Aircraft Dynamic Stability & Design of Stability Augmentation System
Professor A.K. Ghosh
Department of Aerospace Engineering
Indian Institute of Technology Kanpur
Module 1
Lecture No 02
Spring-Mass-Damper System: Underdamped

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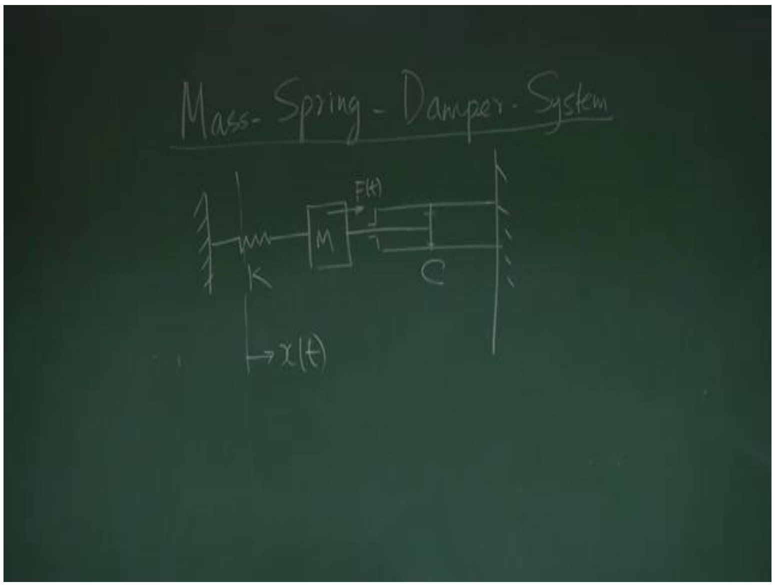

Yes, let us now understand 'Mass spring damper system'. If I draw a typical diagram, most of the textbooks, we find: this is damper and this is stiffness. Somewhere, I am measuring X of T and we understand, X of T means we are talking about the displacement or perturbation about the equilibrium. It is an extension of the mass when I give a disturbance through XT and I am measuring X with respect to equilibrium.

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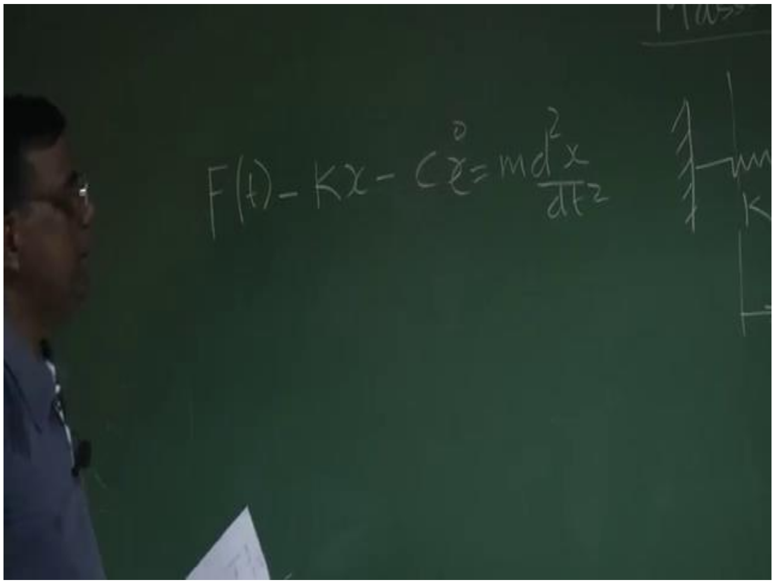

And if I write the equations, I can easily show that F of T - KX - C X dot is equal to M D Square X by D T square.Right?

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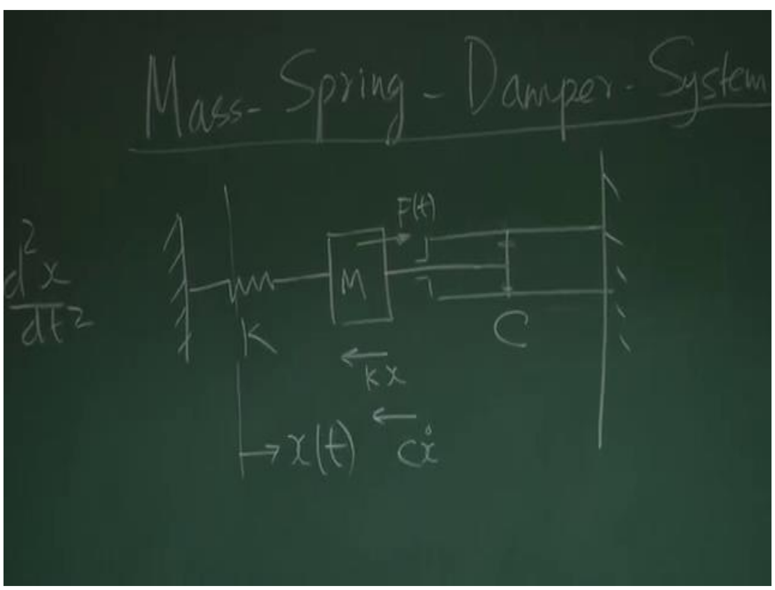

Because if I am stretching the mass like this, there will be force KX, there will be force CX dot. Now that is about linear damping. So naturally net force will be FT - this. That will give us the resultant acceleration, M D Square X by DT Square.

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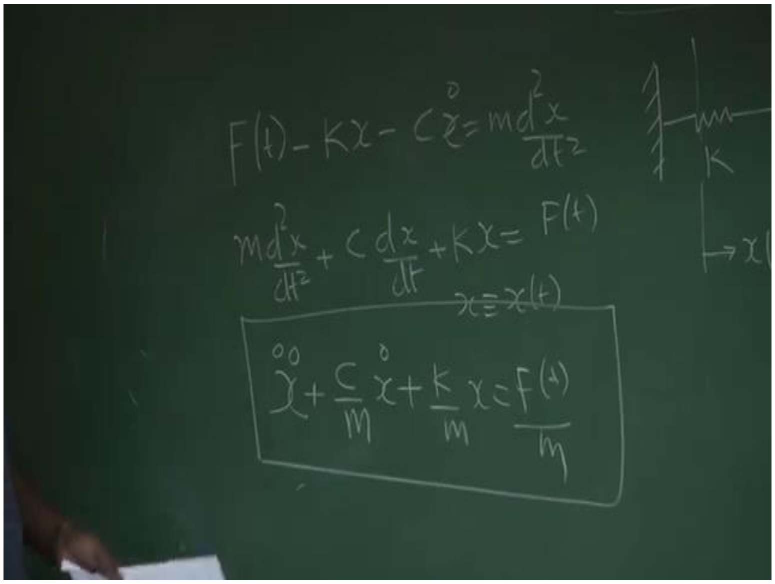

This, I can write it like this M D Square X by Square + C DX by DT + KX equal to F of T. Please understand, this X is also identically, I need to write as X of T because we are talking about response. These are perturbed points. We are trying to model its variation with respect to time. And this I can write in other notation, X double dot + C by MX dot + K by MX equal to F of T by M. This is a very popular equation which you know how to solve it once you identify the type of differential equation it is. We have decided that since we are talking about dynamic stability, please do not forget, we are talking about its response once the disturbance is withdrawn. And that is, in technical terms, we talk about free response. That is F of T is 0.

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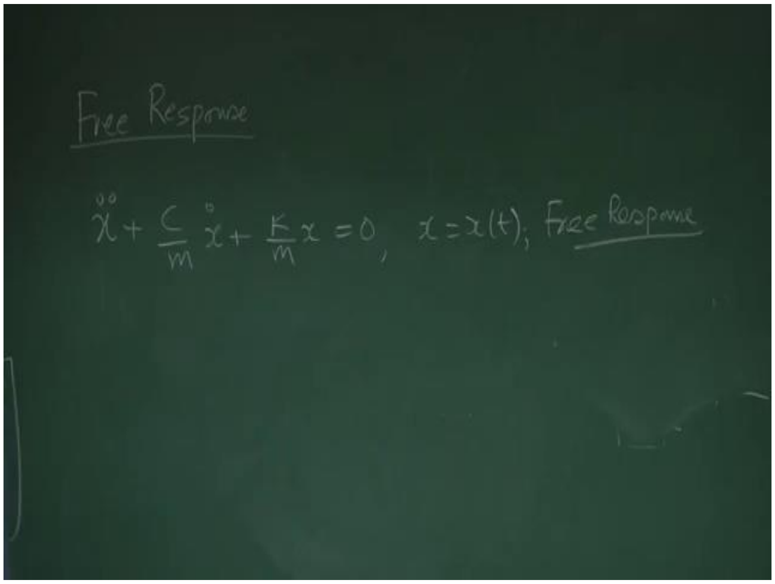

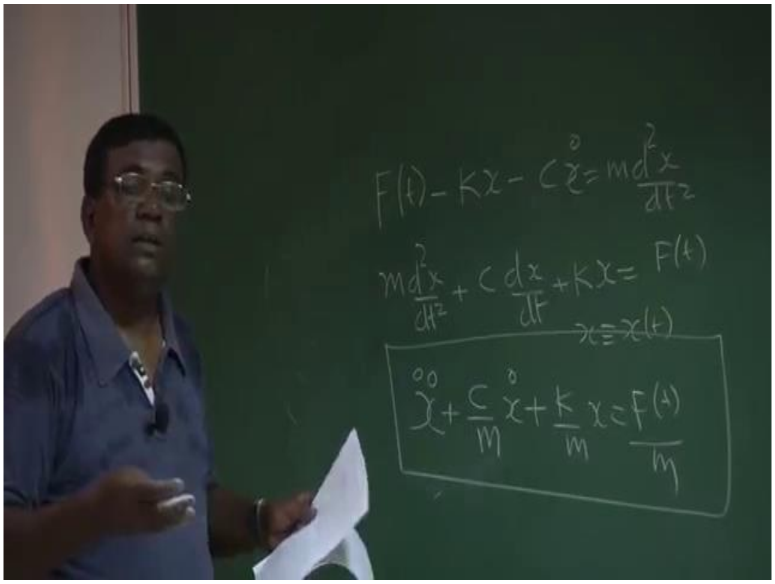

Then, for free response, if you want to really understand it, it is like spring. You have to stretch it and leave it. Now there is no force. Leave it and see how it is moving. So this motion, we are talking about free response. For an airplane, I give an elevated input and withdraw it. So I see how that plane is moving. So we are focusing on free response.

So I can write this as X double dot + C by M X dot + K by M X equal to 0, where I understand X is nothing but X of time. And this is free response. We will analyse the controlled response. That is we will not take the elevator to the null position again. We will hold the elevator and see how finite it is going. But we are talking about free response because for a linear system, if and other free response, I know how it behaves with a controlled response.

For a mathematical understanding, you know that for a complete solution of differential equation, it is a combination of complimentary solution and the particular integration. So that comes from the control part. We are talking about free response.

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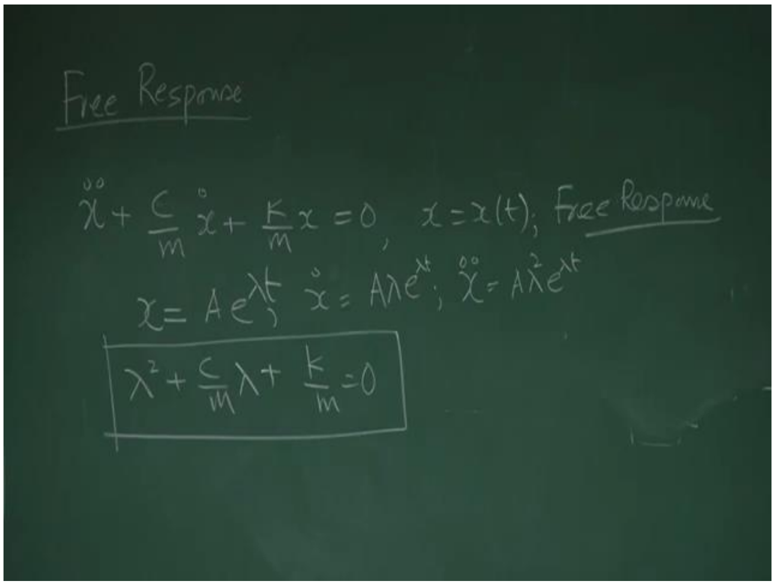

If I do this, now you know for this sort of differential equation, X equal to E the power lambda T. This is a possible solution. And if I put it here, I get lambda Square + C by M lambda + K by M equal to 0. What we have done? Put 5 from X, you find X double dot. X double dot will be nothing but, first we will find X dot, X dot will be A lambda E to the power lambda T. Then X double dot will be A lambda Square E to the power lambda T. And this expression for X, X dot, X double dot, when I substitute here, I get a characteristic equation of this form where lambda is the root of this characteristic equation.

We very well known that if lambda is positive, then this X will divert, X will go on increasing. To ensure that it decays, lambda has to be negative. But you never know what is the nature of lambda? It could be positive, it could be complex, it could be negative. So, who decides that? That is decided by the root of this equation.

And directly you will see that the value of C, M and K, they will decide whether lambda is pure negative, a complex number or if a pure positive number. What is the message? If you want to decide a particular type of transient which is through X of T here, you can easily design the system in such a way that you get the desired lambda through selective proper value of C, M and K. Okay? So this understanding is clear, so let us see what to do next.

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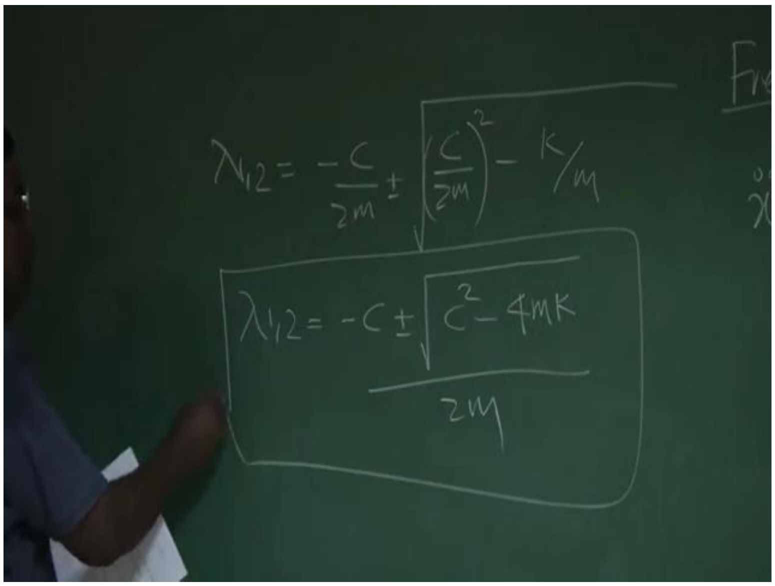

We could easily find out. We have seen the root as - C by 2M + - under root C by 2M Square which equivalently, I can write, lambda 12 as - - C + - under root C Square - 4MK by 2M. They are equivalent. I will be using this form because I will be able to extract some important understanding from this. Okay? Now see the interesting thing.

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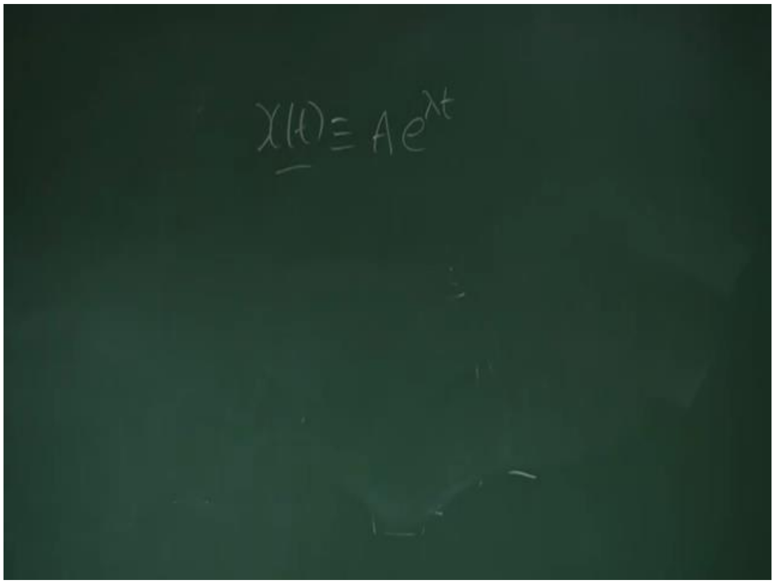

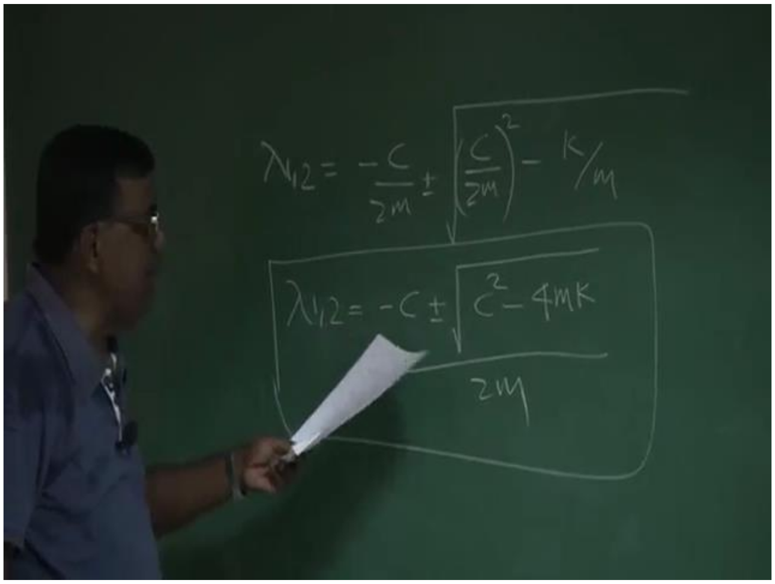

We have understood, since the solution of the perturbed quantity X of T is modelled at A E to the power lambda T, so how X of T will vary, will depend on the nature of lambda. Lambda will have different values depending upon, number one, it has 3 roots.

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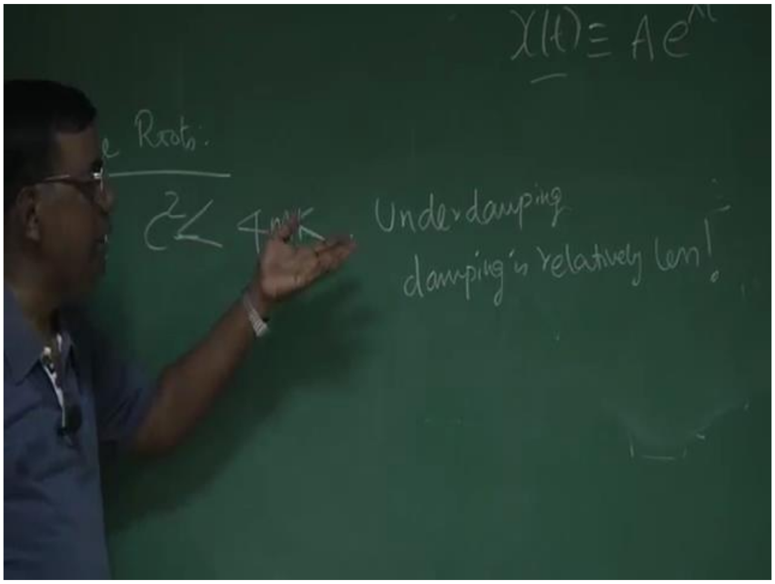

That is, 3 values of lambda are possible. One is, if C Square is less than 4 MK that means this term inside the under root will be negative. Okay? So this is called a typical case we will be talking about underdamping, we will show why it is underdamping or we say damping is relatively less. This C Square is a characteristic of damping. So, C Square less than this, damping is not much. C Square less than 4 MK, it will be termed as under damped, the case is an under damping case.

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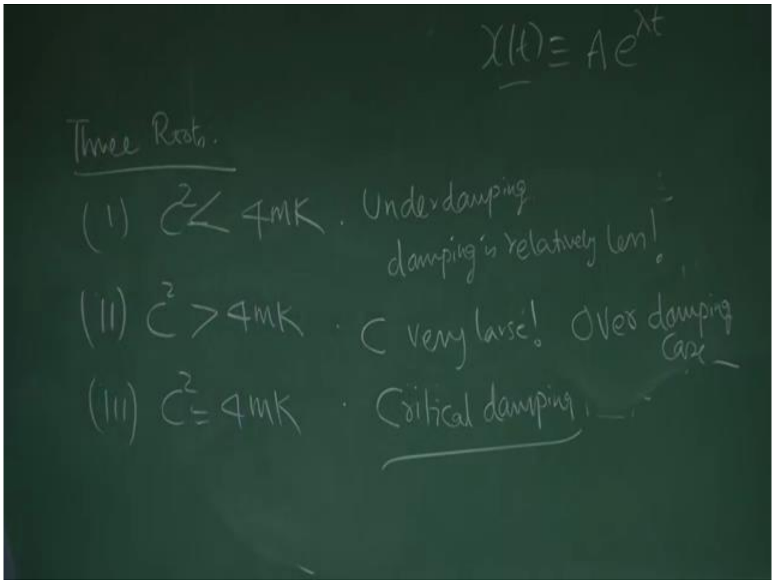

Second case possible is C Square greater than 4MK. The same understanding here. Now we know that damping must be pretty large. Think of a situation. Suppose we take a mass spring shocker system. Mass spring and there is an air around it. So let there be friction and some energy will be lost through air, friction. It will have some damping. But if I replace the air by water or oil, the damping will become large or C value will increase. So it will be referred to as larger damping compared to only having air.

So from that engineering sense, we understand, when I talk about under damping and over damping. So this is C very large relatively. And I explained you what is the meaning of that, very large. So it is over damping. And third one obviously comes to your mind, you all are mathematicians, C Square equal to 4 MK. And we will be referring this as critical damping.

One thing should be very clear at this point. If C is less than C corresponding to critical damping, then it will go to under damping. If C of the system is more than the critical damping C, it will go to over damping. Is this clear? I repeat, if C of the system is more than critical damping, then it is over damping. If C of the system is less than the critical damping, then it is under damping.

This much in engineering understanding, you must have, then naturally a question comes to your mind, what is critical damping? We will discuss it. But mathematically, these 3 things should be clear.

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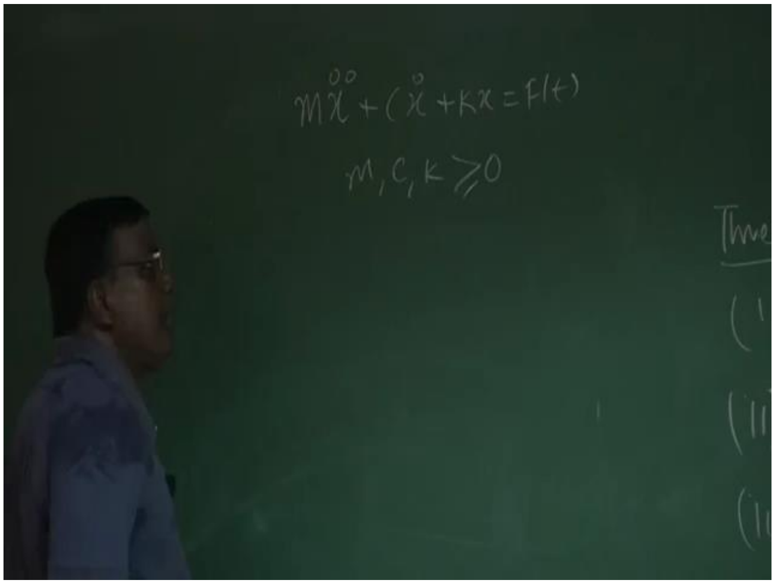

Please also understand one thing. When we wrote equation like this, MX double dot + CX dot + K of X equal to F of T, there was an explicit assumption that M, C, K, all are greater than 0. If C is 0, that means there are no damping but there are positive limits to satisfy this condition. Now, ask yourself honestly, whenever you talk about mass spring damper system, what comes to your mind? You think, it must be oscillating and doing like this.

But is this always true? That means, if I ask the question that once the system has mass spring damper system or a system can be modelled as a mass spring damper system, does that mean it will always have the oscillations. That is the question we are going to ask, analyse from this. Second thing, please understand, the damping is not always some part of a system which is separate from the main system. Even in the mass spring system, it is air, automatic damping will be there. Okay?

So do not think there is some sort of a, it is a modelling approach. We create a system for enhancing the damping by putting a fluid and allow the fluid to fast through an orifice. All such things we do but a normal spring mass system will have a natural damping because it is in a medium. So these two standings were required. And now we would like to check whether a mass spring damper system (())(13:04) it will be always oscillated system.

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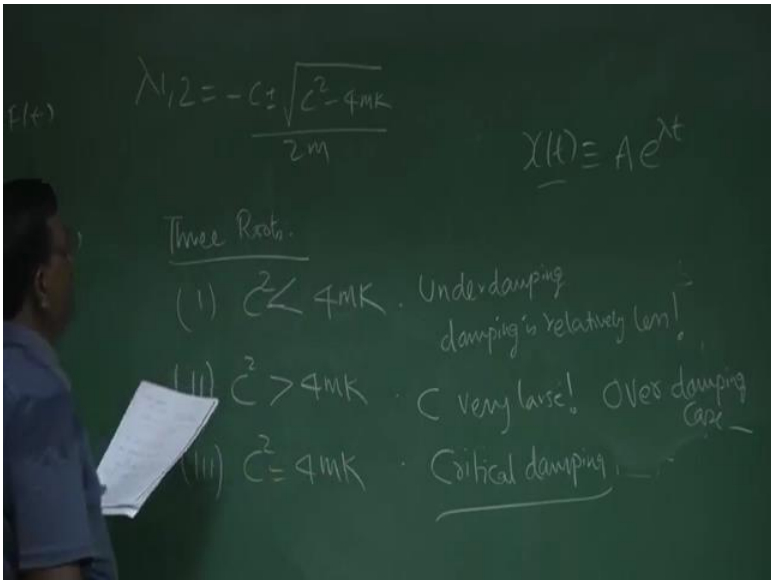

Let us take the first case, under damping. What is this case of under damping? This is a case, C Square less than 4 MK. So the roots of the equation if we recall, lambda 1 2 we wrote as - C + - under root C Square - 4 MK by 2 M. Once I say C Square less than 4 MK that means this argument under the Square Root is negative. So it will give a complex number. Right?

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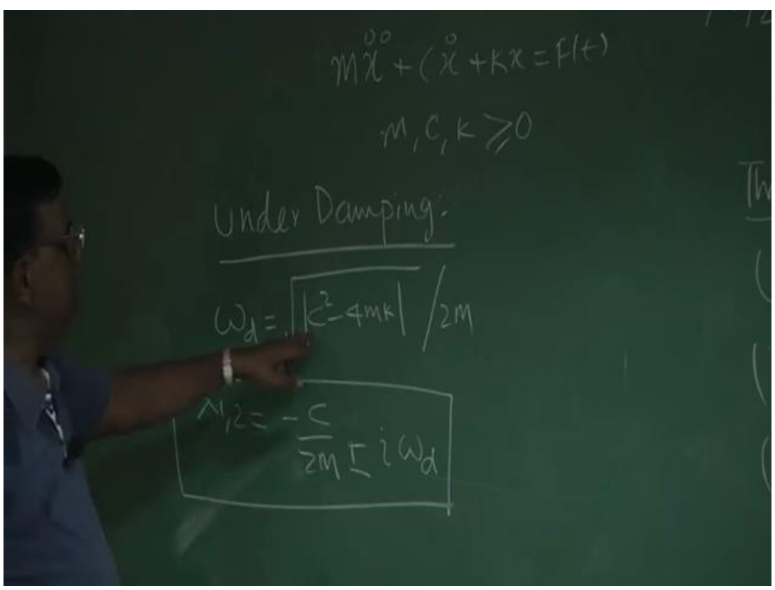

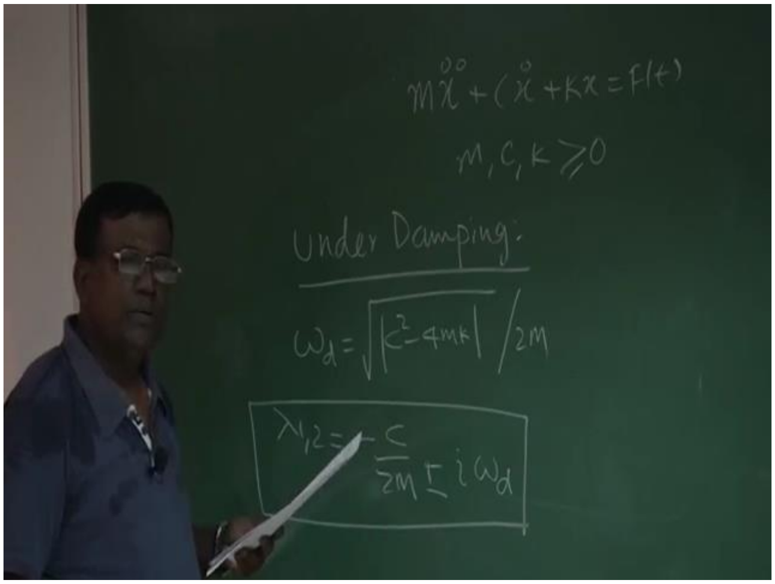

The moment it is a complex number, I write, for notation, I write omega D is equal to under root C Square - 4 MK by 2M. And you understand why this algebra I am doing, so the characteristic roots will become lambda 1 lambda 2 can easily be written as - C by 2M which is nothing extraordinary I am doing. I am only substituting this expression here. And this will give me I complex number unity WD. If you see here, this term is nothing but here, I have taken - 1 root, so I is here. This is a generalised roof, where you will find WD will be termed as damped frequency. We will talk about those. Do not get mixed up at this stage.

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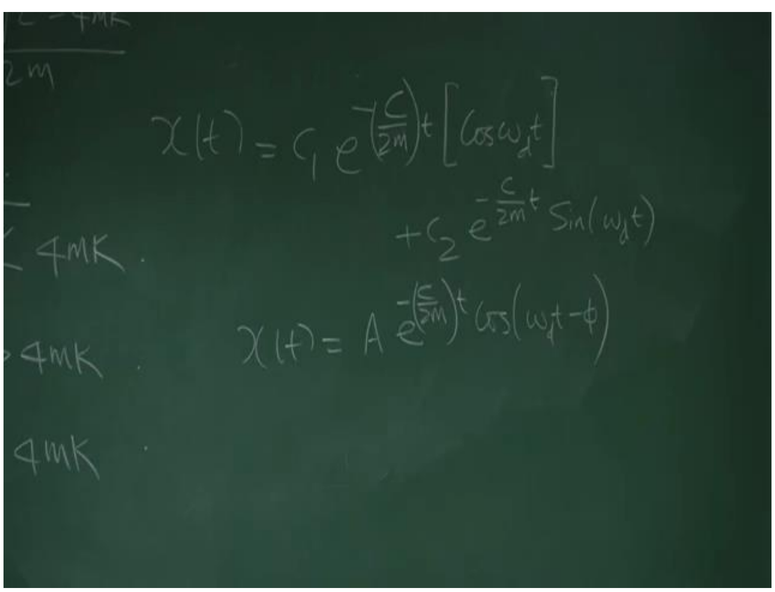

And if you have revised your little bit of differential equation understanding, you can easily write, X of T as C1 E to the power - C by 2 M T, let me write this. Do not get unnecessary disturbed. This expression of course never look very elegant. But you have no shortcut, we will try to get the best out of it. Now X of T, I can write as A E the power of - C by 2M T Cos of omega D - phi.

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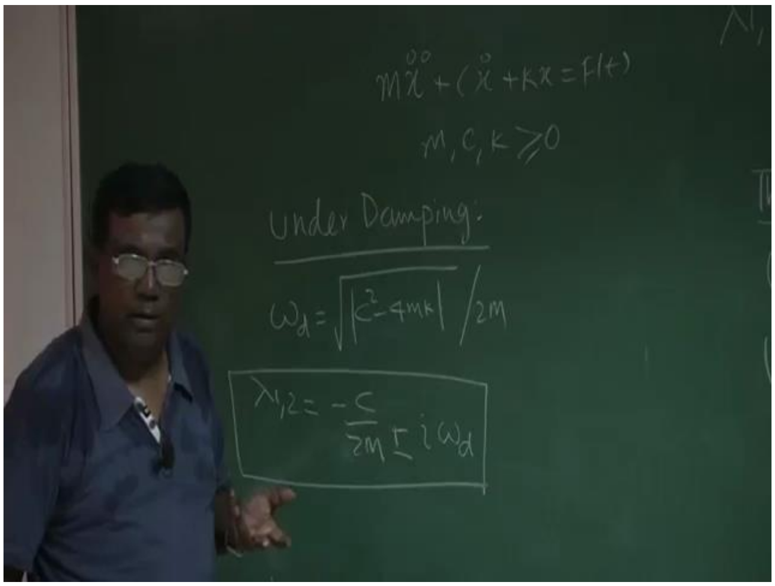

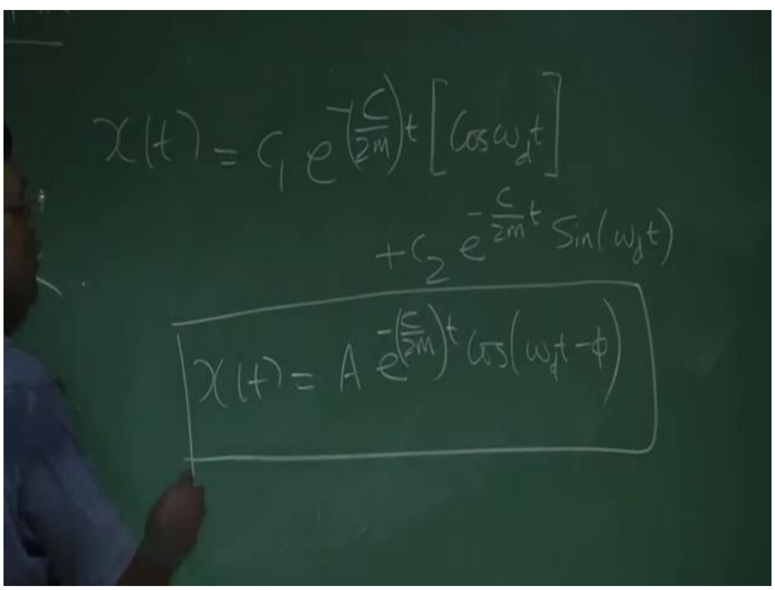

Because you know, if these are the roots, one is - C by 2M + I omega D and the other is - C by 2M - I omega D, they are complex numbers. So there will be oscillations as far as modelling is concerned. And you could see, the final solution is the sum of 2 solutions. It is a sinusoidal part. But try to understand this. You do not require more than this mathematics to understand. One laplace transform we require which is also very simple for our purpose

These are pure standard things. So do not get nauseated as to why this expression is coming. Understand this. What is this expression telling? It tells you, X of T, this perturbed quantity will vary with E to the power see by 2 M - and Cos function. So it is oscillated like this. What is C? C is the damping constant.

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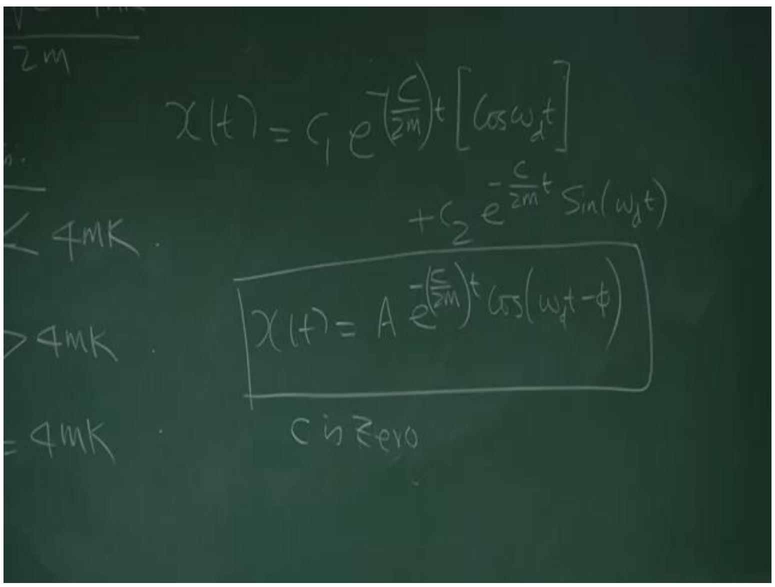

So if C is 0, then there is no damping. How this solution will look like? It will be sinusoidal, a Cos function. Right? If C is 0, this man becomes 1. So, the amplitude will go on in an oscillatory motion with a sinusoidal form. There will not be any decay. Will there be? No. So, no damping. So dynamically, it is neutrally static. That means, what is the message? If you want to make a system dynamically stable, ensure that C is nonzero. Second message is, if you want highly damped, increase the value of C. This is clear?

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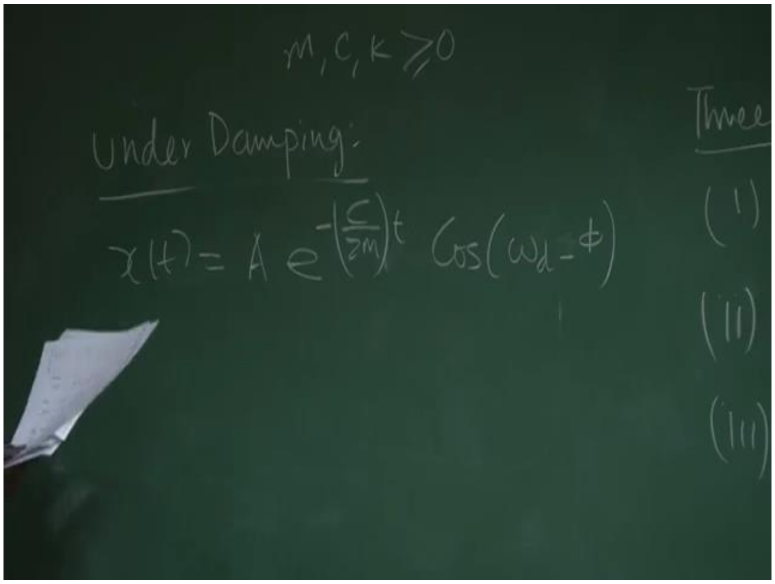

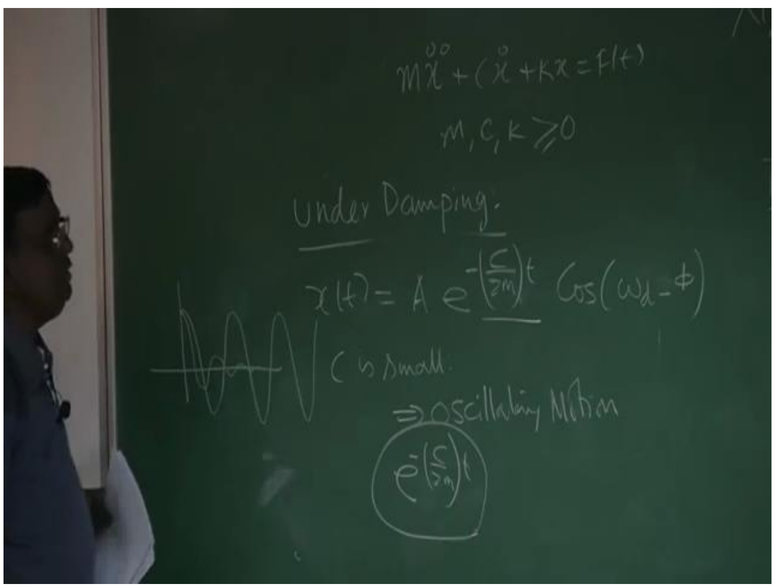

Now, we are discussing about under damping case, so I continue here, I take this as X of T equal to A E to the powers - C by 2 M T into Cos of omega D (+ -) phi. You know that if C is 0, it is sinusoidal. If C is small relatively it will have oscillations. Do you see that? Oscillatory motion. Because E to the power this is negative, amplitude will go on decreasing. That is the important thing.

And this E to the power - C by 2M T, that is actually a damping contribution which will although if C was 0, it would motion like this. With small C, what will happen? It will start and the amplitude will go on decreasing and it will come back to the equilibrium. If C is small and that is typically the case of under damping. What you understood? The under damping case, that is C square less than 4 MK, there will be oscillations and it will be under damped and it will come back to the equilibrium.

But there will be oscillations. But please understand they will not call omega D as real frequency. We will call it as pseudo-frequency. For simple reason, the term frequency, we will call omega D as pseudo-frequency because generally when we talk about frequency, we talk about a periodic function but strictly speaking, this is not periodic. Although oscillatory, not periodic. So we use omega D, the expression which I have earlier given you in the beginning and if I want to write that for your verification.

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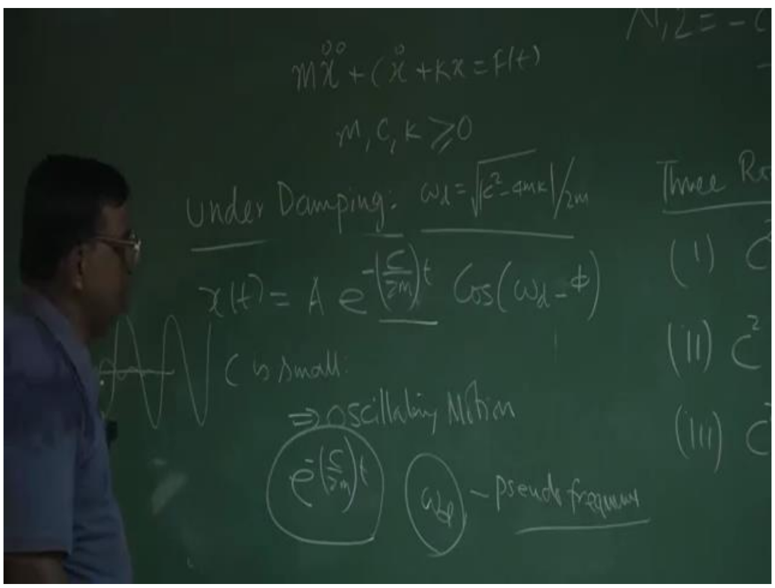

Omega D, we defined as under under root C Square - 4 MK mod by 2M. This is the damped frequency. This is a damped frequency with a caution that we are very well aware, when you define frequency, we talk about a periodic function. However, this is not periodic although there are oscillations. This is just for your information to be more precise.