Aircraft Dynamic Stability & Design of Stability Augmentation System
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Module 4
Lecture No 21
Routh's Criteria and Longitudinal Dynamic Stability

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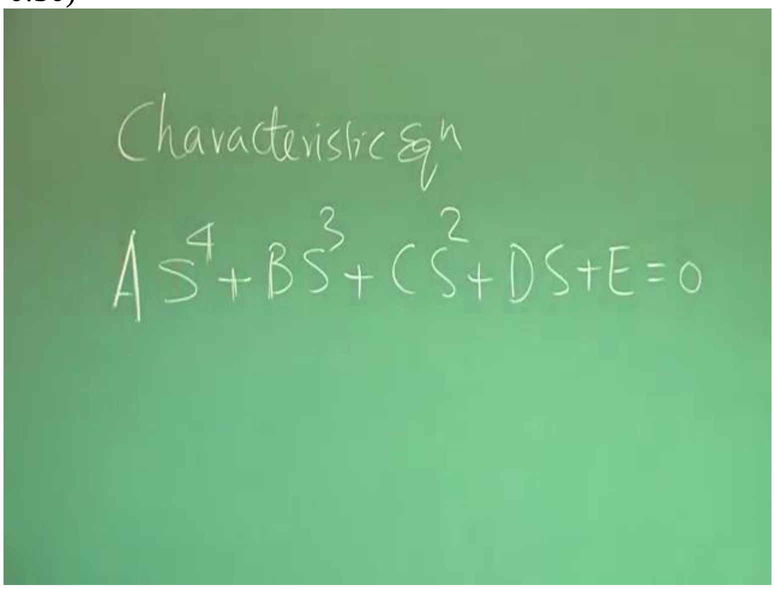

We were discussing about dynamic stability and we have seen the characteristic equation for longitudinal dynamics which was evolved assuming small perturbation and we got equation of the form AS4 + BS cube + CS square + DS + E equal to 0. And how did we get this?

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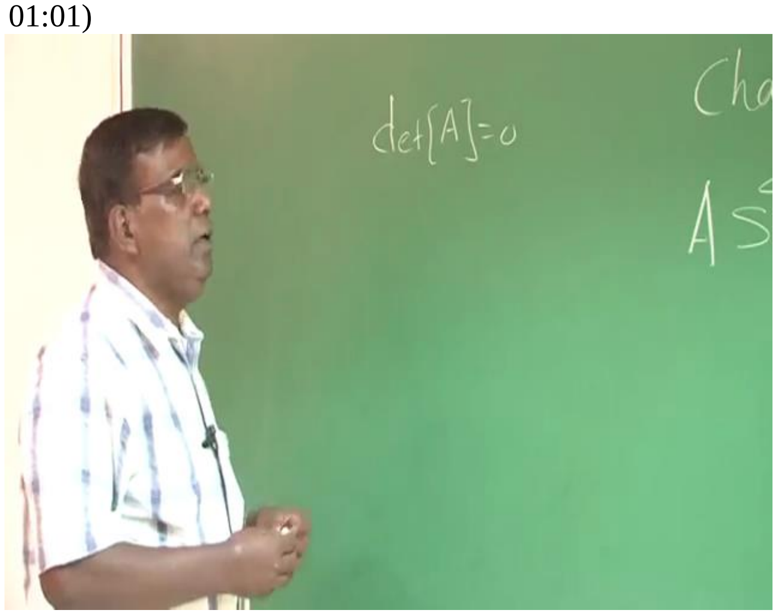

We know that we have to put determinant A equal to 0. And AX equal to B was the matrix equation. And this A, B, C, D, E, they are expanded using the dimensional derivatives, inertia properties and once you see the exact expression, you will realise each one of these have some importance. Specially we will be focusing on E. Before we go to the next part of our lecture, let us understand what are we doing?

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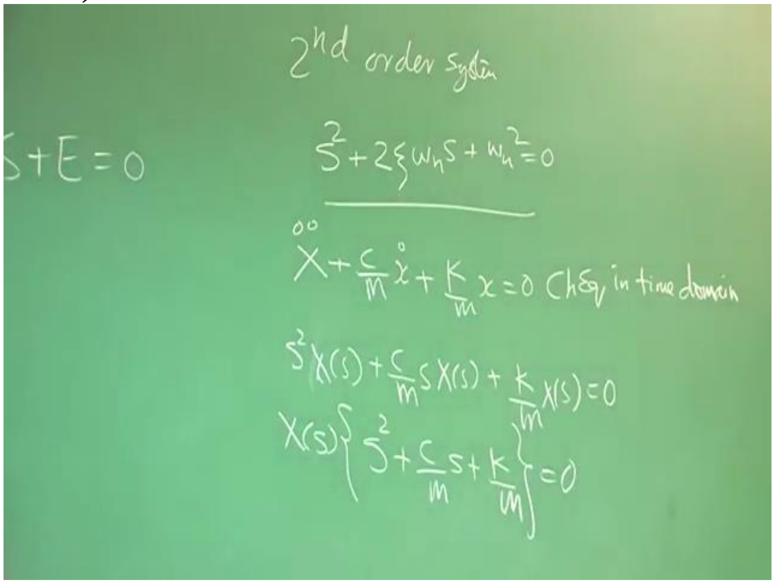

If I take you back to second order system, mass spring damper system, remember we had equation of the form S square + 2 Zeta omega N S + omega N square equal to 0. And we wanted to find the damping ratio and natural frequency. If I go one step backwards, if you remember the equation was of the form X double dot + C by MX dot + K by MX equal to 0 was the characteristic equation in time domain.

And then we took Laplace transform. And then it becomes S square X of S + C by M SX of S + K by MX of S equal to 0. So this is in frequency domain. We have applied Laplace transform this equation. What are the changes we have seen? From differential equation, it has become algebraic equation. And then we say, I take X of S common, S square C by M S + K by M equal to 0. So what was the characteristic equation here?

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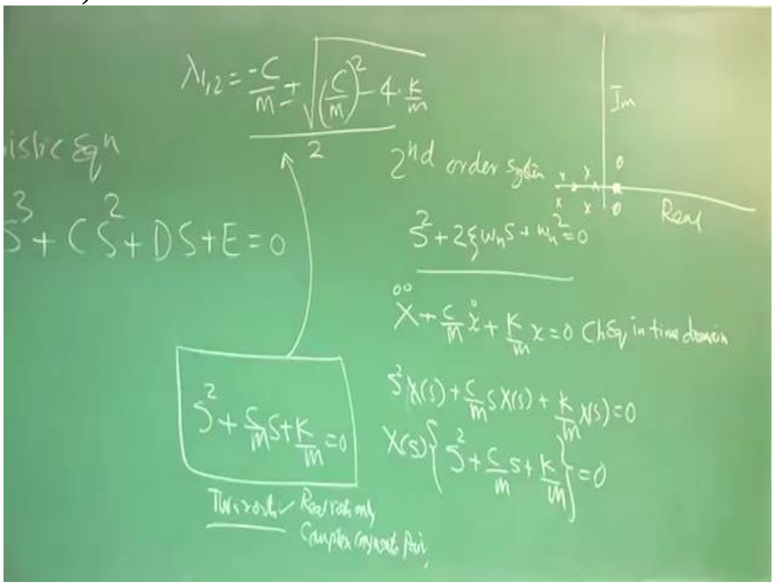

It was S square + C by MS + K by M equal to 0. This is second order equation. This is quadric, 4th order, S4 is here. And here we have realised that there will be 2 roots. We have seen that. And depending on the value of C by M, K by M, remember because if you see the roots forth from this equation, they will be lambda 1 lambda 2 will be - C but M + or - under root C by M whole square - 4 into K by M by 2. That is all.

And we knew that we have already done it but depending on the sign under this square, we will have the real roots or the imaginary roots. And we have seen, the possibilities, real roots, roots only, that could be complex conjugate pair. And also we realise that fundamentally what we want, this root if I plot in real and imaginary axis, then I want the root should be somewhere here or somewhere here.

Because I want the real part of the root to be negative, then there will be a tendency for dynamic stability. If the root comes this side, then I know that the real part has become positive. So it will diverge. So that was the understanding we learnt from this. Similar thing is also here.

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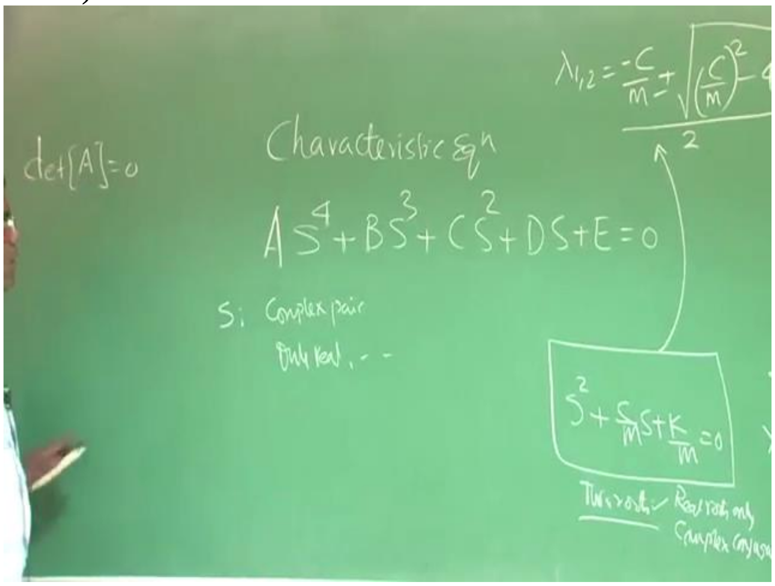

Here also, we have to solve this equation and find the values of S. And S could be complex pair, it could be only real. There are so many combinations possible. And to ensure that the aircraft is dynamically stable in longitudinal, I have to ensure that the real part of the roots whatever is coming by solving this, that should be in the negative X axis here.

But you can understand, as far as understanding is concerned, this is fine. But to handle it mathematically, we need some technique. And that we will be doing. Initially I will tell you in the next lecture how to handle this and the basis for this is to train you for different types of equations. One tutorial will be there and a lot of examples will be given so that you can understand how to handle such conditions.

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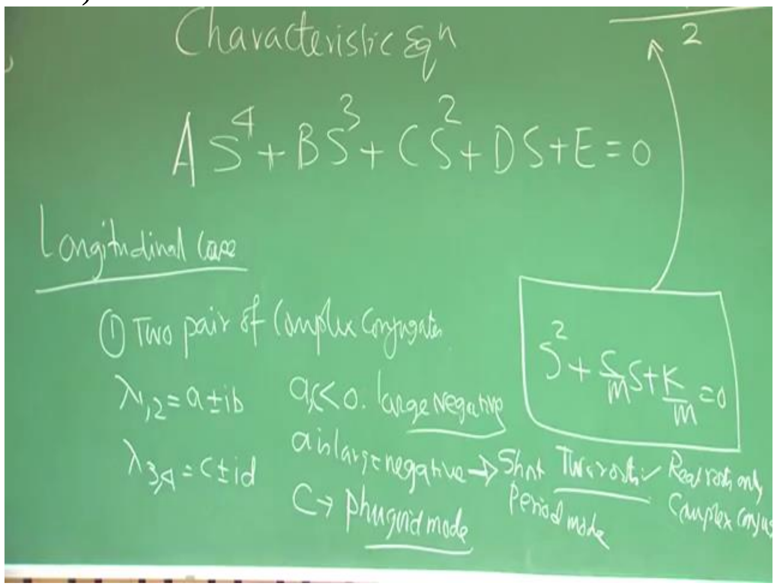

You will find that for longitudinal case, for low speed longitudinal case, and the aircraft typical business, transport airplane you will find that typically when I evaluate the roots of this equation by numerical methods, I will find that it will have 2 pairs of complex conjugates. If I write it, it will be something like this. One will be, typically like this, the 4 roots will be distributed.

And also another interesting thing for longitudinal case you will soon see that this A mostly will be negative. And if it is large negative, it will correspond to a particular type of oscillation. And if I write not only A, A and C, both will be generally less than 0 and it will be corresponding to particular type of oscillations. If A is large negative compared to C, soon you will see that this will correspond to short period mode.

And the others, the root having C, that will go to Phugoid mode. I am sure you have started thinking what is this short period mode? What is Phugoid mode? Before we go for an elaborate analysis, let us come back to this equation, let us come back to the aircraft simultaneously. The aircraft is cruising like this and if I give a small elevated disturbance, lift it up and immediately I take it out like impulse.

You will find the aircraft will just get disturbance like this and it will come back to equilibrium if it is dynamically stable. And that is what we call as a short period excitation. Now second thing, if I hold the elevator up for some time and then release it, you will find the aircraft goes like this, goes like this and then comes back to equilibrium. And that is what is the Phugoid mode.

Now for short period mode, you could understand that A, if it is large negative, that root will correspond to short period mode. The other one will correspond to Phugoid mode. So this is in general, true for most of the transport type airplane. And once you understand how to handle this, you can handle the lateral direction case which we will be doing this time. But do not forget, whatever we have learnt here, exactly same thing we are going to translate here. What we will do?

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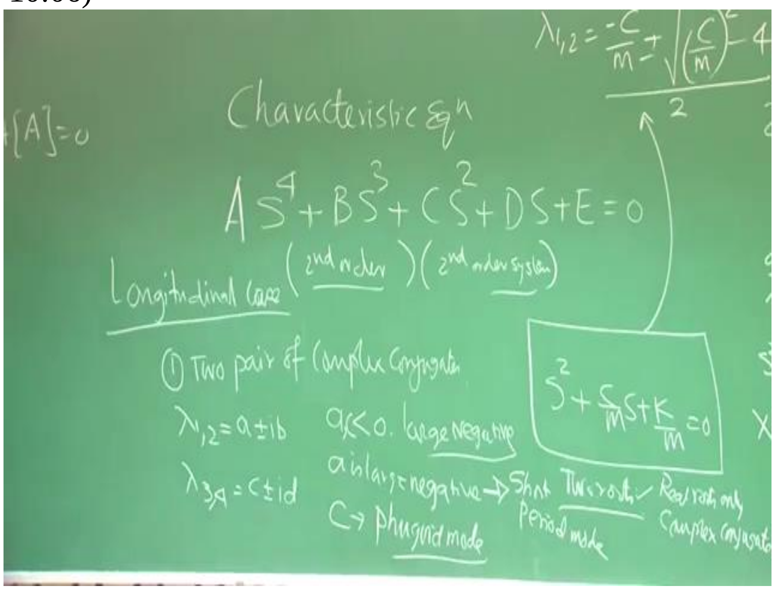

We have found that this AS4 + BS cube + CS square + DS + E equal to 0 can be equivalently written as product of two second order system and that makes our life very handy for a designer. So we can put this equation to be 0 or this equation to be 0 and find out the corresponding damping ratio and natural frequencies. We will illustrate these things through examples and through final treatment.

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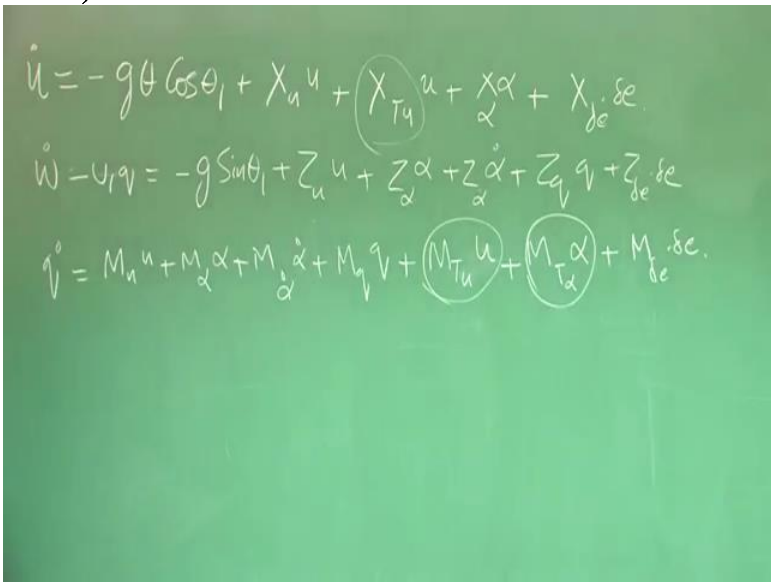

Before I go further, I like to take you back to this equation, short period equation, U not equation which is - G Theta Cos Theta 1 + XU into U + XTU into U + X alpha into alpha + X Delta E into Delta E. Similarly, W dot - U1Q is equal to - G sin Theta 1 + ZU into U + Z alpha into alpha + Z alpha dot into alpha dot + ZQ into Q + Z Delta E into Delta E.

And Q dot is equal to MU into U + M alpha into alpha + M alpha dot into alpha dot + MQ into Q + MTU into U + MT alpha into alpha + M Delta E into Delta E. Remember, throughout the analysis, we said, thrust and drag, they can be handled in a similar fashion. So we said, we are not considering this MTU, MT alpha. Now you will see that you can easily incorporate XTU, MT alpha and MTU.

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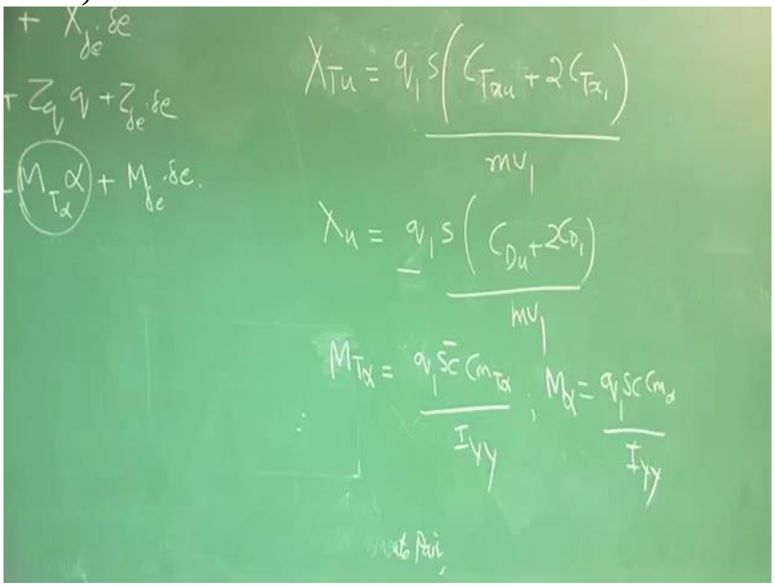

You have to only include that XTU is equal to Q1S CTXU + 2 CTX1 divided by MU1. And what was XU aerodynamic? We see, it was Q1S CDU + 2 CD 1 by MU1. - sign here. You could see that for XTU and XU, this was because of drag primarily and drag and thrust are in the opposite direction.

So this - sign is not here. But exactly the same treatment. Could you see this? Similarly if we check for MT alpha, MT alpha will be Q1SC bar CMT alpha by IYY. And you could see, corresponding aerodynamic M alpha term was Q1SC CM alpha by IYY. All these coefficients, CMT alpha, CTXU, CTX 1 can easily be estimated through analytical methods, not so accurate but generally these things are obtained through experiments, through internal testing.

But the point is, you could see here, if I know M alpha, if I have understood how to handle this, I have to just get MT alpha in the same fashion. And why MT alpha is there? We could understand also the physics behind it.

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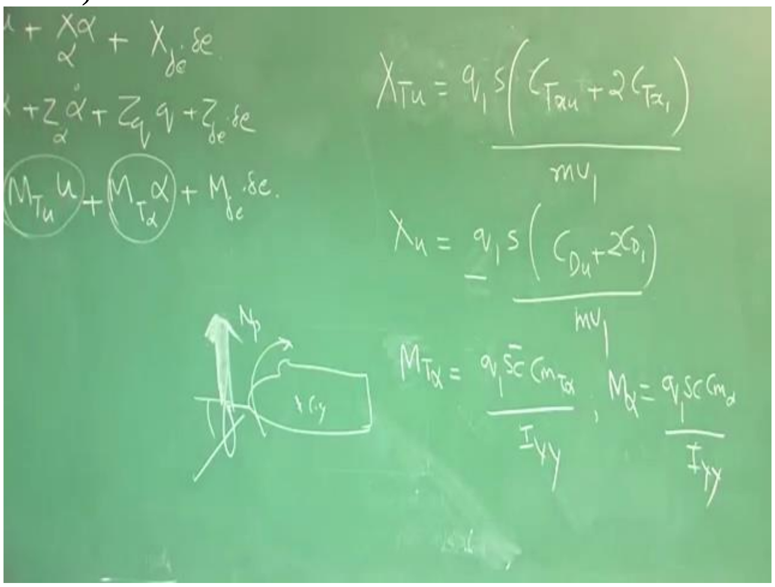

Suppose this is the airplane and let us say this is the engine or the propeller. At an alpha, there will be some normal force, NP acting like this. We know the momentum over here will be push like this. So this into the CG of the airplane will give a moment because of the thrust component. That is how the origin of MT alpha lies.

So I thought I will complete this so that you do not get mixed up when actually you work on the real field. But for further analysis, we will not be considering this as we have agreed that you know, it is straightforward if we just go on adding some terms. So I will now remove this.

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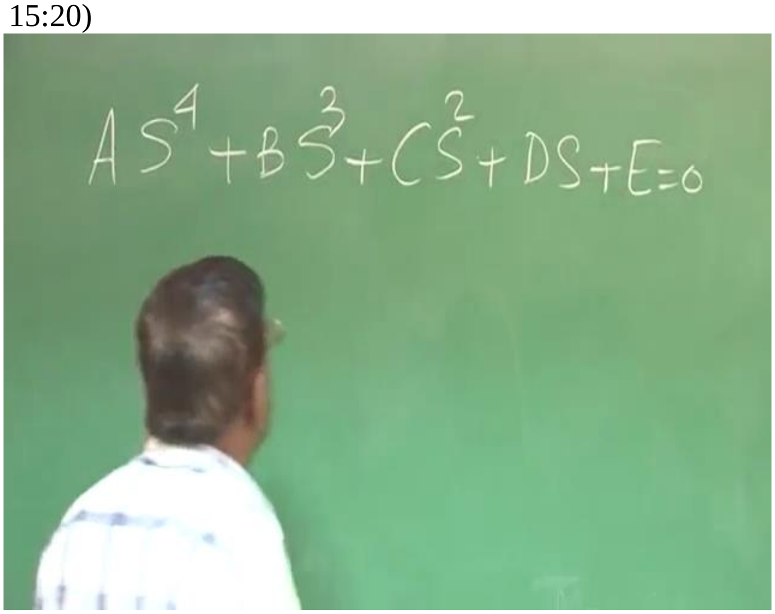

I will start with this characteristic equation of the form AS4 + BS cube + CS square + DS + E equal to 0.

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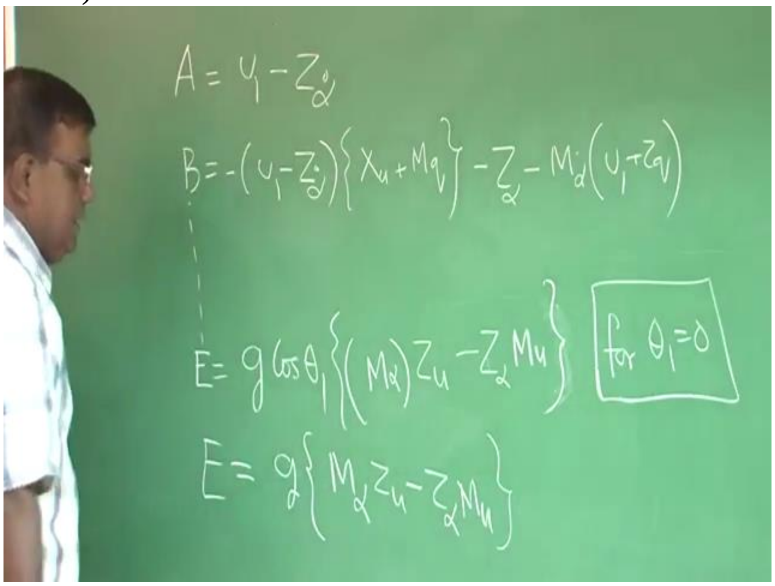

And typically, E and I will write few of this, A is U1 - Z alpha dot and B is - U1 - Z alpha dot into XU + MQ - Z alpha - M alpha dot to U1 + ZQ. This will be displayed separately. And most importantly, E I will write as G Cos Theta 1 into M alpha into Z U - Z alpha into MU. This is for Theta 1 equal to 0. Because there are additional signs, so Theta term will come which I have taken it out.

Theta 1 is 0, Sin Theta is 0. And also, I can easily simplify this as G into M alpha ZU - Z alpha MU. Why I am writing at this stage separately? You will say there are lot of information I can extract from this expression of E. I repeat again. This expression of E is not complete. There will be a sin Theta term.

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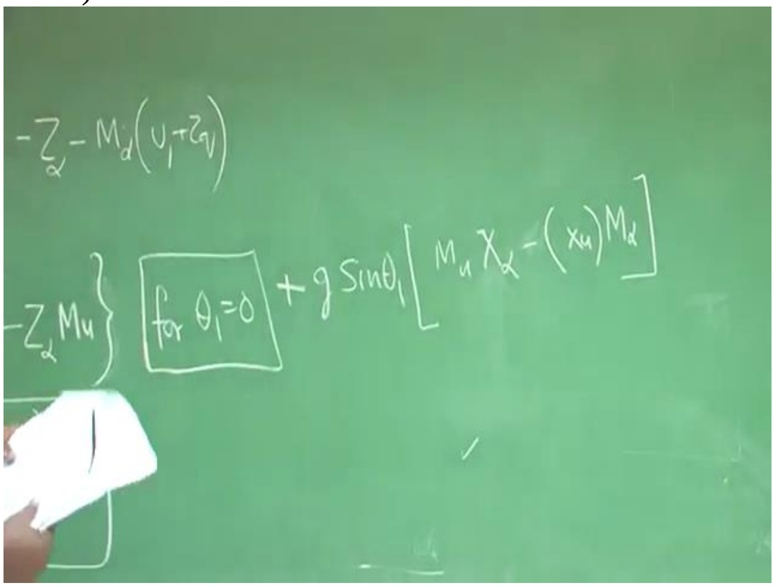

And those who are feeling a little disturbed, let me write for them. This is G sin Theta 1 into MUX alpha - XU into M alpha. Like this. But since I have put Theta 1 equal to 0, so I am eliminating this and I am also putting this value 1. So E is this. Do not worry about this expression. You know I have to put determinant A equal to 0 and find the determinant. You will get all these expressions when you compare with this sort of an equation.

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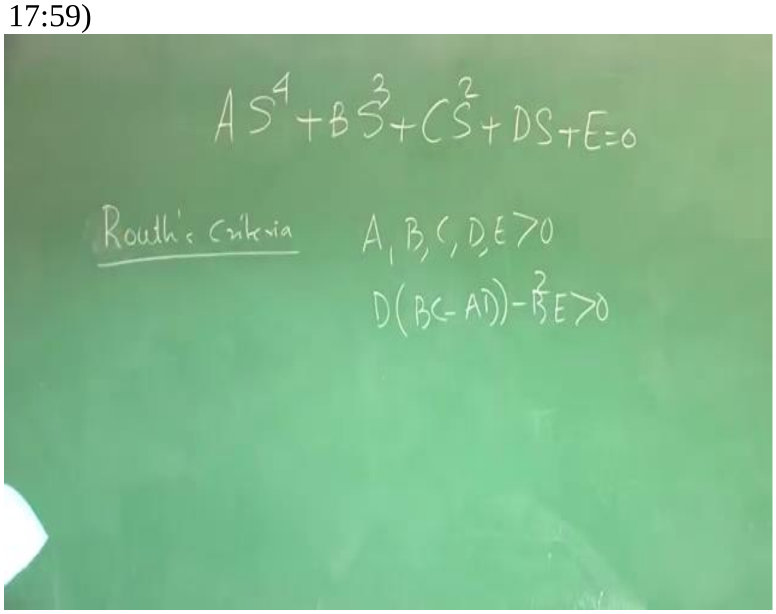

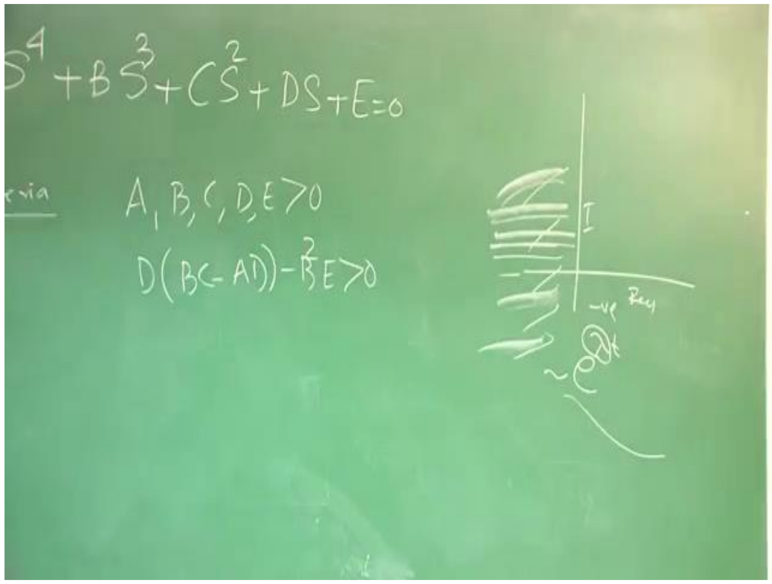

To comment on dynamic stability characteristics of a system having this type of characteristic equation, a quick estimate you can get applying the Routh's criteria and there will be one lecture where actually Routh's criteria will be explained and some numerical will be solved so that you can be handy with it. But I am just writing these conditions, A, B, C, D, E, all the coefficients of this equation have to be greater than 0.

And also D into BC - AD - B Square E should be greater than 0. If these two conditions are satisfied, then I can say that all the roots of this equation will lie in the left-hand side. That is all the roots will be here. This is imaginary, this is real. So the real part be all here. So that we will ensure dynamic stability. Because we recall from second order system also, is this something like lambda T, it goes with that and lambda is negative. Then there is a decay of the amplitude.

So this condition to be met to ensure that the aircraft is dynamically stable in longitudinally mode. What does that mean? It means, since I know A, B, C, D, E are functions of derivatives, dimensional derivatives and inertial properties, I should select those dimensional derivatives based on the configuration.

I will have to change the configuration so that finally the configuration ensures right value of, combination of values of A, B, C, D and E so that these two conditions are met and hence the aircraft will have longitudinal dynamic stability. Thank you.