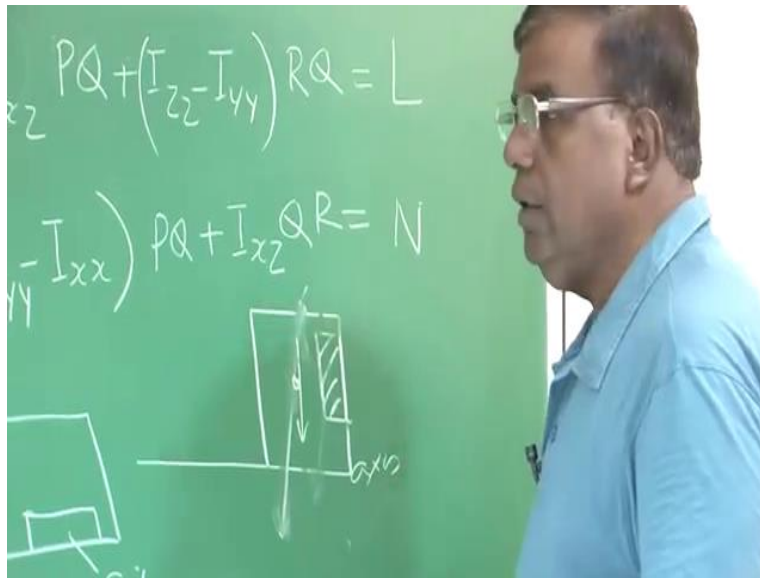
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A greenboard with handwritten equations and a diagram. The equations are:

$$I_{xx} \dot{P} - I_{xz} \dot{R} - I_{zz} P Q + (I_{zz} - I_{yy}) R Q = L$$
$$I_{zz} \dot{R} - I_{xz} \dot{P} + (I_{yy} - I_{xx}) P Q + I_{xz} Q R = N$$

Below the equations is a diagram of an aircraft with ailerons. A man in a light blue shirt is pointing at the diagram.

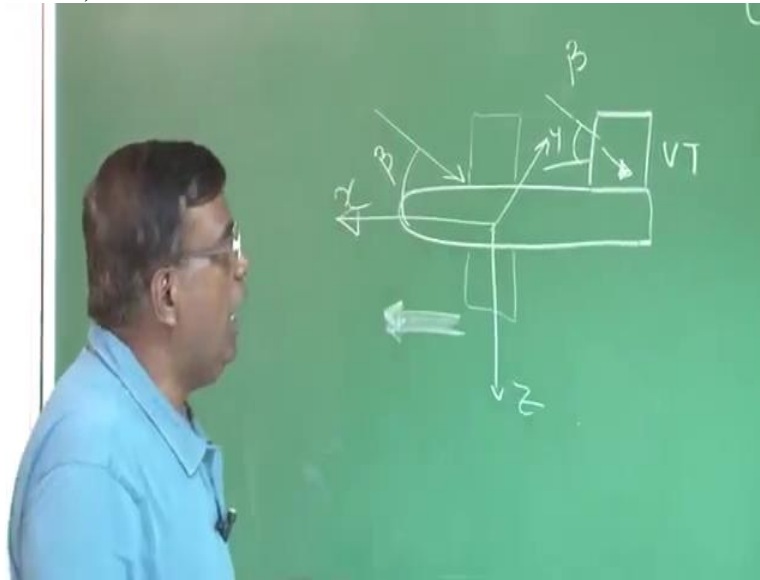


If this is my wing, here this is the flap and here, this is aileron. By deflecting the on, I can generate rolling moment. That is why Delta A. What is important? Also to understand that if I have a vertical tail like this and this is the rudder, if I deflect rudder, this also can generate rolling moment.

Is not it? Because if I deflect the rudder, and if the force is coming out of the black board, like this then this will also give a rolling moment about this axis. Okay? Just to make you more clear, if this is the fuselage and this is the vertical tail, then if I deflect the rudder like this, then as it is moving forward, it will experience a force towards me in this direction and this force into this distance will give a rolling moment.

If you see like this, if I deflect it like this and moving here, force will be towards me. So this force into this distance will give me rolling moment. So rudder also will generate rolling moment in addition to yawing moment. So now let us first check whether side force will depend upon beta, roll rate, yaw rate or not. This is important before we go forward to transient analysis of lateral and directional case.

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I will take simple simple examples. If I say this is the fuselage and this is the vertical tail, if there is a beta, wind is coming from this side, relative air is coming from the side. I am moving actually towards this. Then yes indeed, this gentleman at the vertical tail also will see beta, sideslip angle. We will see that soon.

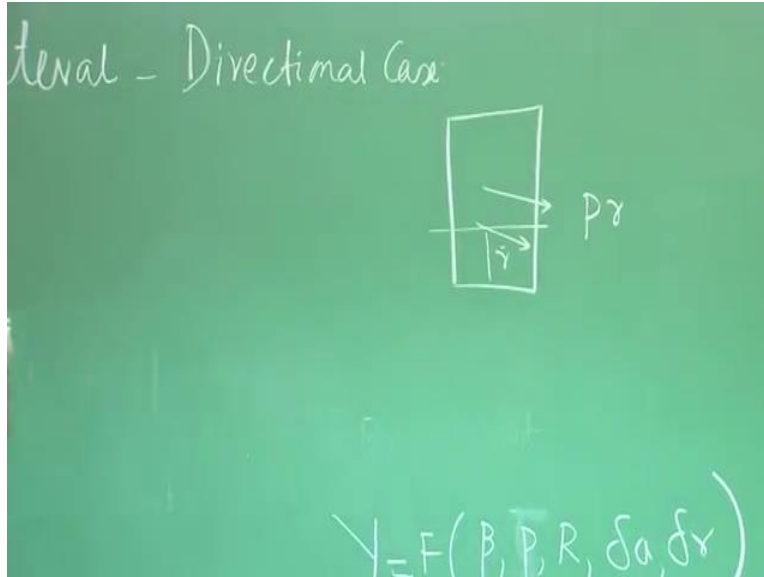
As soon as I put a wing here it will indeed see some angle beta but it will be different than beta in terms of magnitude because of side wash. It is a similar thing as it happened for angle of attack at the horizontal tail. That was different than the angle of attack seen at the wing because of downwash created by the wing. Similarly here, instead of down wash, it will be side wash.

We will discuss about those. But whatever beta comes, that will give a force this side and as far as were axis system is concerned, if it is X, this is why, and this is Z, then you will find there will be for a positive beta, the force along Y direction will be negative by this convention. That is if you see this is X, towards you is Y and this is Z, if there is a beta like this, the force is in this direction and Y is in this direction, so for a positive beta, the side force will have a negative sign.

This is because of our axis system. But there is no doubt about it. If there is a beta, vertical tail will give you side force. Fuselage will also give you side force and depending upon what type of wing, they also contribute. However predominant will be the vertical tail. What about P? Will it be function of P? That is the second question.

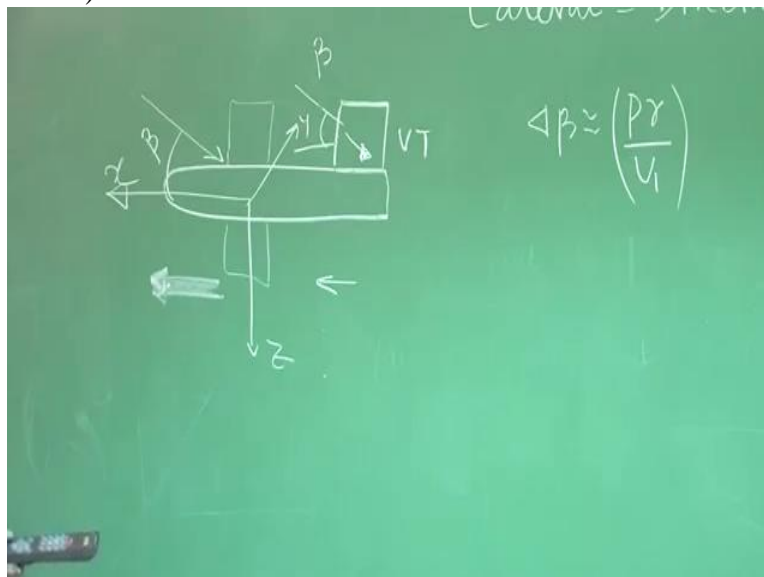
Now you see, if I rotate this body about X axis. Let me demonstrate it here. We are examining whether there will be a side force because of roll rate, that is rotation about X axis. You can see, if you rotate about X axis, this vertical tail will see a relative air which is P into this distance.

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For example, if this is the vertical tail, I am denoting with P . So at any distance R , there will be a relative air speed of which is equal to P into R .

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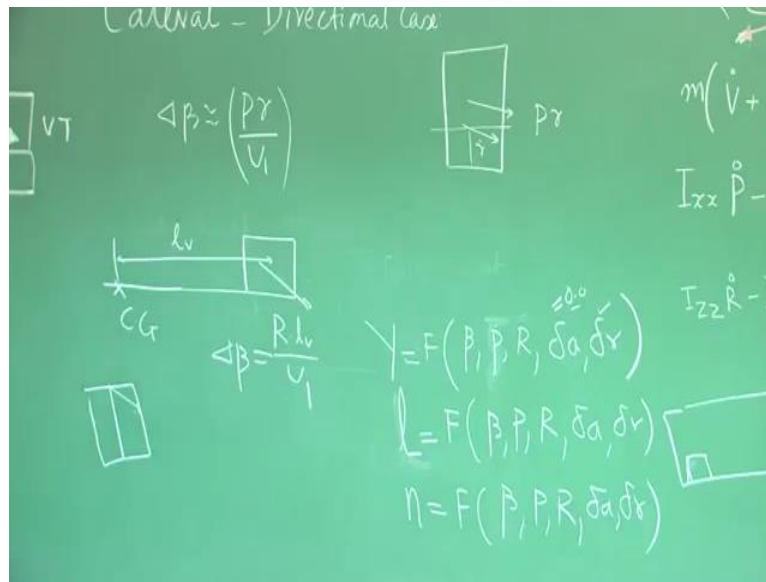


And that in addition with, it is moving with a velocity V . That will give you roughly a beta which is equal to PR by U . I am saying, with U , it is moving forward. The moment there is a

beta which is because of P, this beta will give a side force. Hence, there has to be a side force because of P.

This part is clear? As I rotate it like this, there is a PR, relative air speed, moving forward. So there is a beta and this will give a side force in this direction. So it is correct to assume that because of P, there will be a side force. What about R? If I see like this, R is yaw rate. The tail I have to rotate about Z axis, rotate like this, positive R.

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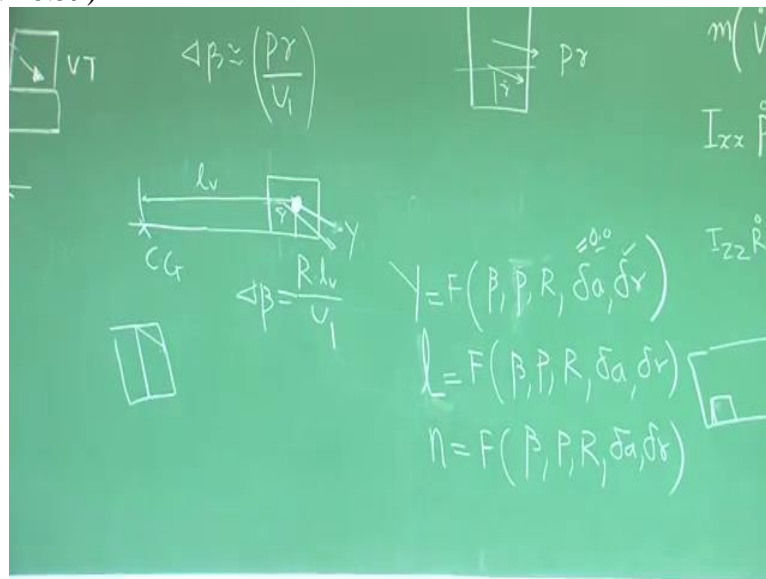


As I rotate like this, you could see, this vertical tail will be seeing relative air speed which is equal to if I draw it like this and I say, this is CG and this length is let us say LV then there will be relative air speed, R into LV divided by U1. Again, that will be Delta beta.

So since there is a Delta beta and you know because of this, there will be a side force. About Delta R, you should not have any confusion. If it is a rudder, if I deflect the rudder for example towards this direction, then there will be a force into the board or if you see this, if this is the vertical tail and rudder is positively deflected towards left.

Can you see this? Straight towards left is Delta R positive. As I am moving forward, there will be a force in this direction. And this is X, this is Y, this is Z. So positive Delta R will cause a side force in the positive Y direction.

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So the point is, Delta R will also cause a force along Y. For time being, Delta A, we will assume that it will hardly cause any force in the Y direction. That is the 1st approximation. Now you could see, this rolling moment and yawing moment are because of this side force, Y. Primarily if there is a force, there will be a rolling moment, there will be a yawing moment. For example central pressure of force is acting at this point, then this force into this LV distance will be the yawing moment contribution.

Let us say this is Y. Y into LV will be contributing towards yawing moment and Y into this distance, let us say R bar will contribute towards rolling moment. So once Y is function of this, automatically rolling moment and yawing moment also gets function of all those motion and control variables. With a difference that for side force, we are telling, aileron deflection will not cause much of a side force.

This is a very first approximation. However for rolling moment, Delta A aileron deflection is very dominating. It creates the active rolling moment. We use as a control. Similarly for yawing moment, Delta R, rudder creates the active yawing moment and we use rudder for directional control. Please understand, in flying, we will be using all these in combination. Okay?