Aircraft Dynamic Stability & Design of Stability Augmentation System
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Module 4
Lecture No 20
Longitudinal Characteristic Equation

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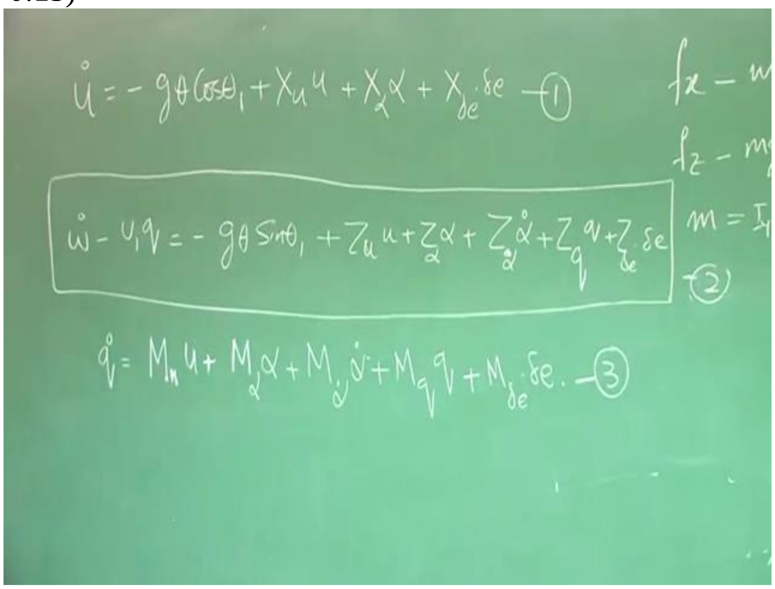

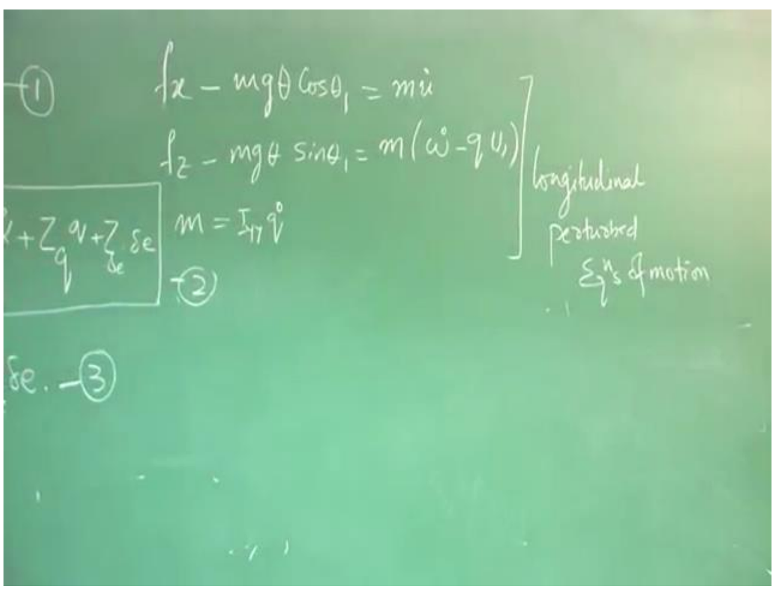

This is the first equation, this is second equation and this is third equation. Let us recall why we are doing all this adventure? Because we realise that this is longitudinal perturbed equations of motion and if I want to comment on the dynamic stability of the airplane, I need to track the value of U, W, Q, all these perturbed quantities and depending upon their responses, depending upon how they are changing, we will comment on whether the airplane is dynamically stable or not.

For example, if I give a small disturbance and U goes on increasing, that is small U goes on increasing, definitely I would know that it is not dynamically stable as far as U is concerned. Like that W and Q, same logic. To solve this equation, we wanted to have the functional form of FX, FZ and M. That is, just ask a question, how this FX, FZ and M depend on motion variable or control variable.

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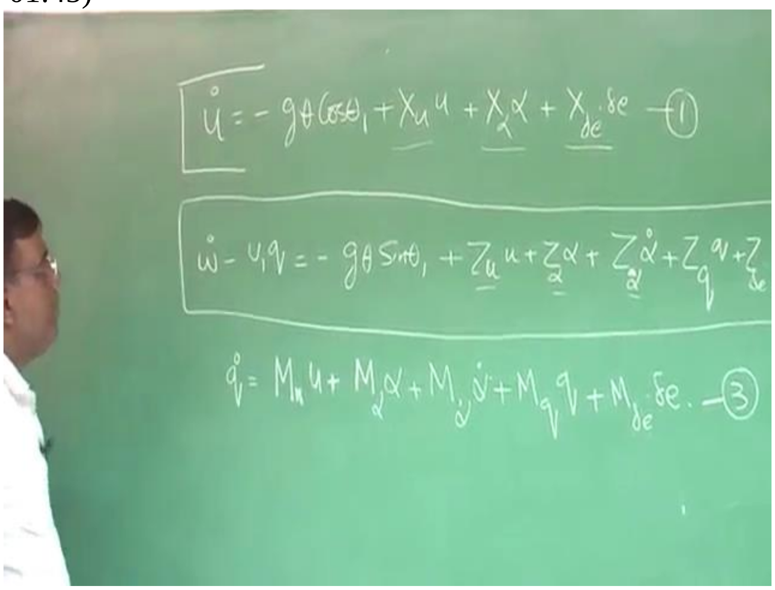

And we have seen this. These are the control variables. Once I know the values of XU, X alpha, X Delta E, all these dimensions are derivatives, I can explicitly write, FX, FZ and M, function of motion variables or control variables, and then my equation is complete or it is in the form where I can solve it. 1, 2, 3 equations I can solve. I can solve using numerical methods. There are so many standard numerical methods out there.

I can solve these equations using numerical methods and comment on whether the airplane is dynamically stable in longitudinal mode or not. But we will be following a different approach. For that, let me go back to the initial example what we have given as a mass spring damper system. Once we agree that we need to solve these equations in time domain to know how a pot of quantity, U, alpha, Q, W they are varying, and once I know that, I can comment on the dynamic stability using a time domain analysis. But we will say that we will be having a different approach.

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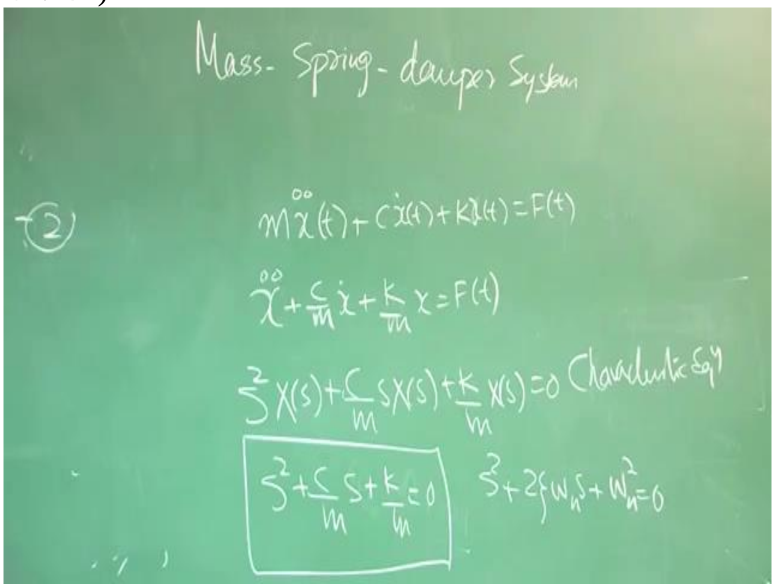

And to make that understanding clear, I will take you back to the lectures what we started initially for mass spring damper system. Remember? The question was MX double dot T + CX dot + KX equal to F of T. Right? And then what we did? We said, this can be written as X double dot + C by MX dot + KMX equal to F of T. Now what we said?

If we want to find the natural frequency and damping ratio, then what I have to do? I have to go for Laplace transforms. So we wrote it as, we took Laplace transform of this. This is SX of S + C by M SX of S + K by MX of S is equal to 0 because we are looking for characteristic equation. We put F of T equal to 0. And by doing this, we found equation S Square + C but MS + K by M equal to 0.

And then we compare this with the standard form second order system, S Square + 2 Zeta omega NS + Omega N Square equal to 0. And comparing this, we found out what is Zeta. What is the numerical value of Zeta and numerical value of the natural frequency. So you can decide the value of C and M, K and M design your system because this also gives you a condition, how to adjust value of C, K and M so that it belongs to maybe a critically damped case, maybe over damped case, maybe an under damped case.

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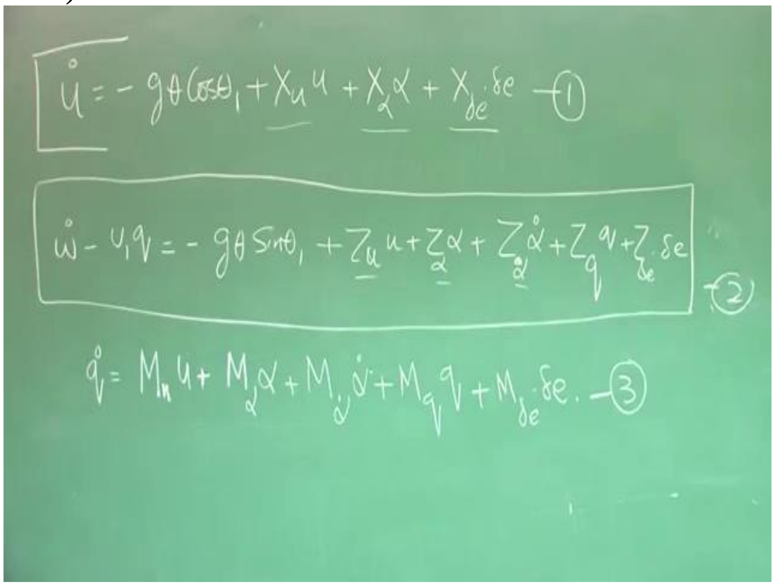

Similar approach also we will be applying here. That is to say, we will use Laplace transform and find the equation in frequency domain the way we have done it here. What is the advantage here you could see? That this was in time domain. So we need to integrate this equation to find out X of T response but once we are doing in frequency domain using Laplace transform then these become algebraic equations. So your computational effort become simpler.

And also as a designer, you need to have some expressions or some numbers which are very handy to use. If every time you need to use a computer, you cannot design an airplane. Because at the preliminary stage, you should be able to have used few well thought expressions and find out the boundaries whether the design will be feasible or whether the design will be not feasible.

From that dimension, we will be using frequency domain but we can always work in time domain. No restriction on that.

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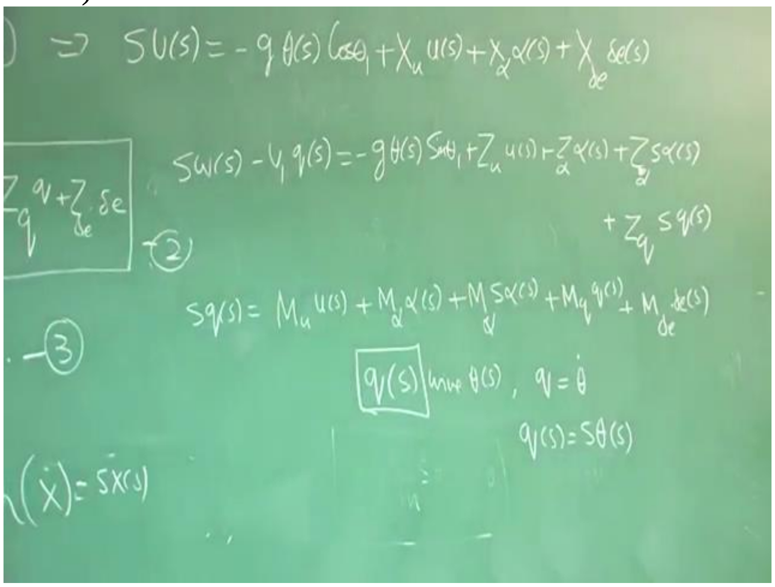

If I now take Laplace transform of these 3 equations, what do I get? first equation what I will get, just see here. I will get SU of S is equal to - G Theta of S into Cos Theta 1 + XU into U of S + X alpha into alpha of S + X Delta E into Delta E of S. Correct? Straight forward. For U dot, it is SU of S. Because we know Laplace of X dot is SX of S if the initial conditions are 0. And since we are talking about a linear system, the stability will not depend upon the initial conditions.

So we can neglect it. Similarly if I see the second equation, I will get SW of S - U1Q of S is equal to - G Theta of S sin Theta 1 + ZU into U of S + Z alpha into alpha of S + Z alpha dot S alpha of S. Please understand, alpha dot is here. So S alpha of S will come. Similarly for ZQ into SQ of S.

And for the third equation, I can write, SQ of S is equal to MU into U of S + M alpha into alpha of S + M alpha dot into S alpha of S. Again alpha dot is there. Similarly MQ into Q of S + M Delta E into Delta E of S. We will do a little bit more trick. We will try to see how can I write Q of S using Theta of S.

For small perturbation, we will assume that which is right also and specially our steady-state is a cruise. There is no bank. So there is a cruise going like this. There is no sigh. So I can always write, Q is equal to Theta dot. And hence Q of S I can write as S Theta of S.

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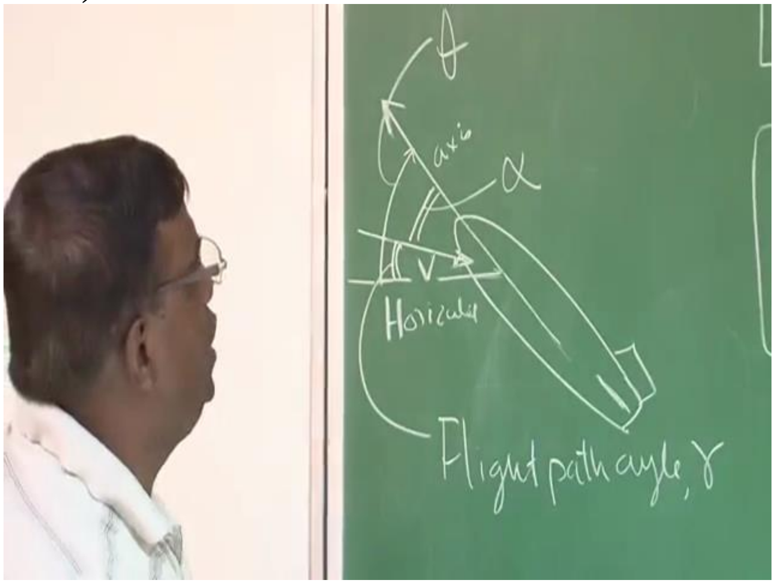

If you have forgotten, let me recall if this is my airplane, if this is my wing chord line and this is the relative air velocity, and this is the horizontal, then this angle is access, chord access to horizontal. This is Theta, pitch angle. This angle between velocity vector and the chord line, we are talking about alpha. And this angle between the velocity vector and the horizontal is called flight path angle, gamma.

So as far as Q is concerned, it is the rotation about Y axis and for a small perturbation stop and specially when we are talking about longitudinal motion, this approximation is fair. So now what do we do with this? What we will do is, wherever QS is there in this expression, say here it is there, here it is there. We will substitute Q of S by S Theta of S.

This, one term here, one term here. I substitute capital Q of S by S Theta of S. And then I write this whole expression in a matrix form which will be very simple as long as you understand this. Now the matrix form equations will look like.

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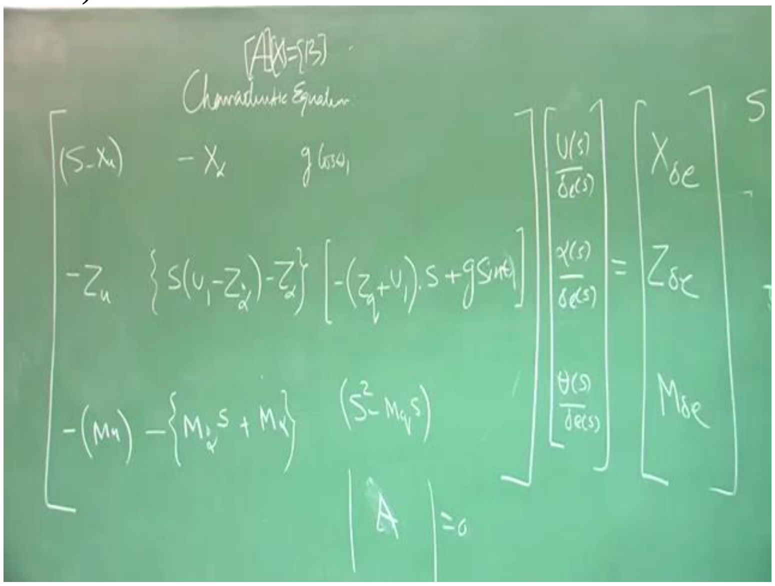

Let me write, once we write, the matter will be finished after that. S - XU, - X alpha and G Cos Theta 1. second will be - ZU, then SU1 - Z alpha dot, - Z alpha. third one is - ZQ + U1 into S + G sin Theta 1 and here, - MU - M alpha dot S + M alpha. And here S Square - MQS. This into U of S by Delta E of S, here alpha of S by Delta E of S, here, Theta of S by Delta E of S. This is equal to X Delta E, Z Delta E, M Delta E. Matter is closed.

Okay, let me check. S - XU, - X alpha and G Cos Theta 1. - ZU, SU1 - Z alpha dot, - Z alpha, right. - ZQ + U1 of S + G sin Theta 1, - MU - M alpha dot S + M alpha. This is something like this. Then this is S Square - MQS. U alpha Theta. That is fine.

You could see here, since we have used this approximation, Q of S is equal to S Theta of S, so when we have substituted it here, so now our motion variable U, alpha and Theta. Right? So this is the, in matrix form, the characteristic equation for longitudinal dynamics and if you want to find out the roots of this equation, the rule is very simple. The determinant of D will be equal to 0 where D is this matrix.

If I write it AX is equal to B then determinant of A is 0 will be the characteristic equation and you will see once you expand it. We can write this whole detriment equal to 0 in a form.

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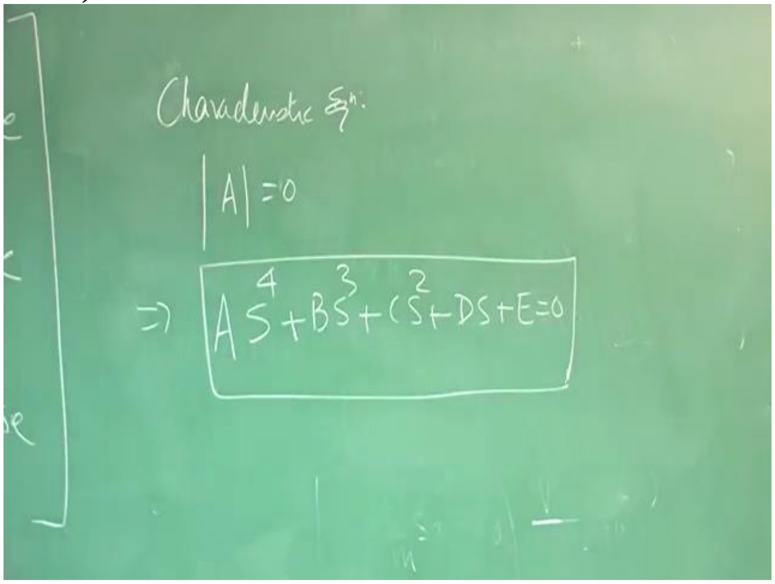

The characteristic equation to find the root will become determinant of A equal to 0 and that will imply equation of the form AS4 + BS3 + CS Square + DS + E equal to 0, where A, B, C, D, E will have the expression once you take the determinant of it which I will just display it in the next class.

But as long as you understand, this represents the longitudinal perturbed equation of motion. The characteristic equation. I need to solve this equation to comment on whether my aircraft is dynamically stable in longitudinal mode or not. What does it mean? You find the root of S and see what is its real part, what is its complex part, whether it is complex root or real root, what is their sign?

And from those, you can find out whether the air pain will be dynamically stable or not.
Basically, you need to know how to solve this equation to find the value of S. Okay, so we will stop it here. Next class, we will start from this point. Thank you very much.