Aircraft Dynamic Stability & Design of Stability Augmentation System
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Module 3
Lecture No 14
Perturbed Aerodynamic Forces and Moments

Good morning friends. We have so far made the first attempt to develop perturbed equation of motion and we are more focusing on longitudinal motion. When I say longitudinal motion, we all understand we are talking about a motion restraint into vertical plane and there are 3 motions that we are considering. One is motion along X, translational motion, motion along Z, which is plunging motion and motion about Y axis which is pitching motion. These 3 motions we are considering.

To restrict our discussion for longitudinal case with an assumption that the disturbances are small. So that there are no influences of longitudinal disturbance of lateral or directional case.

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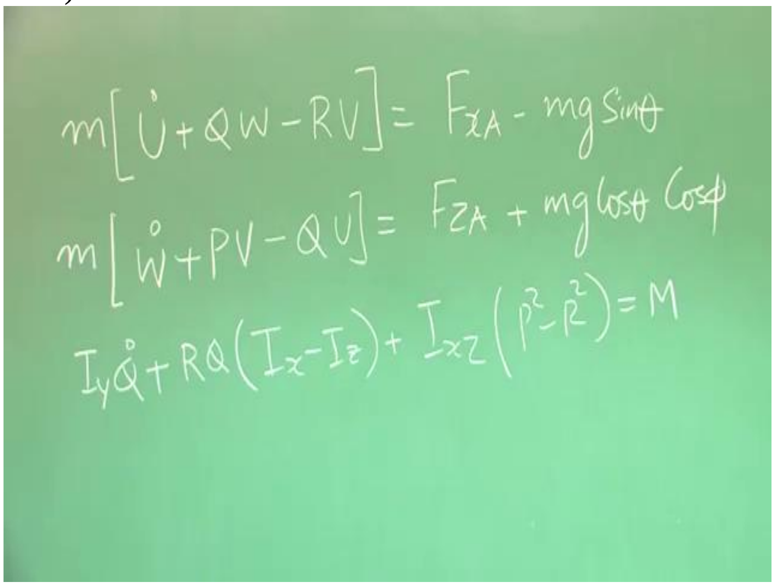

And if I recall, we had equations of motion of this form. MU dot + QW - RV equal to FXA - MG sin theta. This is one of the equations and along Z direction, the equation was MW dot + PV - QU equal to FZA + MG cos theta cos Phi. And the pitching equation was IYQ dot + RQ IX - IZ + IXZ P square - R square equal to M. Now we need to understand the right-hand side.

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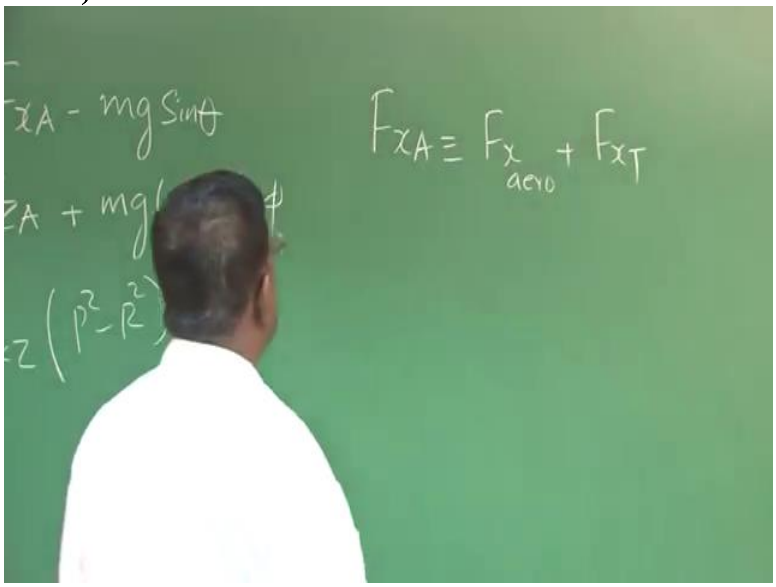

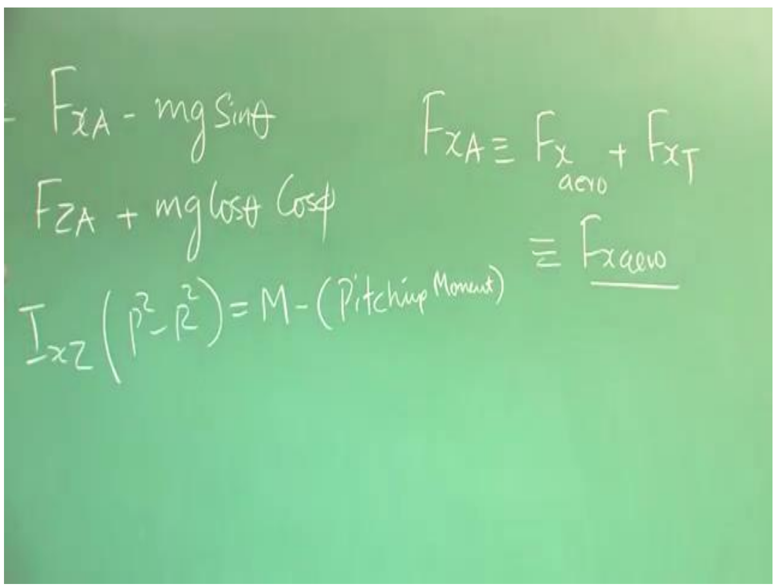

What is FXA? If you see, FXA is actually equal to FX aerodynamic + FX because of thrust. Similarly FZA is FZ aerodynamic + FZ because of thrust and M is nothing but pitching moment. This is nothing but pitching moment. To make our life simple, we will make an assumption that whatever treatment I do for FXA, it assumes that as if FXT is not there. Or whatever treatment I do for FX aero, similar treatment could be made for FXT.

We will give some examples as we mature. But at this point, we will assume that for our FXA, it is basically FX aero. With clear-cut understanding, if I understand how to handle FX Aero, FX because of aerodynamic, I know how to handle FX because of thrust also. We will understand this clear terms as we develop the equations, perturbed he questions of motion. So this is basically longitudinal perturbed equation of motion what we are looking for. And what was the approach?

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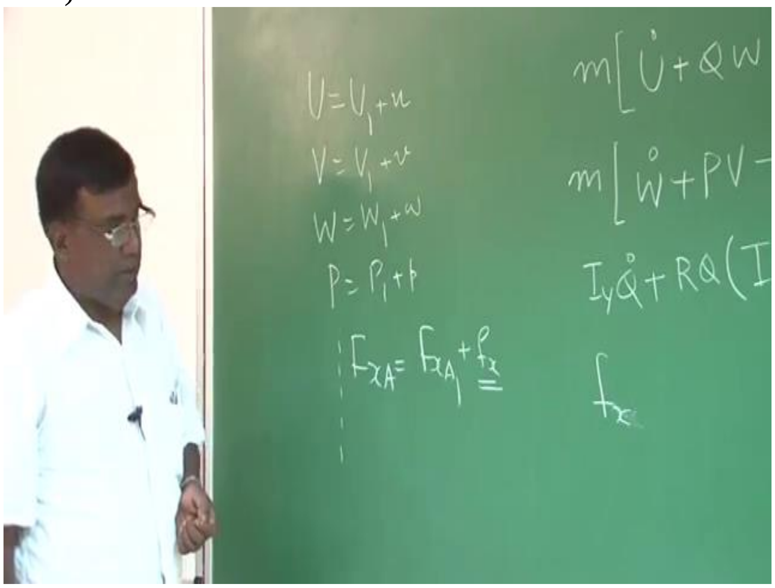

The approach was, we write U, total U after disturbance is introduced. It is U1 + small U. V is V1 + small V. W is W1 + small W. P is P1 + small P and like this with a clear-cut understanding that we are talking about small perturbation. So I can write it like this, that is total velocity after disturbance being introduced along U, V directions can be represented linearly by adding the perturbed quantity to the steady-state quantity. And this is strictly valid for small perturbations.

And as long as this is valid, our approach is valid. If the disturbance is such, I cannot write like this. Then our analysis is not valid. Okay? And what we did? We substituted this here and tried to cancel out few equations and derived finally if I have not mistaken, we got equation of the form, we got FX. This FX is aerodynamic. And where from this FX is coming?

Just for your clarity. We have got this is FXA1 + FX. So what is capital FX? FX was perturbed aerodynamic force. Please understand, this FX ideally will have perturbed aerodynamic force + perturbed thrust force. But we are telling, we are now understanding how to handle this assuming this to be only aerodynamic. And for thrust, we can easily extend this analysis and get the correct expressions.

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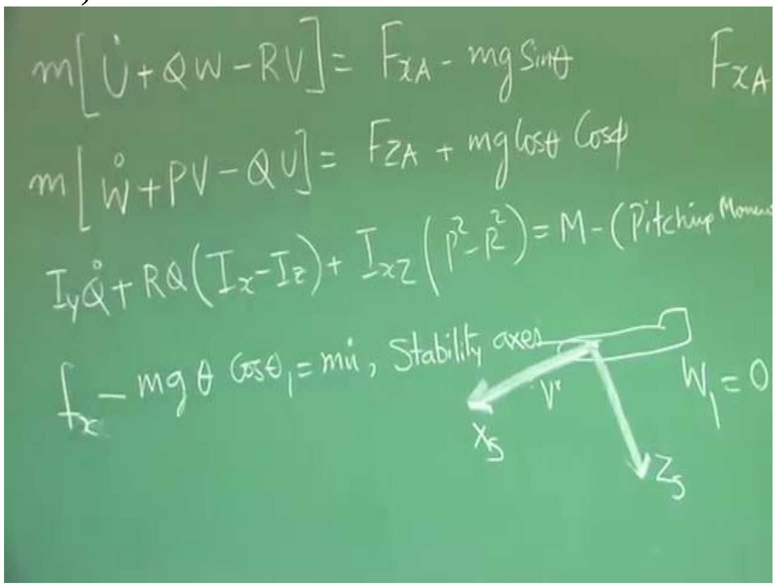

So once we do that, we got equations something like this, FX - MG theta cos theta 1 equal to MU dot. And please understand, here they have also use the concept of, if you see my last lecture, I have used the concept of stability axis and what was stability axis? Suppose this is the aeroplane and at steady-state, let us say this is the velocity vector V, total velocity, V. Now, I align my X axis such that X axis points to the total velocity, V and accordingly, Z also gets tilted.

And the advantage of using stability axis is that now W1, W component at steady-state is 0 because now X is along the total velocity in the vertical plane. So this was one equation we derived and then second equation also we had.

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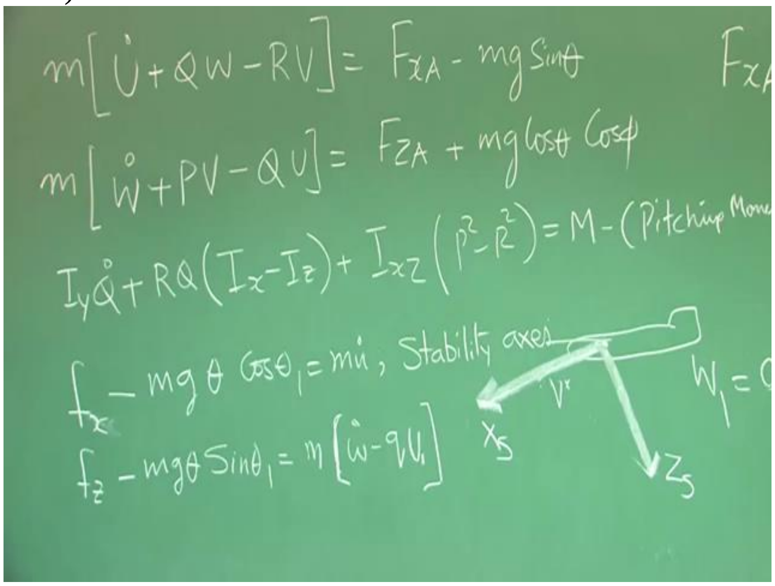

FZ - MG theta sin theta 1 equal to MW dot - QU1. This is another equation which is in Z direction and for pitching moment equation, it was straightforward. You could see again.

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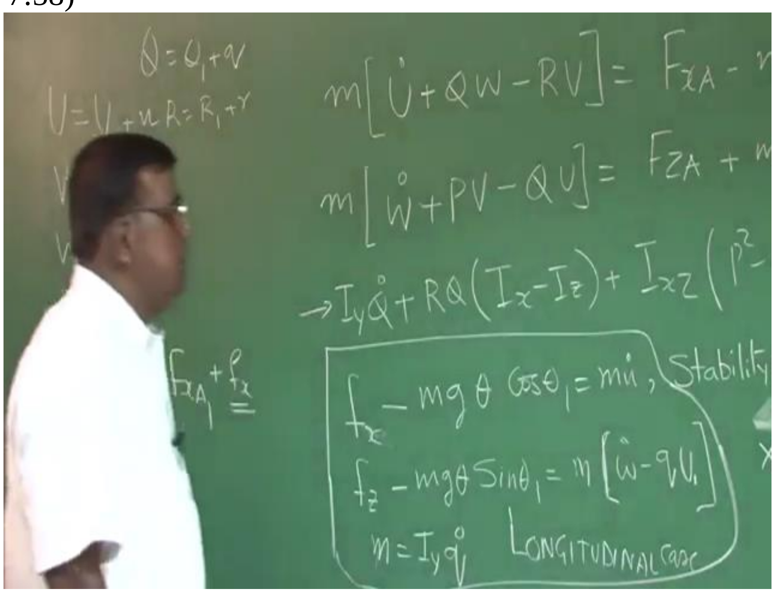

I use this equation for pitching moment and I write Q has Q1 + small Q, R is R1 + small R. Like that, theta is theta 1 + small theta. And all these things if I do here in this equation, I will get equation of the form; M is equal to IYQ dot. So these 3 are my perturbed equations of motion in longitudinal case. This is just to revise whatever we have done in the last lecture.

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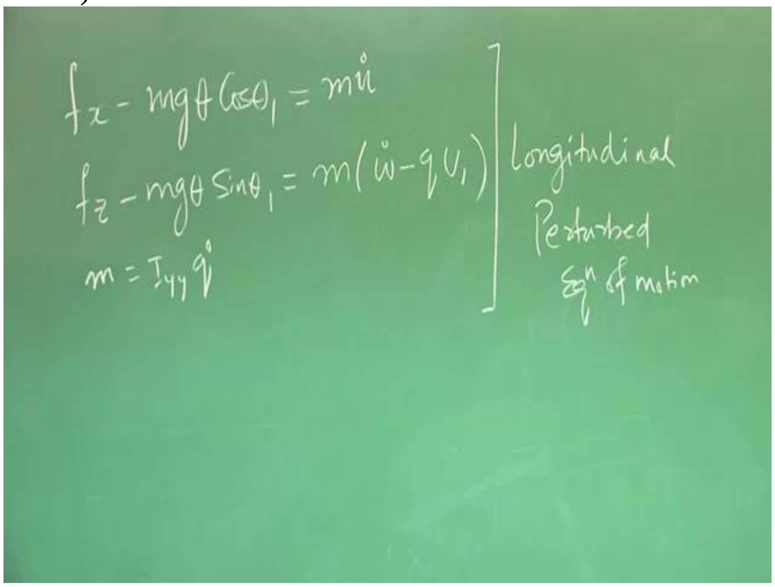

Let us carefully see what are these equations. These are longitudinal perturbed equations of motion. Meaning thereby. What we are saying? What is the small U? Small U is the perturbed velocity along local X direction. What is FX? FX is the perturbed aerodynamic force. What is FZ? FZ is the perturbed aerodynamic force along the local Z direction and this is the perturbed aerodynamic moment about Y direction.

And to see whether the aircraft is dynamically stable or not, I need to track the valleys of U, Q and W and from that variation we should be able to comment whether the aircraft is dynamically stable or not. That is why we have developed these equations of motion. But the question is, do I know how FX, FZ and M are changing because you understand, if airplane is at steady-state like this, and if there is a disturbance, then its angle of attack will change, pitch rate will change.

So the perturbed aerodynamic forces which comes because of lift and drag also will change. So I need to explicitly know how this FX, FZ and M are changing. To know that first I must know mathematically, what are those motion variables? What are those control variables on which FX, FZ, M largely depend.

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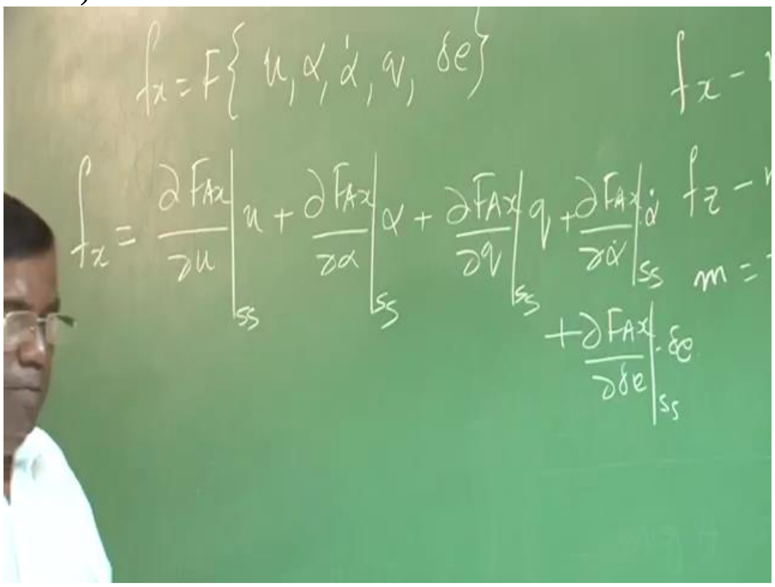

So we will write, at Fx it goes without saying, we are only taking aerodynamic forces because we know that if I can handle aerodynamic forces mathematically, the technique if I understand, I can extend it for thrust also. So we will write it like this, DFAX by DU into U + DFAX by D alpha into alpha + DFAX by DQ into Q + DFAX by D alpha dot into alpha dot + DFAX by D Delta E into Delta E.

Wherefore we have got all this? How suddenly we are writing like this? Because we understand, this FX is function of U, alpha alpha dot, Q, Delta E. This is the first model assumption. And this is more or less true for small disturbances. We know now, FX is function of U, alpha, alpha dot, Q and Delta E. So we can easily expand FX using the partial derivative concept for a linear system but we need to know that these derivatives are evaluated at steady-state.

Physically what does it mean? Suppose I am flying this machine like this and there is a change in alpha. So I need to find out what is the gradient in the sense of partial derivative, DFX by D alpha at steady-state. I evaluate that and then I multiply with perturbed quantities and add it up because we are talking about linear system. So I should be able to model FX as a function of U, alpha, alpha dot, Q, Delta E.

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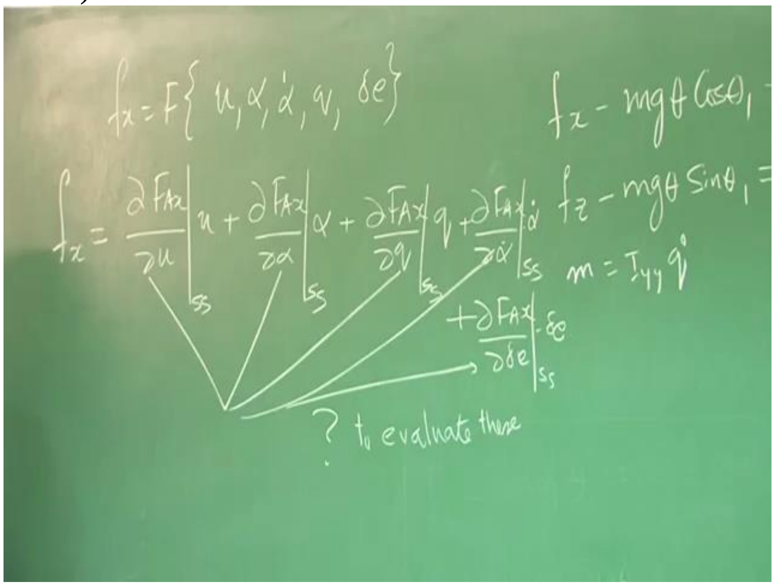

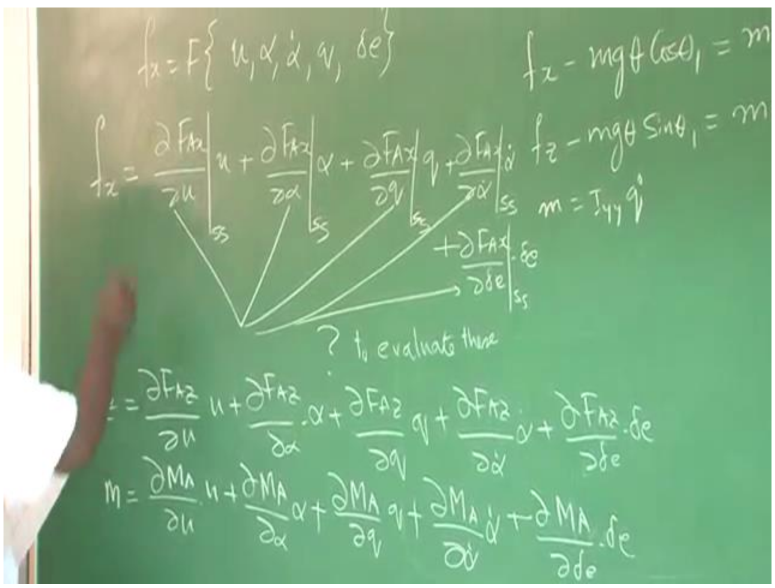

So the next question comes, do I know how to evaluate this? If I know how to evaluate this, I should be able to make our life simpler because now I will be knowing how FX explicitly is changing with motion variables and control variables. So let us take these derivatives one by one and try to see how it can be evaluated.

But before I do that, we also write, similar way FZ as DFAZ by DU into U + DFAZ by D alpha into alpha + DFAZ by DQ into Q + DFAZ by D alpha dot into alpha dot + DFAZ by D Delta E into Delta E. Similarly, you have become experts.

The perturbed pitching moment can be expressed as DMA by DU into U + DMA by D alpha into alpha + DMA by DQ into Q + DMA by D alpha dot into alpha dot + DMA by D Delta E into Delta E. So mechanical. Once I know this, I write this and once I want to write this, I also assume that FZ is also a function of these variables. Similarly, once I derive this, I also assume that M is also function of U, alpha, alpha dot, Q and Delta E.

So this is the first part. Now let us take one by one and see what are the little bit of mathematical manipulations required to get some meaningful answer.

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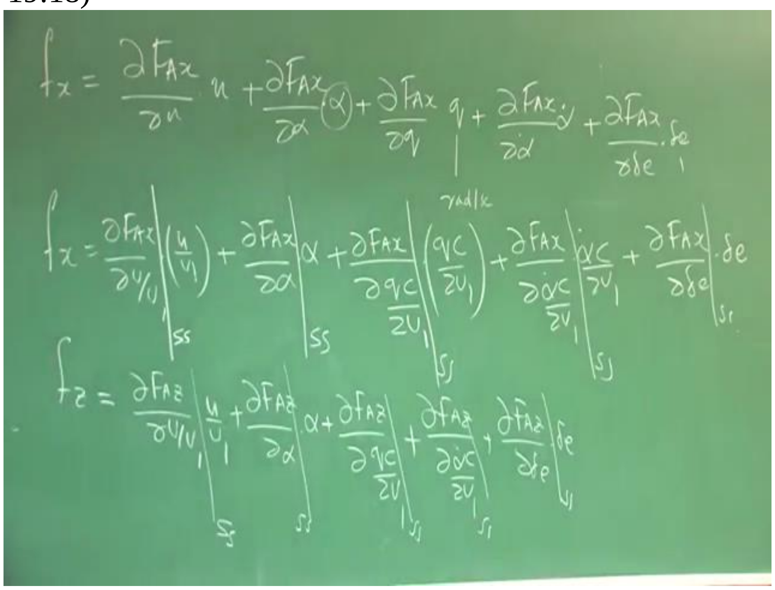

Let us say, we take FX. FX is what? It is DFAX by DU into U + DFAX by D alpha into alpha + DFAX by DQ into Q + DFAX by D alpha dot into alpha dot + DFAX by D Delta E into Delta E. Let us take first this case. Let us also see, what are the dimensions of alpha? Alpha is angle. It is dimensionless, radian. Q is radians per second, alpha dot is radians per second, Delta E is dimensionless, U is m/s.

Now, it creates a complicated mathematical inconveniences. What we want to do? We want to express FX using motion variables in a non-dimensional form. So what we will do? We will write FX as DFAX by DU by U1 into U by U1. We are trying to non dimensionalise the motion variable or control variable. Anyways, control variable is non dimensional. So what we have done?

This motion variable, perturbed U was m/s. So we have divided by U1, the velocity at steady-state X direction. And the derivative we have changed, DFAX by DU by DU1. So it is all the same. We are not changing the actual nature. Similarly here, we write DFAX by D alpha. Alpha is already dimensionless. Nothing to be done. But here when I come for Q, I find Q is a dimensional quantity.

So we do it like this which you are familiar, from Q radian per second and we are changing it to QC by 2U1 which is dimensionless. And similarly we do it here, DFAX by D alpha dot C by 2U1 into alpha dot C by 2U1. And here, I remain same and I do not do anything for the last term. That is already dimensionless, angle is dimensionless.

So what does this tell us? That yes, we agree that now these are all dimensionless but now you need to calculate these derivatives defined in this fashion at steady-state. So now our approach will be how do I find this derivative, DFAX by DU by U1? We will be referring this as U derivative. This, we will be referring as alpha derivative, this will be referred as Q derivative, this will be referred as alpha dot derivative, this will be Delta E derivative.

So if you understand that, it is not difficult for you to write FZ as DFAZ by DU by U1 into U by U1 + DFAZ by D alpha into alpha + DFAZ by DQC by 2U1 + DFAZ by D alpha dot C by 2U1 + DFAZ by D Delta E into Delta E. No problem. What is the catch point? Catch point is, do not forget, all these derivatives, partial derivatives, I need to evaluate at steady-state.

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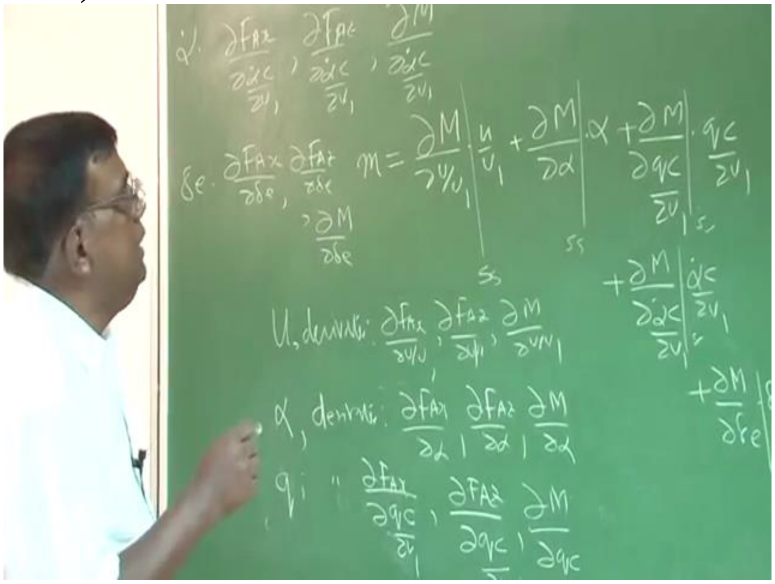

Similarly, now you can write M as DM by DU by U1 into U by U1 + DM by D alpha into alpha + DM by DQC by 2U1 into QC by 2U1 + DM by D alpha dot C by 2U1 into alpha dot C by 2U1 + DM by D Delta E into Delta E. It goes without saying that we have to again evaluate these derivatives, partial derivatives at steady-state, all of these. Once we know how to evaluate this partial derivative, I will know explicit form on which FX, FZ and M depend.

That is our motivation. And how we are going to categorise them, for our own understanding, we will say, DFAX by U, DU by U1, DFAZ by DU by U1, DM by DU by U1. They will be referred to as U derivatives. So, there are U derivatives like DFAX by DU by U1, DFAZ by DU by U1, DM by DU by U1.

There are alpha derivatives which will be DFAX by D alpha, DFAZ by D alpha, DM by D alpha. Similarly, Q derivatives will be DFAX by DQC by 2U1, DFAZ by DQC by 2U1, DM by DQC by 2U1. We will have alpha dot derivatives and Delta E derivatives.

So alpha dot derivatives will be DFAX by D alpha dot C by 2U1, then DFAZ by D alpha dot C by 2U1, and DM by D alpha dot C by 2U1. They are alpha dot derivatives and we have Delta E derivatives. That will be DFAX by D Delta E, DFAZ by D Delta E and similarly DM by D Delta E.

Please understand, when I write Delta E, I am talking about elevator. If the perturbations, because of any other control surfaces which are affecting longitudinal motion, let us say flap deflections, then the Delta E has to be replaced by capital Delta F for flap. This is to give you a basic idea. There will multi-control, longitudinal control.

So we have to go on adding those influencers. And another thing, what about these derivatives? We need to evaluate these derivatives at steady-state. And that is the catch point. Why we are doing all these things, let us again understand. We want to solve those 3 equations which represent longitudinal perturbed equations of motion where we need to know what is explicitly the expression for FX, FZ and M.

If I somehow calculate these derivatives based on the geometry of the aeroplane, based on the flight conditions, based on the environment conditions, if I could find it out, then I know explicitly how FX, FZ and M are changing with alpha Q, alpha dot, Delta capital E. Those I will plug in the equations of motion and try to solve it to see what is the behaviour of U, alpha Q, etc, those perturbed quantities to comment whether the aircraft is dynamically stable or not. Next lecture, we will be finding out. Thank you.