Aircraft Dynamic Stability & Design of Stability Augmentation System

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Module 3

Lecture No 15

U-derivatives

Good afternoon.

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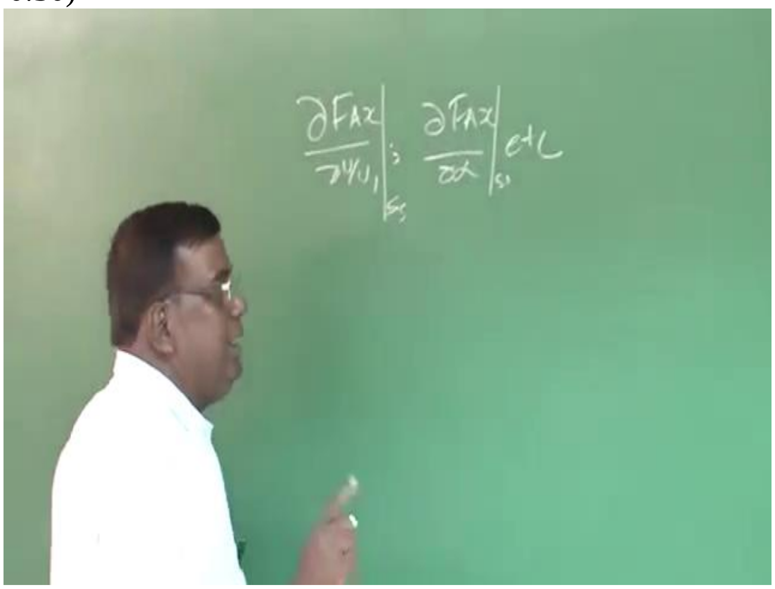

Now we will be deriving expressions for those derivatives which are last class you remember, DFAX by DU by U1 OR DFAX by D alpha, etc, which are U derivatives, Alpha derivatives, Q derivatives, like that. And we understand that these derivatives are evaluated at steady state. This is a very important thing. And second thing we should understand is that we are using stability axis system and by now you know what is stability axis system.

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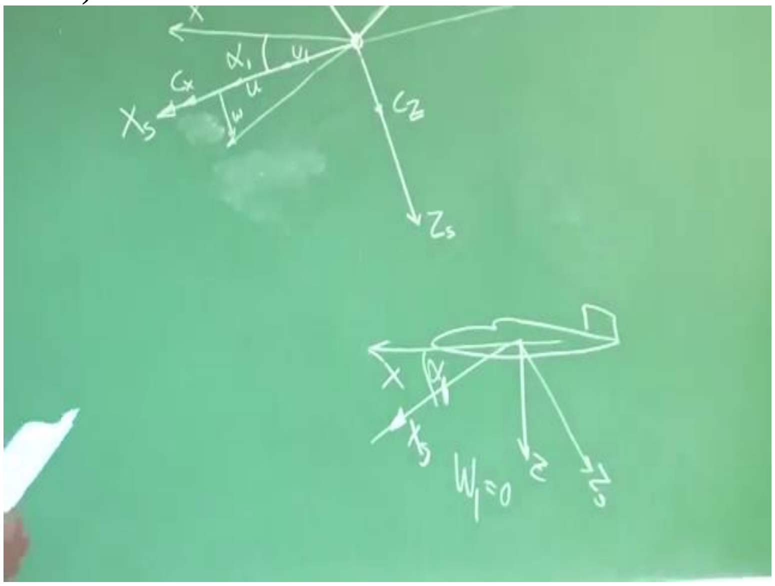

To revise your understanding, we know that this is an airplane which was having X axis like this, Z axis like this and it was having steady-state angle of attack let us say alpha1. It is cruising at alpha 2 degree. So I say alpha1. When I am defining stability axis system, I am saying I am aligning this X axis along the total velocity in the vertical plane and I am calling it XS. So naturally, your Z will become ZS. So you could see that now I am not working in X and Z axis.

I am still working on XS and ZS and the consequence of that is, W1 becomes 0. Because there will not be any component W1 when I am talking about vertical motion restricted to vertical plane. And since it is very simple that this is the airplane, this is the velocity vector and if this is the X axis, I align with the X axis. This is stability axis. Now, there cannot be a component perpendicular to the axis.

So, component along ZS will be 0. So W1 is 0. So we define stability axis at steady state. We confine our or redefine our axis system aligning with the velocity vector in the vertical plane so that everyone is 0. And we have to evaluate this derivative keeping those things in mind.

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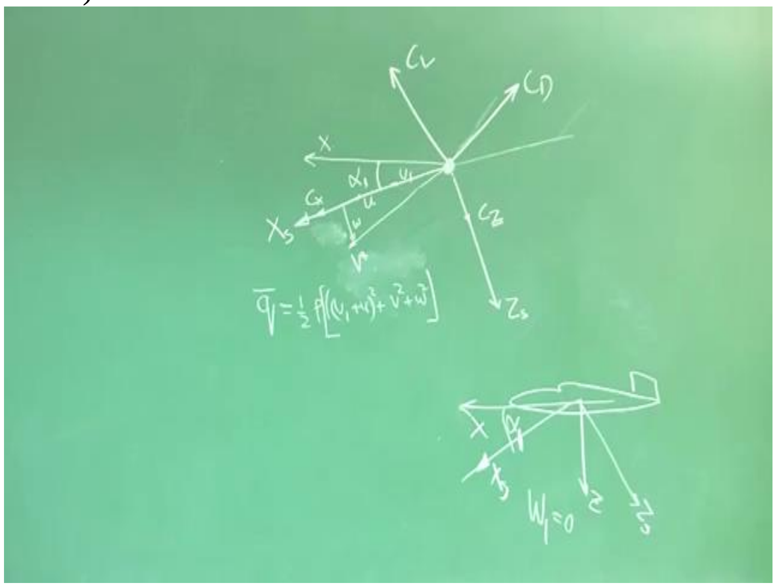

So if I draw the diagram for more clarity, you could say that this is alpha1, this is X what I was showing here, this is the perturbed U, this is the steady-state value U1. With that is the XS direction. And now because of disturbance, total velocity direction will change which is given here, let us say V Star. Total dynamic pressure will be now, if you see, ideally speaking, total dynamic pressure will be half Rho U1 + U square + V square + W square.

Half Rho U square, right? That is, U square is resultant of total velocity having perturbed quantity U, V and W and since at steady state, there were no V1 component, no W1 component so in general form, I can write it like this. Now I want to find out this derivative.

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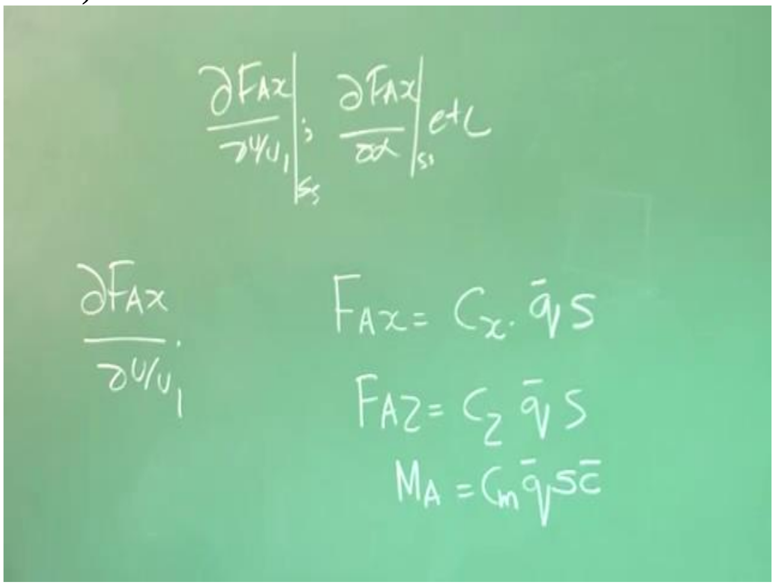

Let us say I am trying to find out DFAX by DU by U1. Let us see how it can be done. I can write FAX is equal to CX into Q bar S. Similarly I can write FAZ equal to CZ into Q bar S. I can write a aerodynamic moment as CM Q bar S C bar. Clear? There is no problem. I am just using a definition of FAX, FAZ and MA through the definition of CX, CZ and CM as we did for CL, CD and CM. Now we want to find DFAX by DU by U1. C, how do we handle this?

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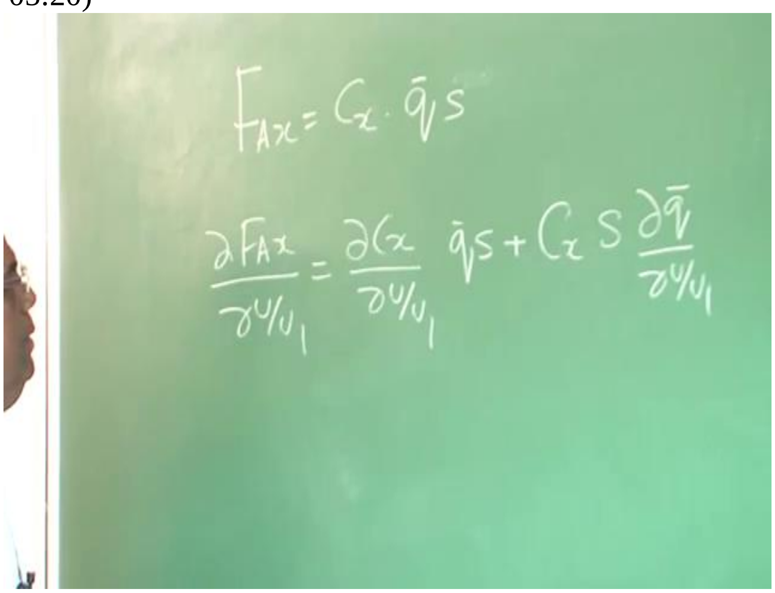

So let me go like this. FAX is equal to CXQ bar S. This is very mechanical. So follow my steps. So DFAX by DU by U1 will be equal to DCX DU by U1 into Q bar S + CX into S DQ bar by

DU by U1. No issues. Class 11th, 12th. What next? Let us see what is DQ bar by DU1. DQ bar by DU by U1. What is it? Let us check this.

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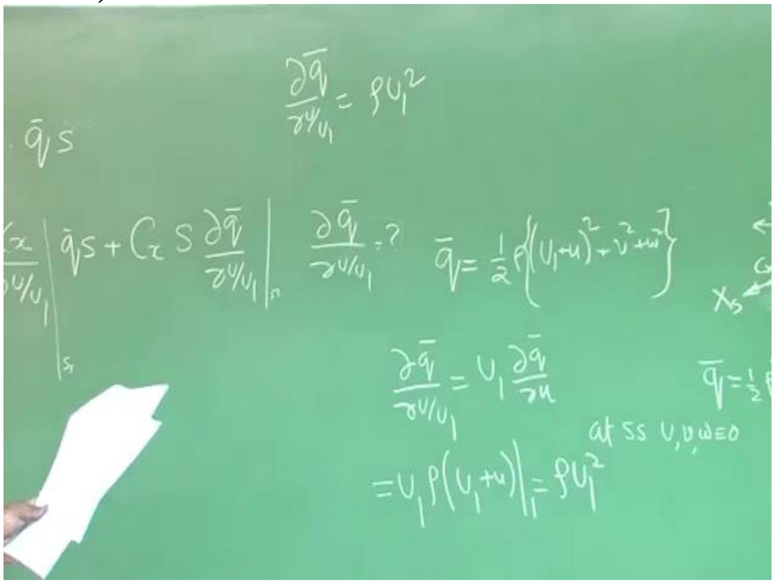

You know what is Q bar. Q bar, we have already understood. That is half Rho U1 + U whole square + V square + W square. Half Rho total V square. What is the important thing we should remember is that this is to be evaluated as steady-state. So all these things are to be evaluated at steady-state. So DQ bar by DU by U1 has to be also evaluated at steady-state.

So if I write DQ bar by DU by U1, I will write it as U1 constant, DQ by DU and I can differentiate this and put a condition that at steady-state, all the U, V, W are 0. Is not it? At steady-state, U, V, W, all are 0. So I will take the differentiation with respect to U. So this will give me U1 into Rho U1 + U at steady-state and that will be nothing but Rho U1 square. So what did I get?

I get DQ bar by DU by U1 is nothing but Rho U1 square. As simple as that. So if I want to write DFAX by DU by U1, what will be my result? I have to change this term because I have evaluated this by this term. first step. So I can write, DFAX by DU by U1 is equal to DCX by DU by U1 into Q1S. Now, I write 1 because I have to evaluate these at steady-state. For Q, I write Q1S + CXS Rho U1 square.

Now the question comes, can I find an expression for DCX by DU by U1. That is the question. And then also since I am evaluating and steady-state, what is this CX? CX is what? CX is a component which is coming because of drag, lift, etc. all the forces resolved along the X direction. But once I say evaluated as steady-state means, at a condition 1. So I write this CX has to be at condition 1. So CX is evaluated at steady-state. That is the understanding.

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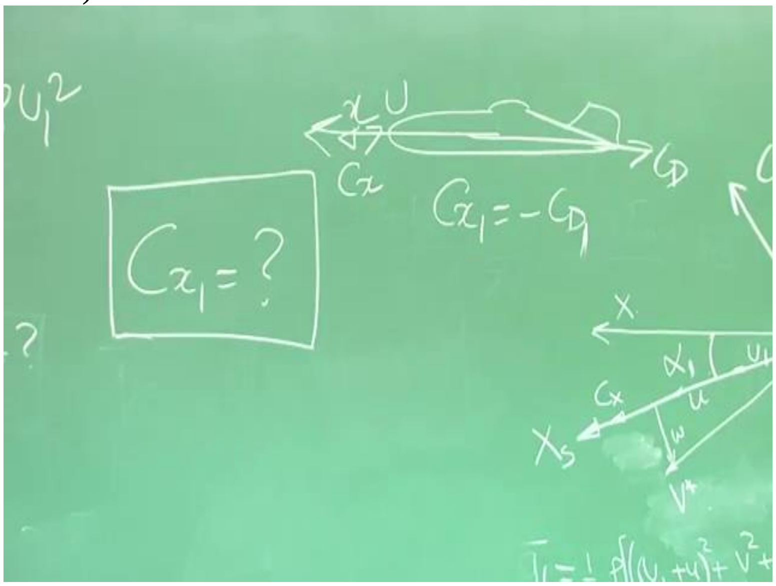

What is CX1? Let us see like this. Suppose this was X and the velocity was in this direction. Then CD would have been like this. CX is defined like this, in this direction. If the flight was like this, the condition was like this, what would have been the CX1? CX1 would have been nothing but - CD or CD1. Now let us see, in the generic sense of understanding, how do I find out what is CX1 in terms of CL and CD? Please understand we are trying to find out everything on CL and CD because we are using stability axis system. Right? That is what I was mentioning in the beginning also.

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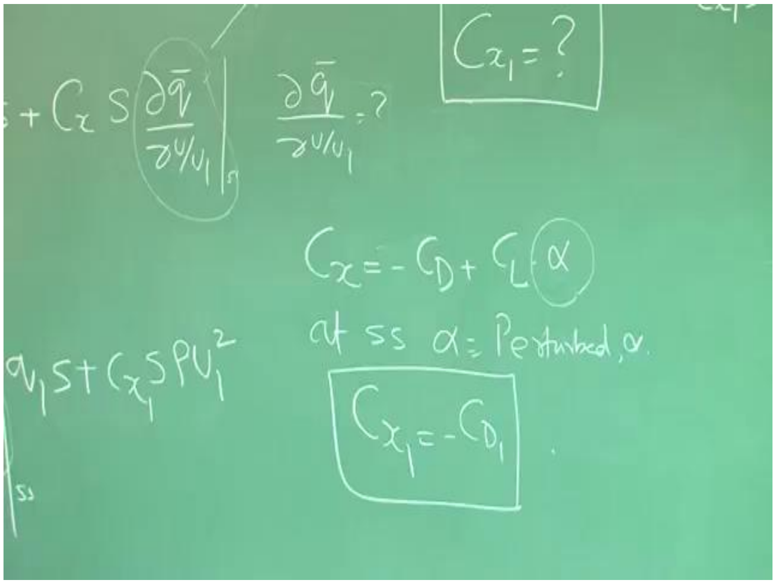

So let us see what happens. You know the CX, for small angle, I can write it as CD + CL into alpha. You can use this CX. CL is here, CD is here, Alpha is there. So you can easily find out how do I define CX and CD? So this is your alpha. Put the geometry and you can find this. So at steady-state, the alpha here which is perturbed alpha that means perturbed alpha at steady-state is 0. So in this case, CX1 is - CD1.

You get the same answer as we could see from simple understanding. If you see my last course lecture, I have developed this equation assuming this sort of an understanding. But in this course, I am trying to go a little bit more. We go all the way so that you understand the basic mathematic or the basic algebra or the basic technique, not even, I do not call it algebra or mathematic.

How to manipulate the expression? How to camouflage physical understanding into the expression to make life simple? So this is CX1 is handled. We know this. The next question comes, what is DCX by DU by U1? Let us see that.

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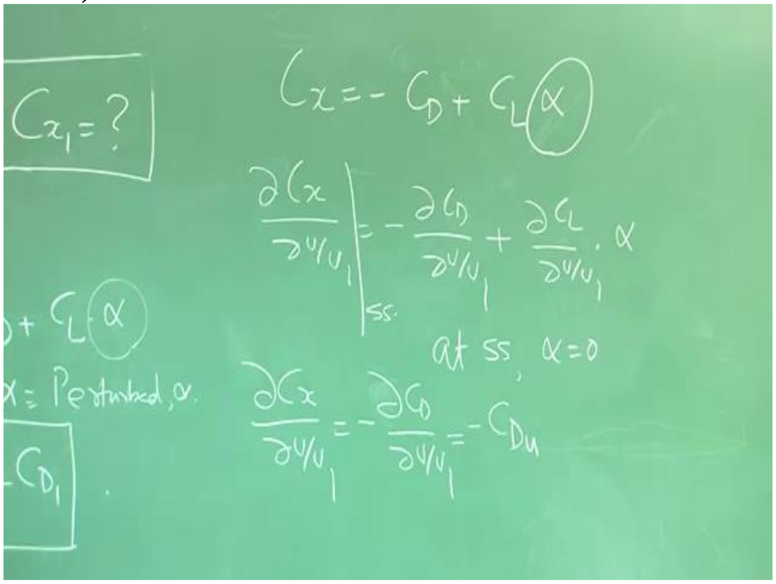

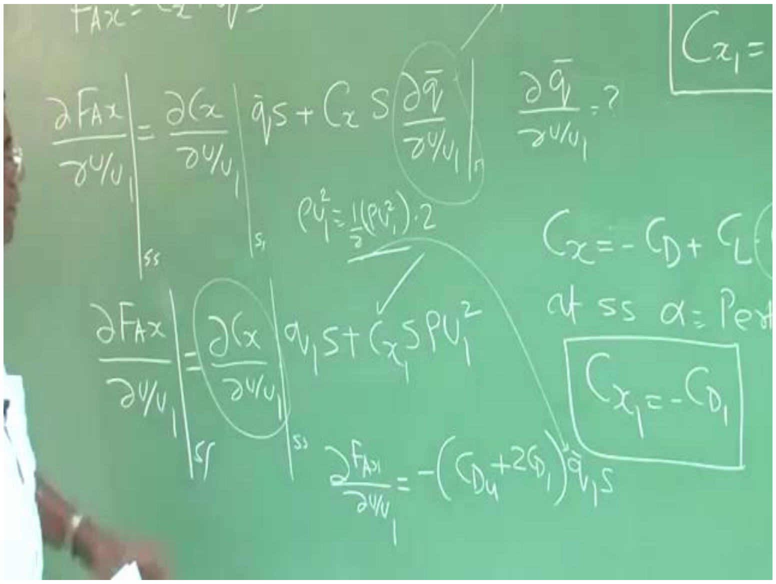

We again start with this expression, CX is - CD + CL alpha. Please understand, once I have written this, I have assumed that alpha, the perturbed quantity is very small. That is why you know that there will be a Cos alpha term, a sin alpha term. Sin alpha becomes alpha, Cos alpha becomes 1. So we are again using the advantage of small perturbation.

So, CX is this. What is our aim? Our aim is to find DCX by DU by U1. So now we write DCX by DU by U1 is equal to - DCD by DU by U1 + DCL by DU by U1 into alpha. Right? But what is it that we cannot afford to forget is that this derivative has to be evaluated at steady-state. It has to be evaluated as steady-state means what?

As far as this expression is concerned, because I know alpha is perturbed quantity, but at steady-state what is the value of perturbed alpha? It is 0. At steady-state, Alpha which is perturbed is 0. So DCX by DU by U1 is - DCD by DU by U1 and that is also sometimes written as CDU. Okay? We will try to give an understanding as to what this CDU means but as far as the expression is concerned, now I know that DFAX by DU1X, I can easily write as - CDU + 2CDU1 into into Q1S.

Is it okay? You should not get surprised where from these 2 terms came? Please see that this is Rho U1 square. So we have taken a common with Q1. So this Rho U1 square has been written as half into Rho U1 square into 2. And this is Q bar, Q1 bar that has been taken care of. That is how you got this term 2. Okay? This simple manipulation, you should be able to do.

So we have seen primarily that these derivatives can be evaluated if I know what is the value of CD1? What is the value of CD 1 means what? What is CD? CD is you know.

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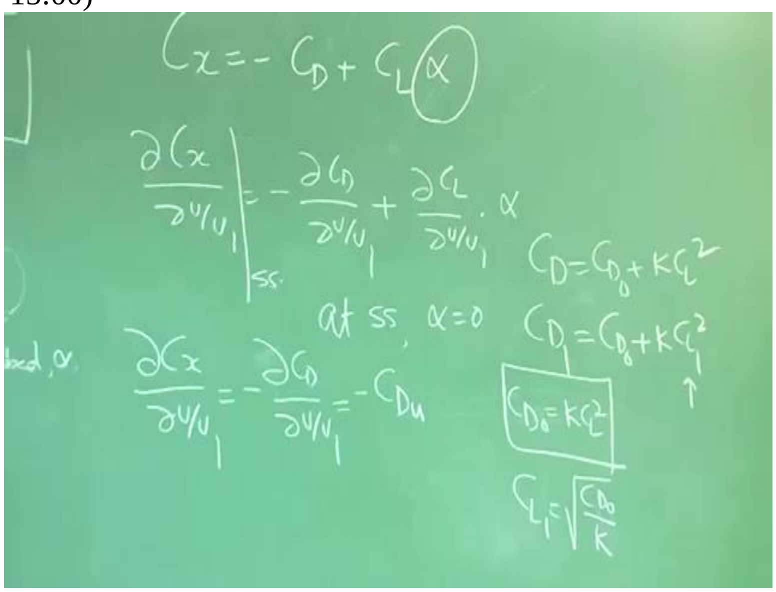

For an air aircraft, CD is CD not + KCL square. Let us say I am talking about LO subsonic airplane. So what is CD1? CD 1 is the value of CD at steady-state. What is a our steady-state we considered here is Cruise. So at Cruise, the question is what was CD not value and what are the values of CL1 with which it was cruising. For example, if it was cruising at a minimum thrust required condition, then you know that CD not is equal to capital KCl square and you can easily note, CL1 is nothing but under root CD not by K.

That is the cruise condition. That is the condition at steady state for our case. And what is CDU then? The question comes, what is CDU? Let us see what is CDU?

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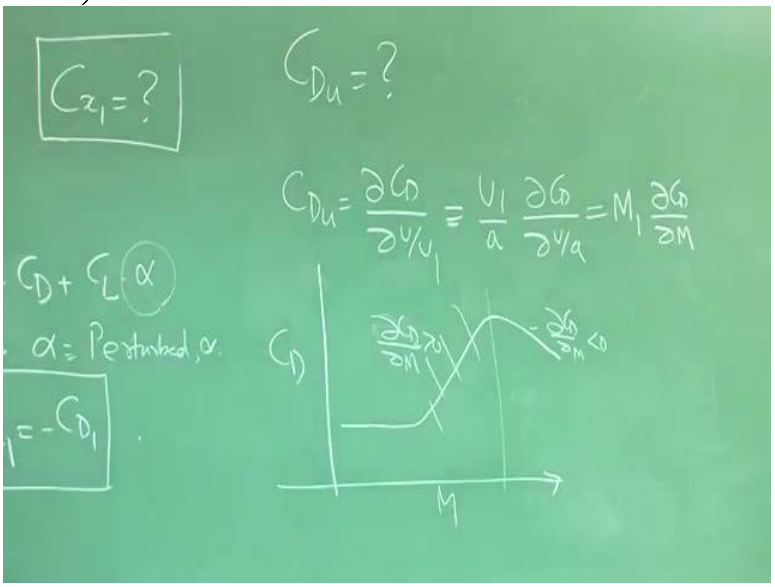

CDU, I can write it as DCD by DU by U1 by definition. And I can write this as U1 by A into into DCD by DU by A. What is A here? Anybody's guess? U is the velocity of sound at that altitude. And what is U1 by A? It becomes the mass number at steady-state. So it becomes M1 into DCD by DM. Now see, very interesting phenomena, this CDU is immediately giving us some sort of a very very significant understanding. If you recall, CD vs Mac number for an air aircraft it looks something like this.

Typically this is transonic zone, maybe 1.1, 0.98 or to 1.1 or 1.2 depending upon the configuration. Now you could see, if you are in this region, here DCD by DM is greater than 0. Whereas here, DCD by DM is less than 0. So depending upon what is your flight regime, this DFAX by DU1 will change depending upon the value of CDU.

Sometimes it becomes positive if it is subsonic type. It becomes negative if it is supersonic. So this plays an important role. You will see, why I am stressing this? Overall stability, overall dynamic stability when you want to study, we have agreed that we will study the response of U, alpha, Q, perturbed quantities. To know U, alpha, Q, we need to solve perturbed equations of motion and to solve perturbed equations of motion, we need to know the values of FX, FZ and M. And FX, FZ and M depend on the U derivative.

So depending on subsonic supersonic, the responses will change and it may affect the dynamic stability character of the airplane. So it is extremely important to understand this in a real, physical sense. So you have understood what is DFAX by DU by DU1. Now let us go to the next.

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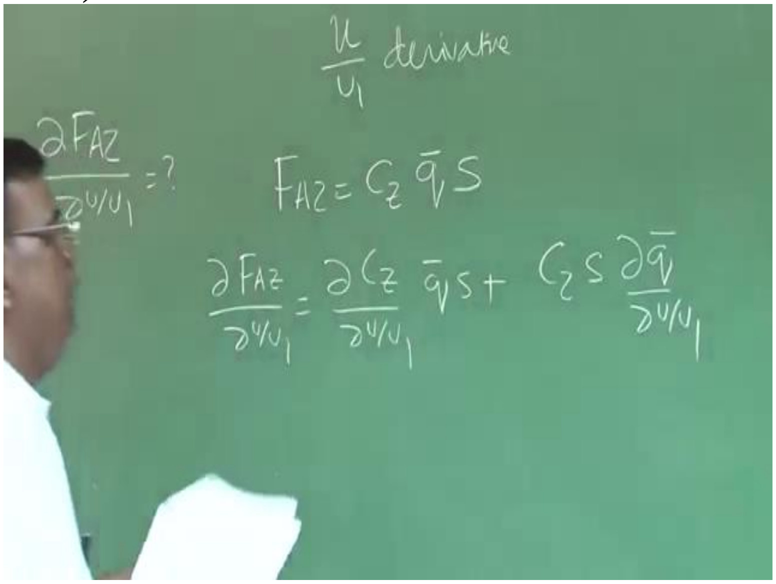

We are continuing with U derivative or we say U by U1 derivative. We have finished DFAX by by DU by DU1. Now we want to do something with DFAZ by DU by DU1 and the method is similar to what we have done. So let us write FAZ as CZQ bar S. That is where from we started and we want to find its derivative. So DFAZ by DU by U1 will be equal to DCZ by DU by U1 Q bar S + CZ DQ bar by DU by U1. By now you should be able to do yourself actually.

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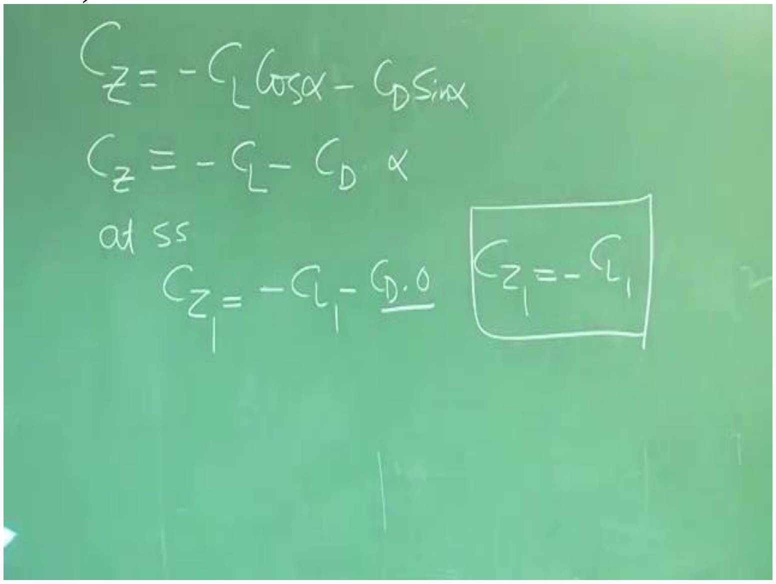

Now let us see from the earlier diagram, we can write, CZ is - CL Cos alpha - CD sin alpha and if I apply small angle approximation, this becomes CD into alpha because sin alpha is equal to Alpha and Cos alpha is equal to 1 for small angle approximation. After all we are using small perturbation theory and alpha is the perturbed quantity.

Now if that is true, I can write at steady-state, CZ1 by definition of steady-state and the notation we were using, so this will become - CL1 - CD into 0 because alpha perturbed is 0 as steady-state. So what finally we get is CZ1 is - CL1. This is one we have got.

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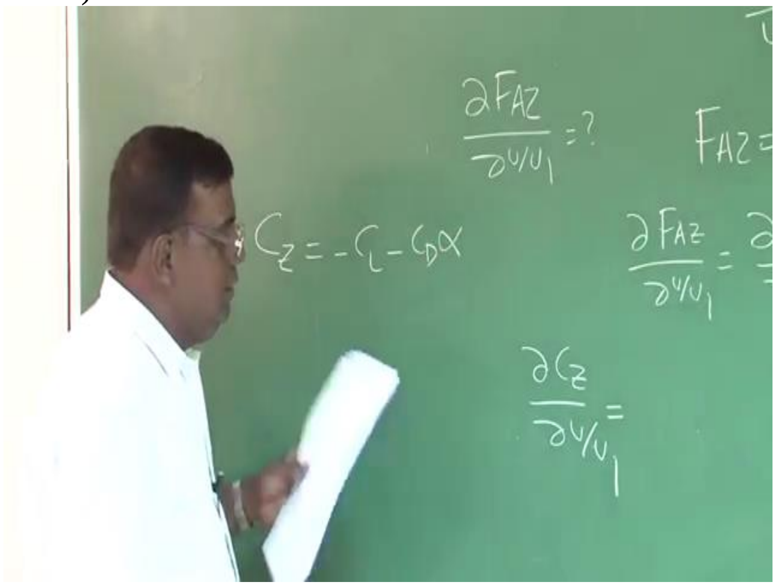

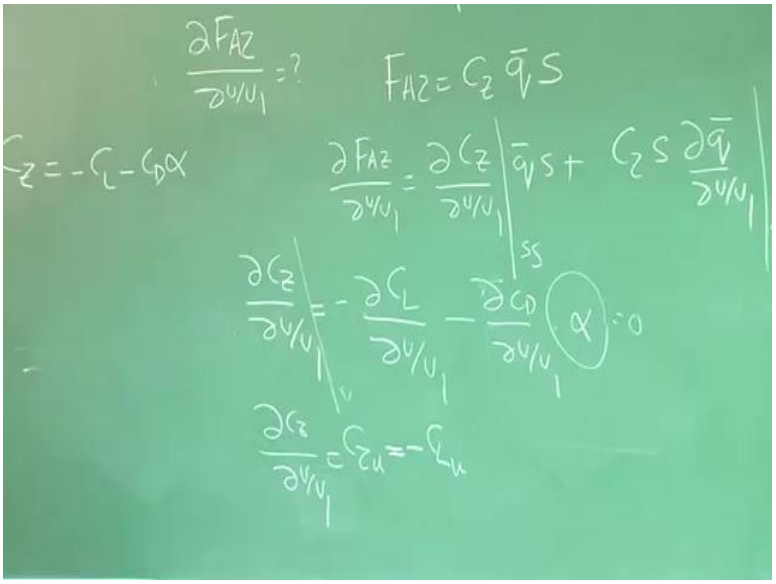

second thing, now we do DCZ by DU by U1 which is, this term we are trying to evaluate, please remember, we have to evaluate at steady-state. Do not forget one thing. Already we know how to handle this term, DQ by DU by U1. So let us see, this DCZ by DU by U1 and that I can write as bar from that expression, I use this. So let me write this here for your comfort.

CZ is - CL - CD into alpha. So this DCZ by DU by U1 I can write as - DCL by DU by U1 - DCD by DU by U1 into alpha. Right? Now you know that this has to be evaluated at steady-state. So what happens? At steady-state means, alpha is equal to 0. So what is it that you are getting? You are getting DCZ by DU by U1 is also known as, it can be denoted as CZU which is nothing but - CLU. So simple. So you know what is this term? Nothing but - CLU. You know what is this CZ1. It is nothing but CL1.

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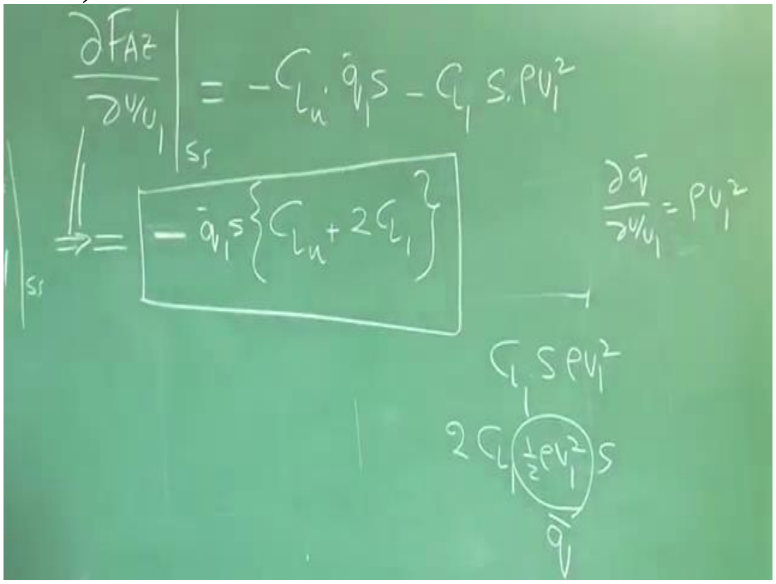

So if I want to calculate or if I want to derive the expression for DFAZ by DU by U1 at steady-state, I get, this is DCZ by DU by U1 which we have seen, - CLU into Q1 S + for CZ, we put - CL1. What is our aim? Our aim is to find out this expression, DFAZ by DU by U1 evaluated at steady-state is DCZ by DU1 evaluated at steady state is - CLU, then Q1 bar S + CZ. CZ is CL1S and DQ by DU1 already we have seen, we have derived it as Rho U1 square.

So if I now take Q1S common, so I get this is, I write it like this, - Q1S CLU + 2 CL1. That is what is this expression. Again, if you understand where from this term 2 came? Because it is here, this was CL1 into S Rho U1 square. So what we have done? We have written CL1 into 2 into half Rho U1 square S and for this, we have put Q bar. So that 2 is here.

Clear? No issues. So we have got these 2 terms, DFAZ by DU by U1, we have also got DFAX by DU by U1. Here, what is CL1? CL1 is the CL at steady state. What is our steady-state? It is the cruise. So what about the cruise CL? That is CL1. What is CLU? Let us understand that.

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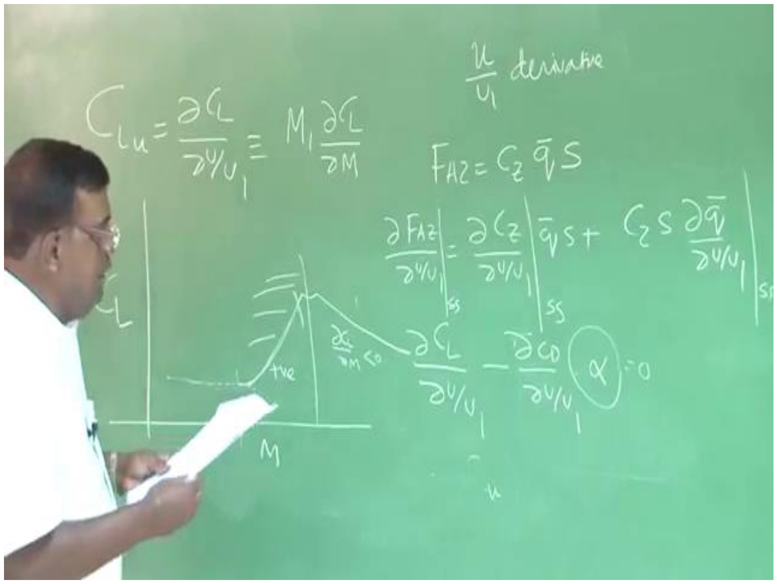

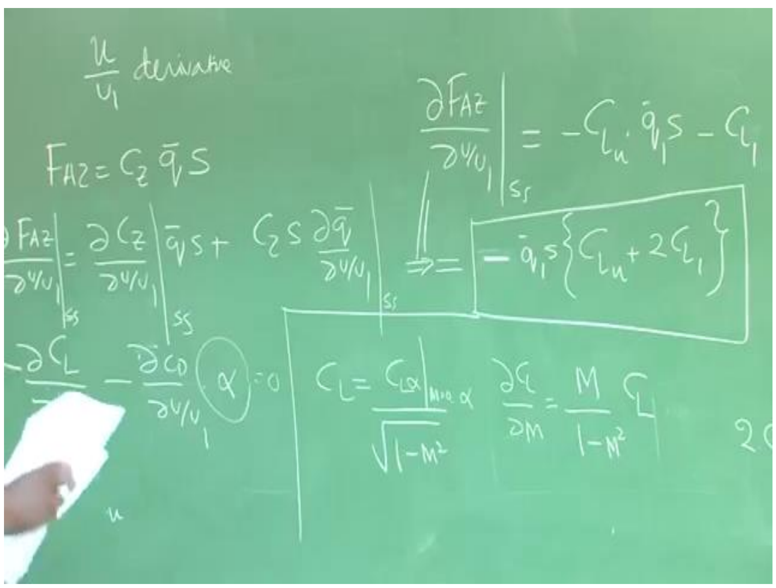

This is DCL by DU by U1 . As we did for CD, I can write it as M1 into DCL by DM. And if you see the variation of CL versus Mac number, also something goes like this. After 0.6, it remains almost constant. Then there is a dip, then it goes like this. You know this typically follows frontal gloriate transformation. And here also you could see, here in this region, beyond this transonic and up to subsonic, this slope is positive, DCL by DM is positive and here, DCL by DM is negative.

So again depending upon which flight regime you are flying the value of DFAZ by DU1 is also going to change. We could also see that there is an analytical way to handle this frontal gloriate transformation. And if you see, CL is defined as CL alpha under root 1 - M Square and this is at M equal to 0. This is the compressible case, this is a compressibility correction.

And now if we simply take a derivative, DCL by DM, we get an expression, M by 1 - M Square into CL. so analytically also, as a first-hand approximation, you can easily find the value of DCL by DM which will be required in finding the value of CLU. But for practical purpose, you need to do it in your own way through different testings. And when I am writing DCL by DM, please understand, I am talking about, I am conscious about the fact that depending upon the flight regime, this derivatives are going to be different.

The sign of the derivative is going to change. Okay, the final derivative in terms of pitching moment in longitudinal case which is a very very important derivative one that expression also we will be developing today in this module. Other expressions, we will develop in the next module. In that sense, this is the final.

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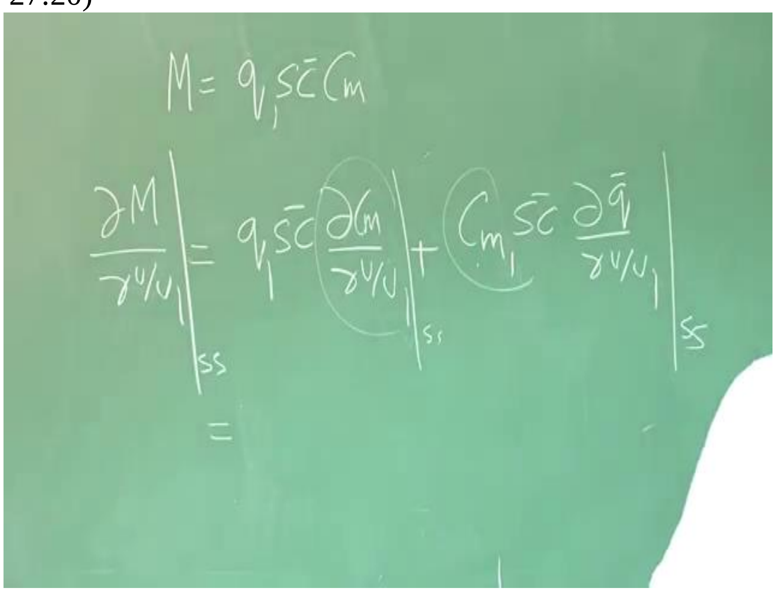

It is basically, if I say the pitching moment is written as Q SC bar into CM, we want to find DM by DU by U1 television nothing but Q1 SC bar DCM by DU by U1 + CM Smith Chart bar DQ bar by DU by U1. You have become experts now. You know that I need to develop this at steady-state. That is why, 1 I have put here. So I need to put 1 here as well. And also, this is at steady-state. This is at steady-state.

As far as D Q bar by DU by U1 is concerned, we know that we have 2 things to understand, what a CM1 and what is this CM by DU by U1? If you understand that, then our job is done.

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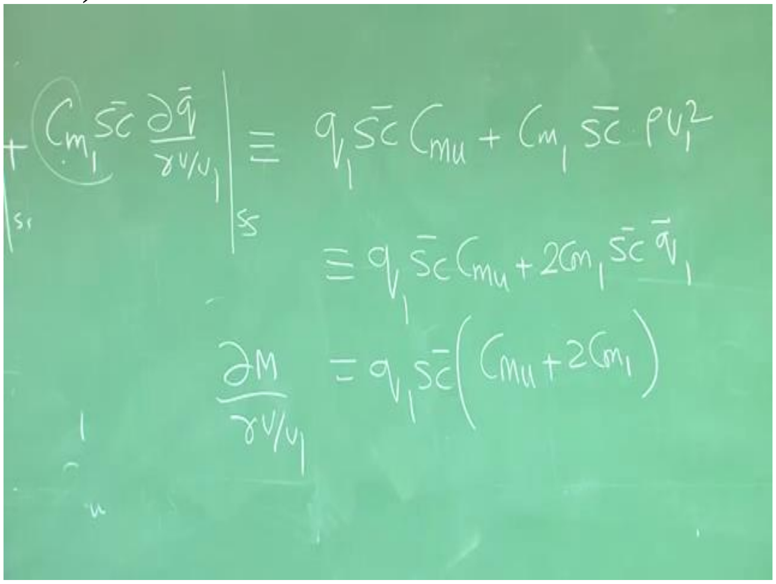

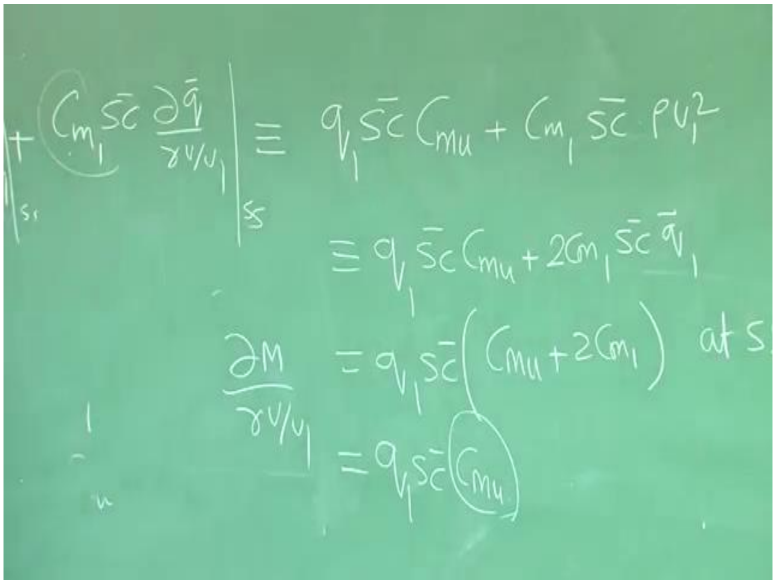

Here I do for the simplification. In terms of expression, I write Q SC bar CMU + CM1 SC bar into Rho U1 square. Because I know DQ bar by DU1 is Rho U1 square and which I can write again as Q1R SC bar CMU + 2CM1 into SC bar Q1 bar. Same theory, divide and multiply by 2 to incorporate Q1.

So I have Q1SC bar into CMU + 2 CM1. This is nothing but DM by DU by U1. Since we are not considering any cross contribution, CM 1 is CM at the, when the aerodynamic moment is 0. Otherwise for a trim, if thrust is there, then the moment due to thrust + the aerodynamic, the sum should be 0. But here since we are not considering that, so the aerodynamic moment has to be 0. So CM1 I am putting as 0.

At steady-state CM1 is 0. So I have got this expression as Q1 SC bar CMU. Please understand, once I put CM1 is equal to 0 because there are no thrust contributions and if suppose the thrust line was such that it was also giving a pitch down moment, then to trim the airplane, that pitch down moment and another pitch up moment, total CM has to be 0, the aerodynamic + moment due to the thrust.

But since we are not considering thrust, for trim, total CM has to be 0 means, the aerodynamic moment has to be 0. That is the understanding. And here, it is important to know what is CMU. And let us understand that.

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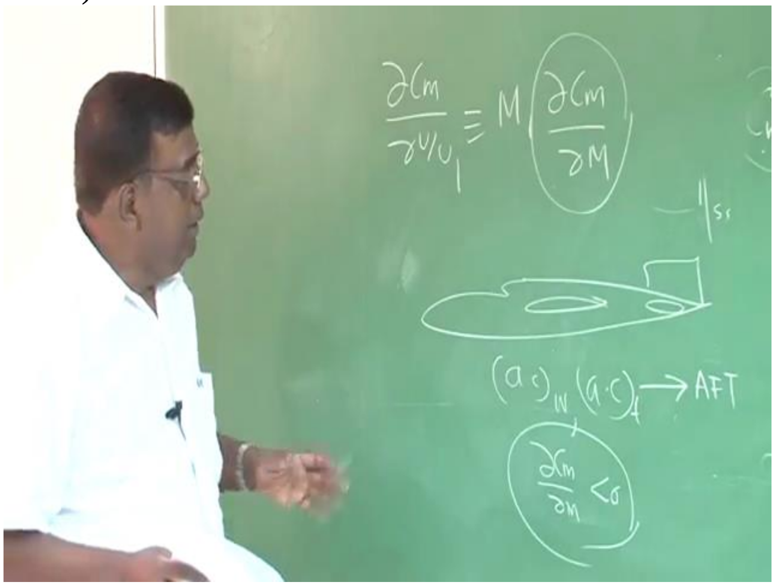

DCM by DU by U1, this again I can write it as, M1 into DCM by DM. Now focus here. Think of as plain. As the airplane is accelerating towards high-speed, the supersonic Mac number, what happens? The aerodynamic centre of the wing, aerodynamic centre of the tail, they move aft. So if you try to see what is the effect of aerodynamic centre moving, of the tail moving aft, that means the tail moment is increasing.

So there will be a pitch down moment. So DCM by DM will become negative. In the sense, it will give a pitch down moment. As I am going to accelerate, it is coming nose down. And that phenomena has to be known as tuck under. Accelerating and this unless the pilot is aware of this and he puts an appropriate elevator correction.

So from that point of view again you see whether you are at a lower subsonic speed or going to a supersonic speed, you need to be very careful about this derivative DCM by DM which you can estimate once you know how the aerodynamic centre is changing with respect to the Mac number.

So that is also a very important derivative. But all of these, you can estimate. Either approximately in analytical method, fairly, accurately through internal testing and more reasonably well through flight testing or all together in a collaborative manner, in an excellent way, all these derivatives can be captured within 10% to 15% depending upon what regime you are operating.

So today what I tried to explain you, how do I find these derivatives which are useful for characterising dynamic stability through solving the equations of motions which are perturbed the questions of motion and we try to understand how do I find these derivatives using the geometry of the airplane, using the flight regime of the airplane whether it is subsonic or supersonic, knowing the characteristic of the airfoil.

Once I know these, I know how to model the equation of motion to evaluate whether the airplane is dynamically stable or not. Towards that aim, we are inching forward. Thank you very much.

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