Aircraft Dynamic Stability & Design of Stability Augmentation System
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Module 3
Lecture No 17
Alpha dot derivatives

Good morning my friends. You must be tired by now. We have been deriving derivatives and you have to be patience. We will be doing it for another maybe 1 or 2 lectures. Then we will come back to a session which is our popular session, *Man ki Baat.* We will summarise what we were doing so far. In the meantime, one of my TA has solved the problem of second order system.

These things are done to make you habitual of solving some standard equations. Do not give too much weightage on that. Try to understand the result because those we will be using in our future discussions.

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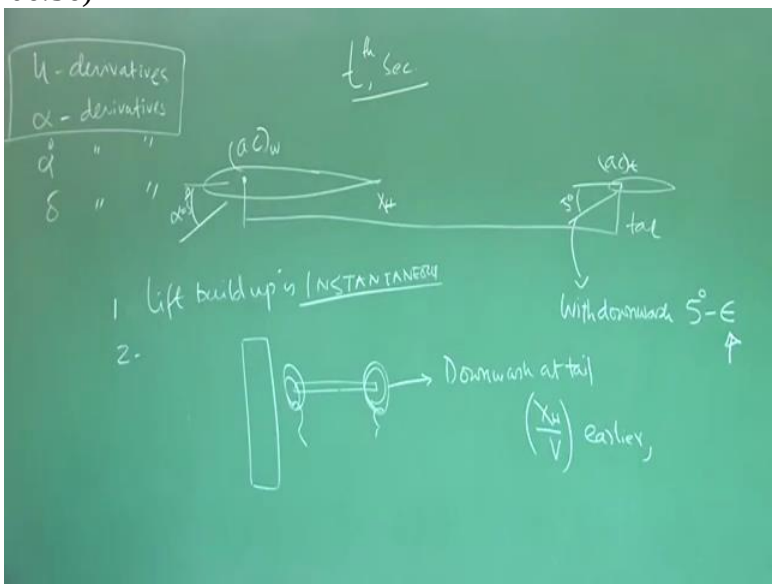

If you recall, we had U derivative, we have Alpha derivative and then alpha dot derivatives. Then Delta derivatives. We have completed up to this. Today we will be discussing about alpha dot derivatives. What is this alpha dot and why it is important? Please understand. If this is the wing and if it sees angle of attack alpha, theoretically speaking, for normal type of manoeuver, the lift buildup is not instantaneous.

It takes time for pressure distribution to happen. And then the lift gets activated. However, in this case of alpha dot derivative, we will assume that as soon as the wing sees or any components sees angle of attack, the alpha buildup or strictly speaking, lift buildup is instantaneous. Okay. This is an assumption which is true for all the types of motion that we are talking about.

But if there is a highly oscillatory motion, then this statement may not be very correct. However for our case, in small perturbation, we will assume that lift buildup is instantaneous. Second thing which you should also understand, as there is a lift over the wing or as there is a pressure distribution over the wing, which causes lift, you are aware that there will be wing T vertices. Okay, remember?

Because of pressure distribution or pressure differences between the lower and the top surface of the wing and these vertices will induce a down wash at tail. What these vertices will do? These vertices will give down wash at tail. That means, if there were no such wing T vertices and suppose this alpha was 5 degree seen by the wing, then the tail also will see 5 degree. No down wash if it was like that.

But because of this down wash, there will be a downward component of relative air velocity. So this angle now with down wash, it will be 5 degree - Epsilon and Epsilon is the down wash angle in degree. This is one. But there is another salient point which we need to understand, if I ask this question, who is responsible for this Epsilon? We know that it is because of wing T vertices.

And if I take the distance between AC of the wing and let us say AC of the tail, this distance if I take, I say XH. So whatever is happening at the tail, is because of something has happened at the wind at a time, XH by V earlier. Is this part clear or not? Let us understand. Suppose I am analysing the phenomena at time, T, Tth second. What will happen? At Tth second, some wing T vertices will be generated.

They will travel and a fair assumption to believe, they travel with the speed V of the air plane. Now it will take finite time which is roughly distanced by speed. So after that time only, this alpha of the tail will get modified. That means, if I am analysing at time, T, I should be careful.

When I take this Epsilon, I should know that this Epsilon is caused this much of time earlier and that is why this alpha dot derivative becomes important. And they will not have to formulate it. And what are those derivatives?

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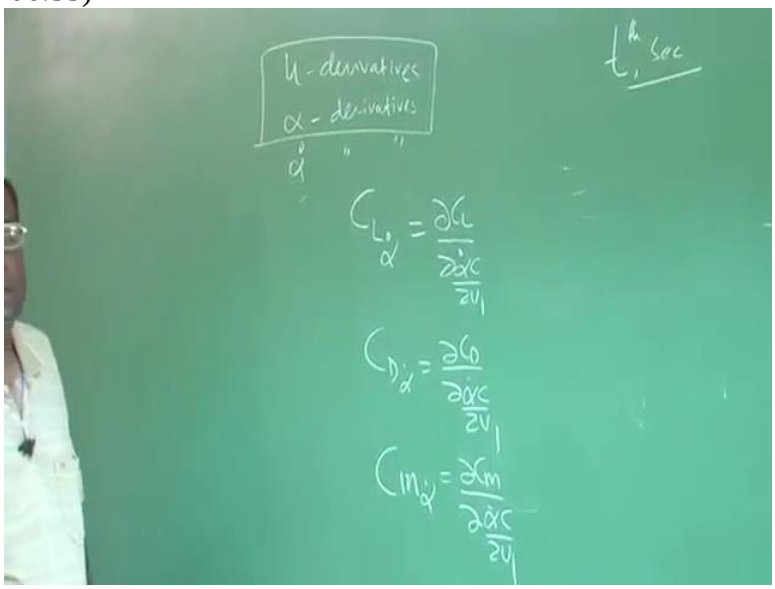

We will try to find alpha, CL alpha dot which is nothing but DCL by D alpha dot C by 2U1. CD alpha dot which is DCD by D alpha dot C by 2U1. And CM alpha dot which is DCM by D alpha dot C by 2U1. By now you know why we are putting alpha dot C by 2U1? Because alpha dot has a dimension. We want to operate in a non-dimensional quantity. So alpha dot C by 2U1 is a non-dimensional quantity.

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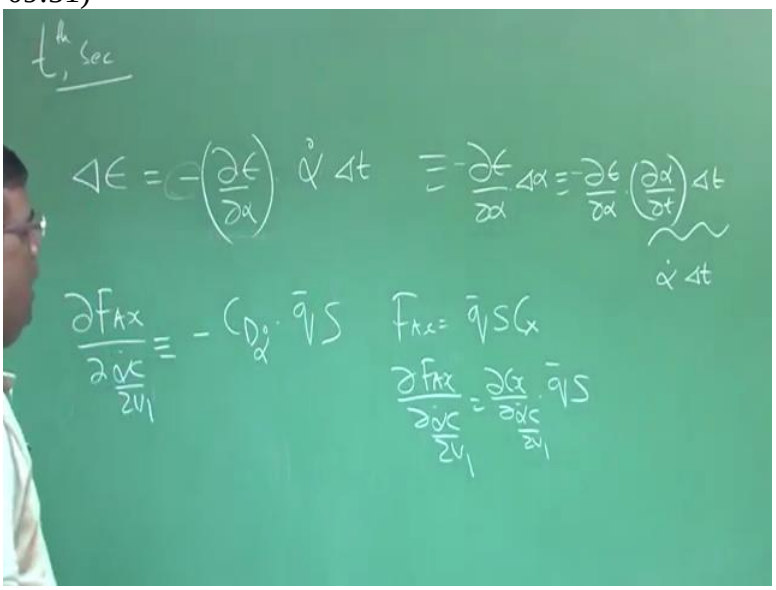

With this understanding, let us see how do we modulate? If I try to know what is Delta Epsilon, I can write it as - D Epsilon by D alpha into alpha dot into Delta T. If you see this carefully, this is D Epsilon by D alpha into Delta Alpha which I am writing as D Epsilon by D alpha into D alpha by DT into DT. And this is nothing but alpha dot Delta T. All these linear approximations helping us to write like this.

Why this - sign? Because we know, down wash will actually change the angle of attack. Now let us see. If I tried find out DFAX by D alpha dot C by 2U1, this will be nothing but - CD alpha dot into Q bar into S. I hope you understand this. Because we are writing FAX, go back to earlier lecture, this will be, FAX will be Q bar SCX and CX. So DFAX by D alpha dot C by 2U1 will be DCX by D alpha dot C by 2U1 into Q bar S.

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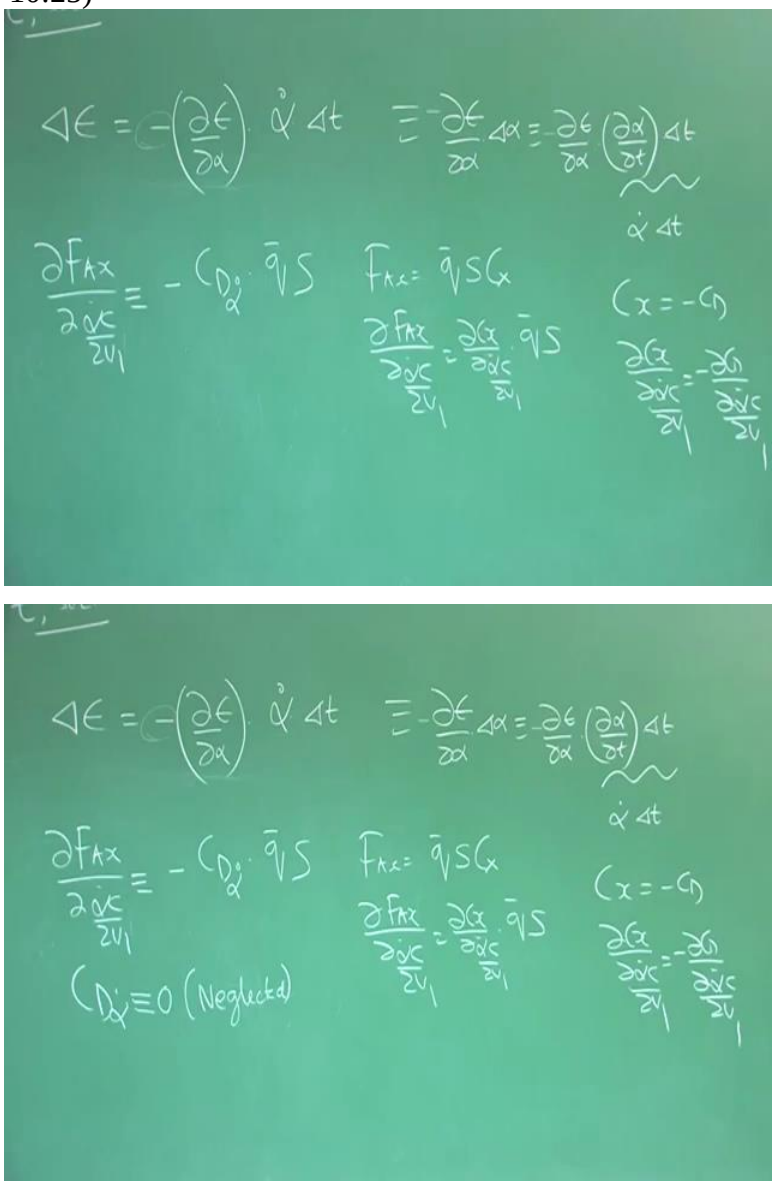

And you know, approximately CS is equal to - CD. So DCX by D alpha dot C by 2U1 is nothing but - DCD by D alpha dot C by 2U1. That is how this CD alpha dot is coming here. Fortunately, for most of the conventional alteplase, it is not a bad idea to assume that CD alpha dot is almost 0 or can be neglected. So our life becomes simpler.

So this derivative, DFAX by D alpha dot C by 2U1, we can put it to 0 for most of our conventional air planes. Now it comes to CL alpha dot. Let us see what is CL alpha dot. Since we are working with CL and since we are working on X, Y, Z axis, I will start with FAZ.

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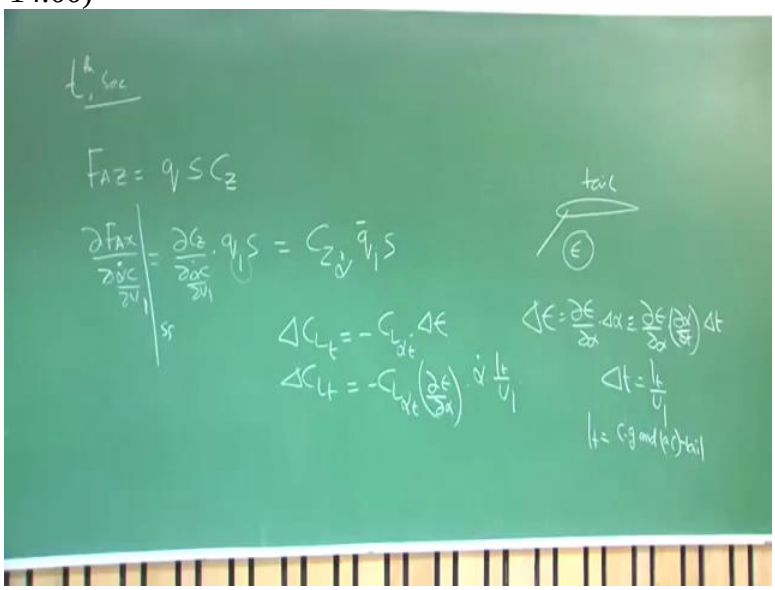

FAZ will be equal to Q bar SCZ and DFAX by D alpha dot C by 2U1 would be equal to DCZ by D alpha dot C by 2U1 into Q1 into S. Why this Q1? Because I know, this has to be evaluated at steady state. At steady-state, Q becomes Q1. What was the steady-state? For our analysis, it is the cruise.

So whatever the cruise speed was there, the dynamic pressure, corresponding to that cruise speed, the given altitude at Cruz is Q1. So these things are well known to you. And after this what we do, we write this as CZ alpha dot Q1 dot S. And now see what is Delta CL tail. That will be - CL alpha tail into Delta Epsilon. Remember, I told you because of wing T vertices, the tail angle of attack will be reduced by Epsilon.

So the moment it reduces by Epsilon, so this reduction in the lift, the tail, that is why this is - sign, - CL alpha tail into Delta Epsilon. No issues. Now this I can write as - CL alpha tail. And for Delta Epsilon, we have derived an expression. We can write it as, D Epsilon by D alpha into alpha dot to LT by U1.

Delta CLT. Correct? CL alpha tail into Delta Epsilon. Delta Epsilon is nothing but D Epsilon by D alpha into Delta Alpha which I can write as D Epsilon by D alpha into D alpha by DT into Delta T and Delta TVR assuming as capital LT buy Q1. Although initially we took the distance from the AC of the wing to the AC of the tail but we are doing an approximation.

This LT is distance between CG and AC of the tail and since CG of the is plain and AC of the wing are fairly close, so this approximation is also not bad. Okay? Remember, whenever we talk about CL due to alpha dot in terms of whole aircraft.

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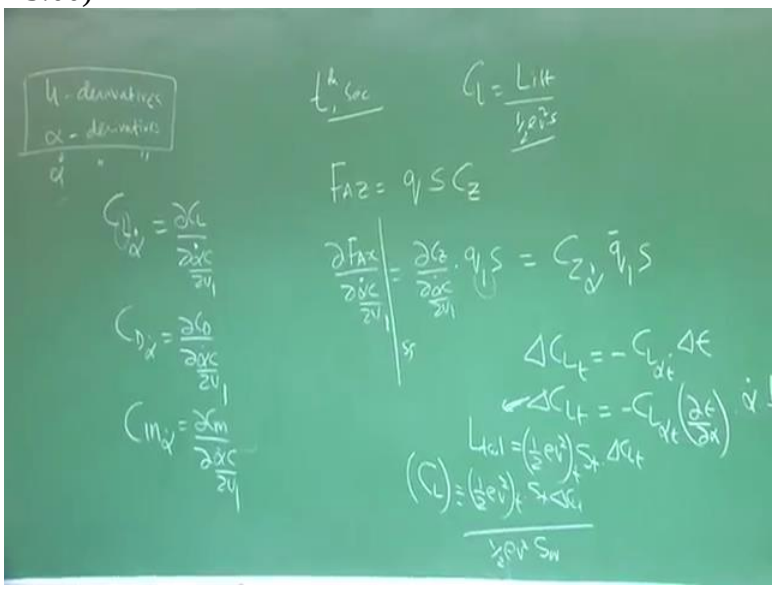

This is at the tail. For the whole aircraft, you know, CL has to be defined by lift by half Rho V square S with respect to free stream. So what will happen? The lift at the tail because of Delta CLT is half Rho V square had tail, S tail into Delta C LT. Correct? Half Rho V square at tail. But when I try to talk about CL based on the whole aircraft, the contribution of tail, I have to nondimensionalise this with respect to free stream dynamic pressure and wing area.

So this will become half Rho V square tail into S tail Delta CLT divided by half Rho V square free stream into S wing. Correct? So this is CL because of tail.

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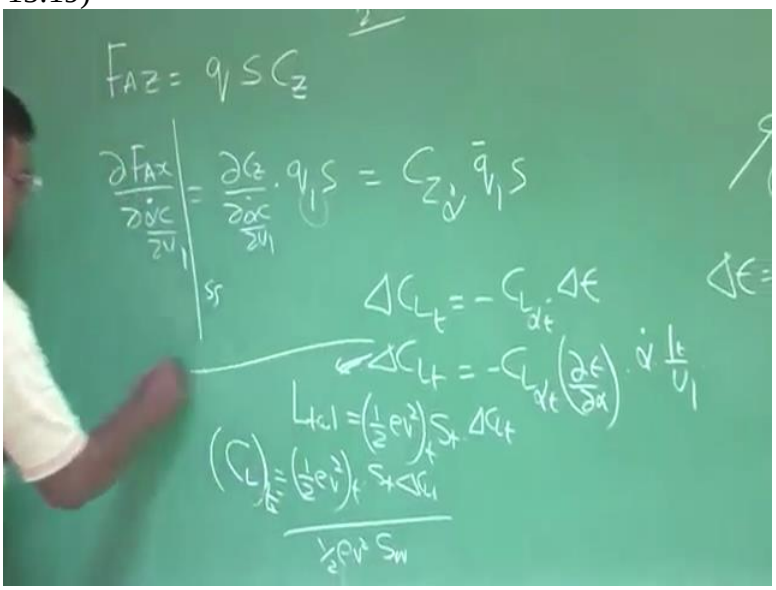

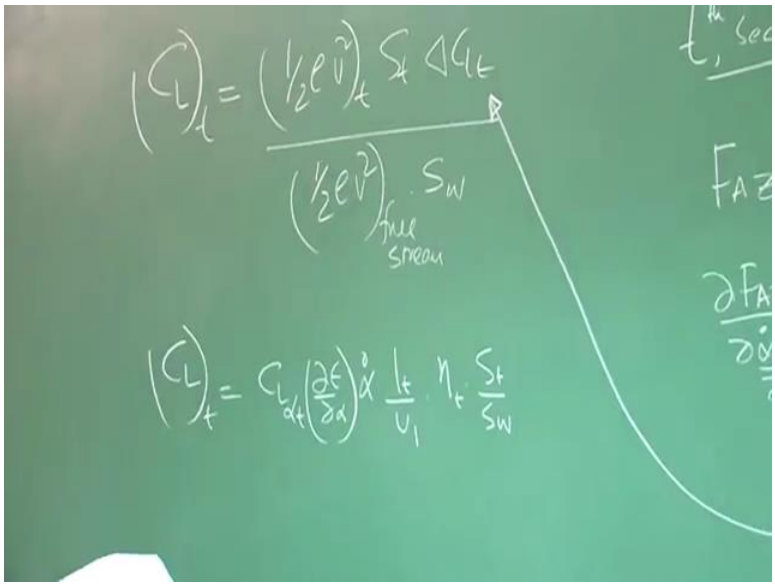

Now from here what we do is if I write CL in terms of tail which is a nondimensionalised standard form, we write half Rho V square tail into S tail Delta CLT by half Rho V square free stream into S wing which is standard into S of wing. And I substitute the expression for Delta CLT from here, then I can write CL tail as CL alpha tail into D Epsilon by D alpha into alpha dot into LT by U1 into Neta T into S tail by SW.

You may be wondering where this - sign has gone. Please understand, the expression of Delta Epsilon had a - sign which I missed it here. So there is a - sign here. So reduction and that is how the - sign got absorbed. Clear?

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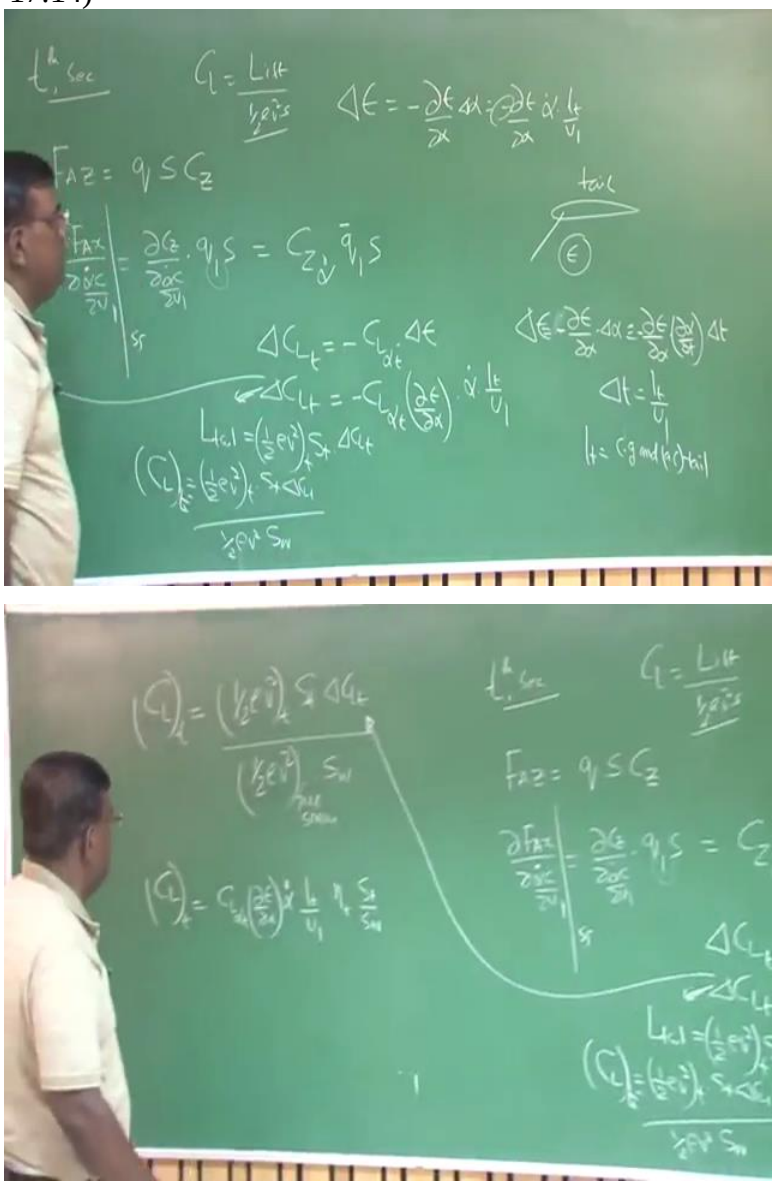

This Delta Epsilon if you see, earlier I had just shown that it is - D Epsilon by D alpha into Delta alpha which is - D epsilon by D alpha into alpha dot into LT by U1. And this - sign and this - sign gets cancelled and you get an expression like this. Now please understand, this CLT is due to alpha dot. That is very important. And we were working in terms of DFAZ.

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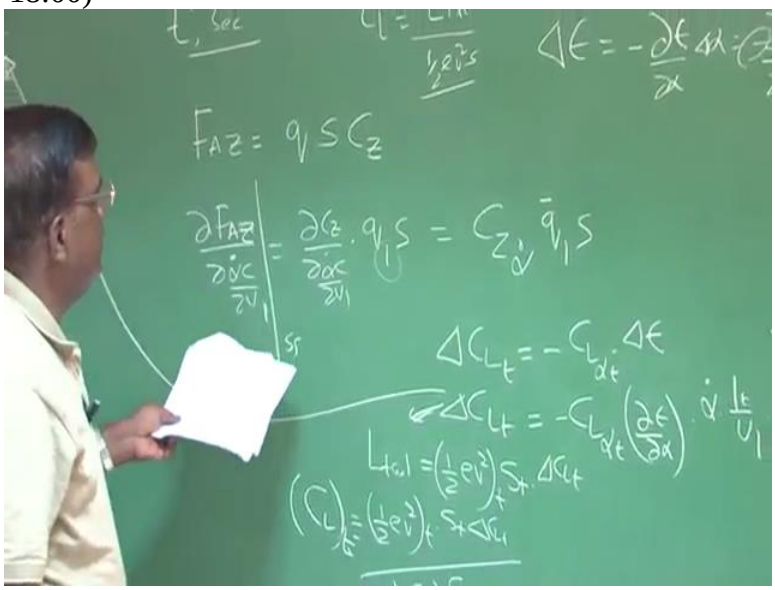

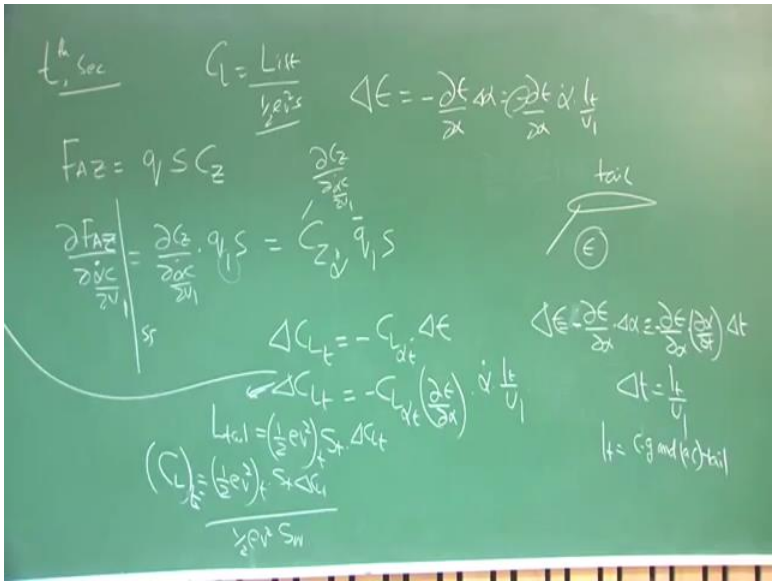

This should not be X. This is DFAZ by D alpha dot. So we are looking for DZ alpha dot or CZ alpha dot to be more precise which is nothing but D CZ by D alpha dot C by 2U1. So we have to work in terms of capital CZ alpha dot.

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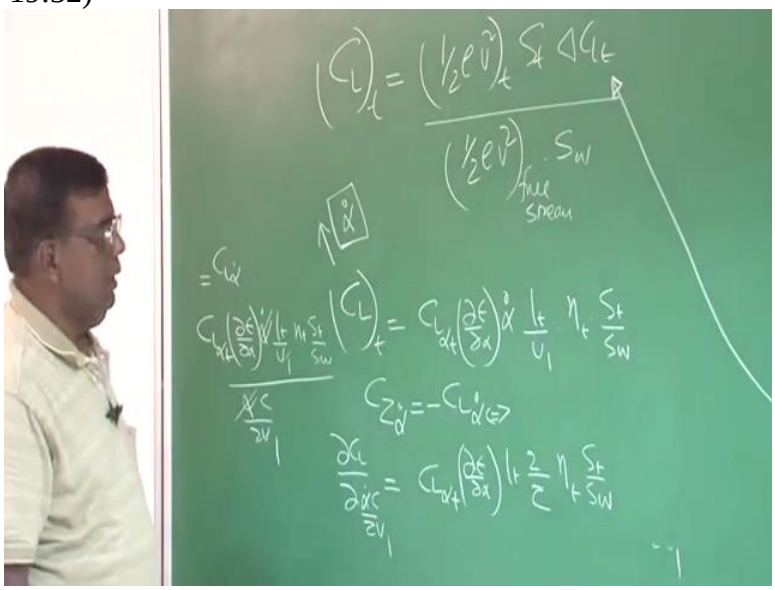

But we know that CZ alpha dot is approximately CL alpha dot. So this will be used and I can differentiate this to get DCL by D alpha dot C by 2U1 and CL alpha tail into D epsilon by D alpha into LT 2 by C Neta T ST by SW. Let me check. If it take derivative with respect to alpha dot C by 2U1 in this case. And that is if I quickly check, CL alpha tail into D Epsilon by D alpha then alpha dot LT by U1 Neta T ST by SW divided by alpha dot C by 2U1.

This is nothing but CL alpha dot. It is linear. I can just divide. So I see that this man goes off. 2 goes to numerator. U1, U1 get cancelled. So I have LT by C Neta T. 2 comes here, LT by C is here Neta T ST by SW. Fantastic. It is so nasty. But once you get the correct result, you start loving it.

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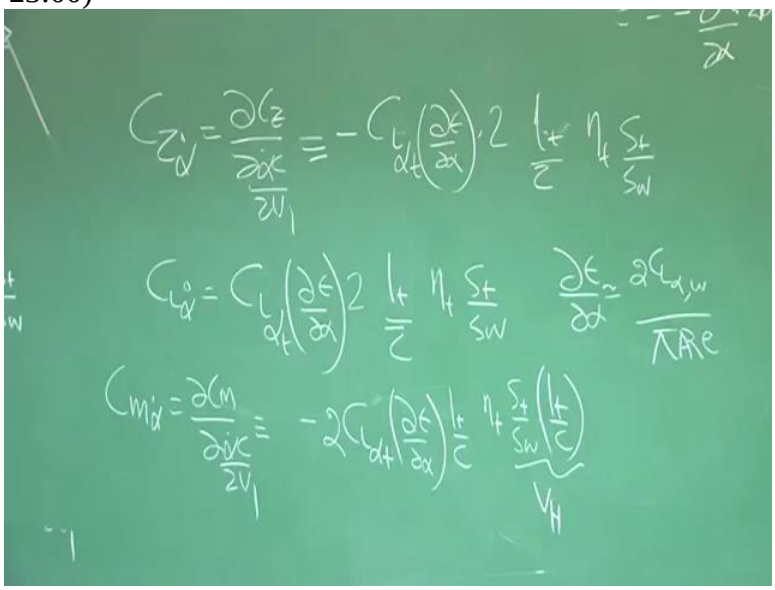

Now, once I know this is CL alpha dot, my aim is to find CZ Alpha dot. So I will write CZ alpha dot which is equal to D CZ by D alpha dot C by 2U1 which will be equal to just - C alpha dot. So - CL alpha tail into D Epsilon by D alpha into 2 into LT by C Neta T S tail by SW. Very good expression. This is fine.

From here if you see, this CL alpha dot which is nothing but - CZ alpha dot, that will be CL alpha T D Epsilon by D alpha into 2 into LT by C bar into Neta T ST by SW. Usually, the - sign is no more here. Because CZ alpha dot is - CL alpha dot. And let us see how to compute this. CL alpha tail we know once we know tail. D Epsilon by D alpha for low speed, this can be written as CL alpha wing by pi aspect ratio capital E.

Or you can get this value through internal testing. LT you know, Neta T you know, ratio of dynamic pressure, so everything is known. So at the design stage, you know the value of CL alpha dot. That is very important. Similarly if I want to find out CM alpha dot which is DCM by D alpha dot C by 2U1. If you see who is causing CM alpha dot? CM alpha dot is coming because of CL alpha dot.

It is a force in nondimensional sense, this is a moment in nondimensional sense. So I have to just multiply this with the moment arm. So I will write this, this is CL alpha tail D Epsilon by D alpha. So 2 I put here. LT by C Neta T ST by SW into LT by C. And we have to put a - sign here. You can easily see from here that multiply by LT by C and take appropriate sign.

So CM alpha dot will be - 2 CL alpha T D Epsilon by D alpha LT by C Neta T ST by SW.
And you should also understand, this is nothing but tail volume ratio, VH. So as a designer, you know how to find out the value of CM alpha dot and that is - 2CL alpha tail which you know. You know D Epsilon by D alpha and rest of the parameters you know.

Why this sign is negative? That question has to be answered. Because we have agreed that whatever we are doing when we are trying to calculate this alpha dot derivative in a down wash lag approach that something has happened at time LT by V earlier. That means at that time, the tail had a larger angle of attack corresponding to that time. That larger angle of attack will give a nose down moment or which down moment. So this is a - sign. Are you clear with it?
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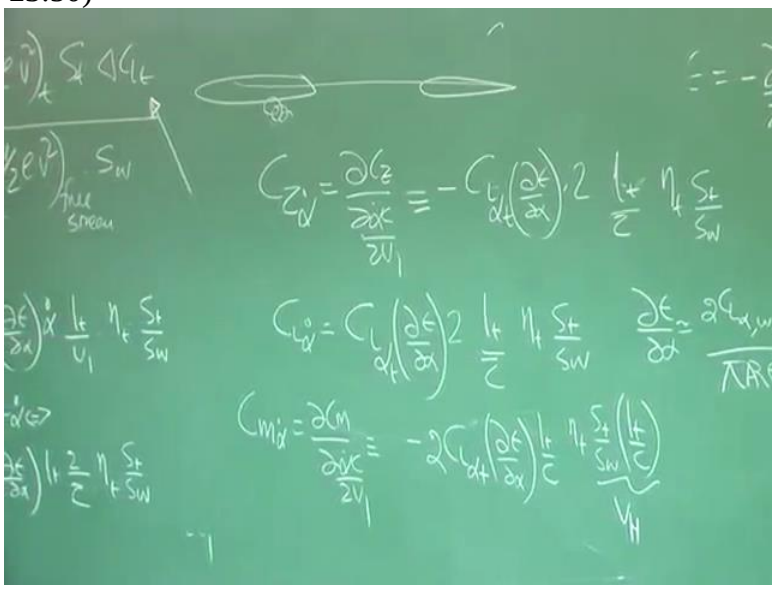

This is a tail and this is a wing. All these things what you are deriving, we are saying whatever happening here in terms of shedding vertices, it takes time for LT by U1 to reach here. That means if I am analysing at time T, that means the influence has now reached here. So that means, this angle is more than what it would have been.

So that additional force will give a nose down moment, nose down pitching moment. So this - sign we have to put here. Correct? We have now done CM alpha dot, CL alpha dot and typically you will find this CM alpha dot value for most of the low speed airplane around 20% to 30% of CMQ derivative. Okay?