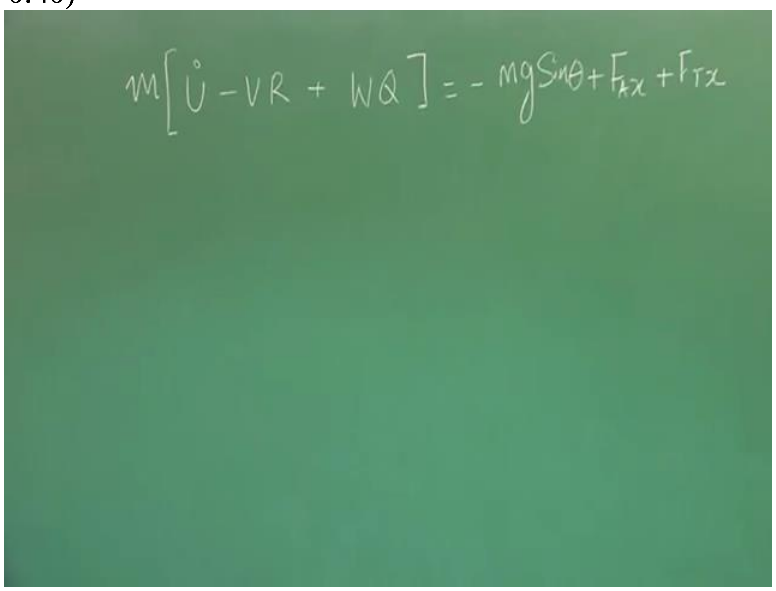
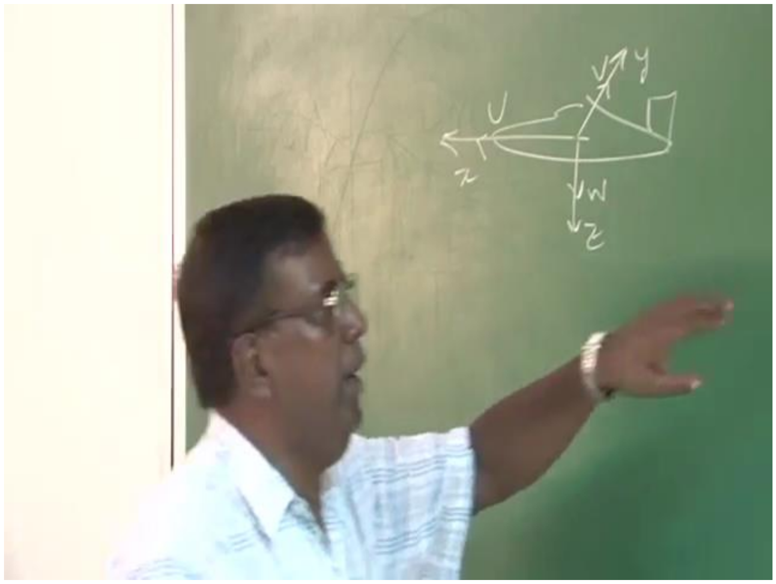
Aircraft Dynamic Stability & Design of Stability Augmentation System
Professor A.K. Ghosh
Department of Aerospace Engineering
Indian Institute of Technology Kanpur
Module 3
Lecture No 13
Small Perturbation Theory

Good morning friends. We are again back and in the last class, we decided that we will now develop perturbed equations of motion. When I say perturbed, it explicitly means that we are talking about small perturbation.

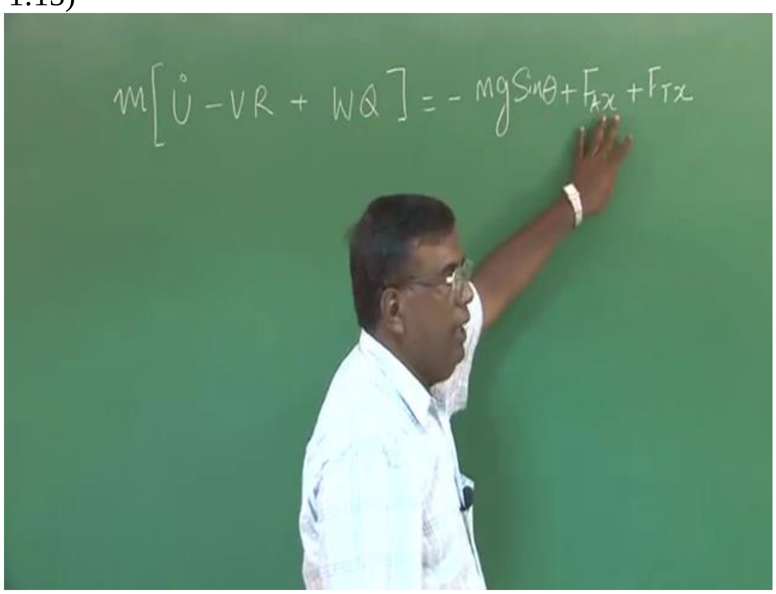
(Refer Slide Time: 0:40)

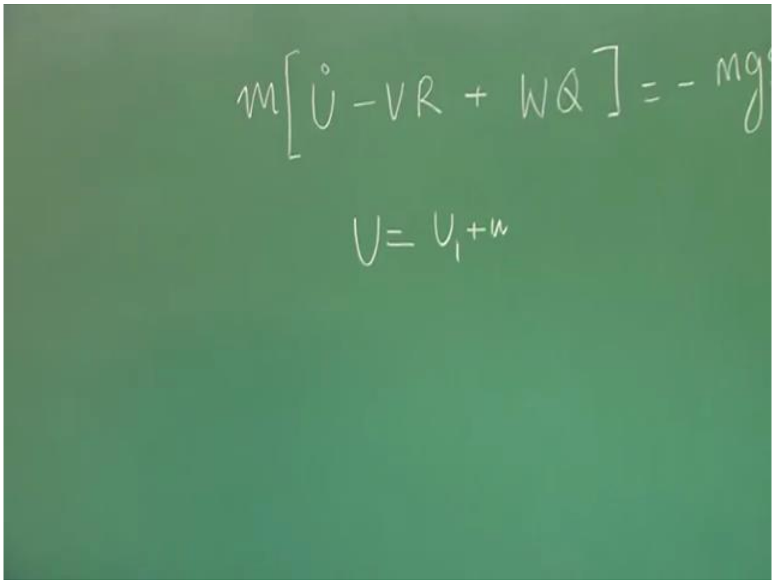




And to show you an example, I picked one equation that is along only fixed X direction. This is X, Y, Z, body fixed and U is the component of total velocity resolved along X. This U, this is V, this is W. And we are writing equations along X, Y and Z axises. One equation we are presenting here that acceleration along the X direction is represented by this equation.

(Refer Slide Time: 1:15)

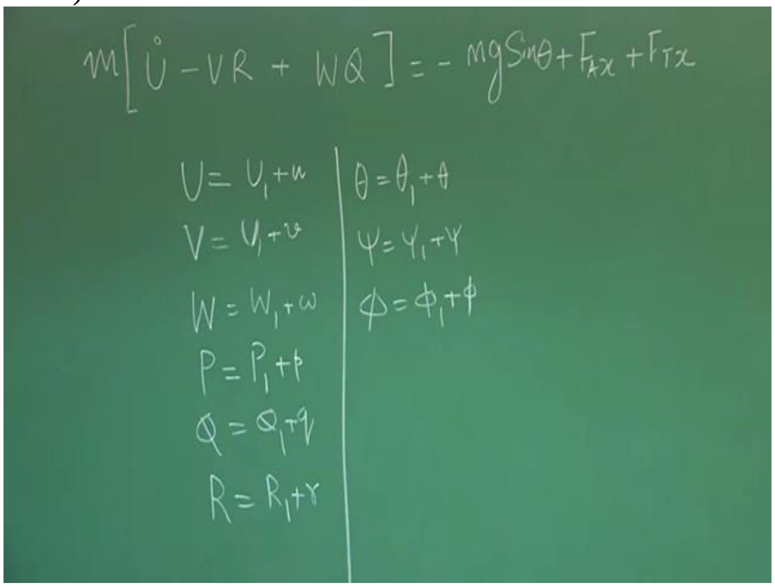




We very well know, these are the external impressed forces, these are the gravity, these are the dynamics and these are thrust of propulsion. We are looking for how to develop perturbed equation of motion and our understanding was U equal to U1, once the perturbation of small U is introduced, this perturbation is small. So we can use the (())(1:47). We also understand when I talk about small perturbation, the product of the perturbed quantity is negligible.

We are introducing this perturbation about the steady-state. Physically it means if it is flying level Cruise which is our steady-state in this case and let us say, I just give it a (())(2:06) deflection and I withdraw it. The velocity will change, the angle of attack may change. All those things, we are representing the change in the perturbed quantity equal to small U, small V, small W, like that.

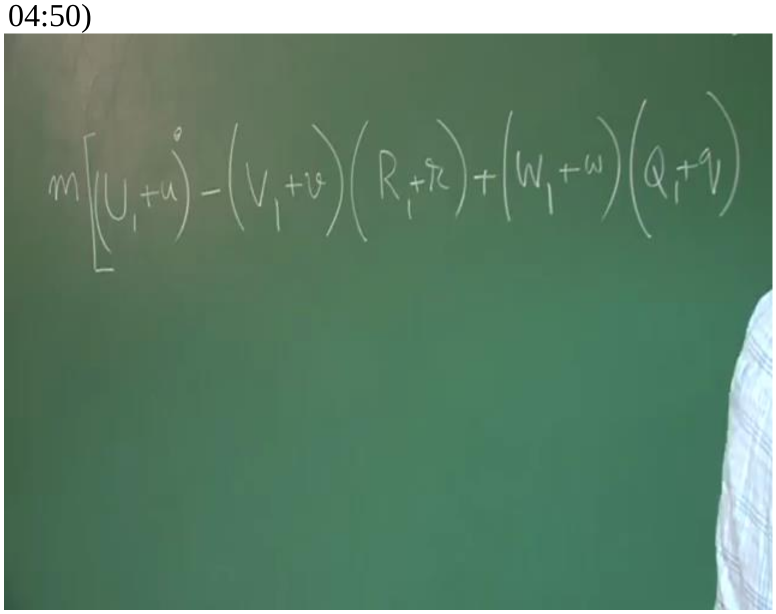
(Refer Slide Time 03:08)

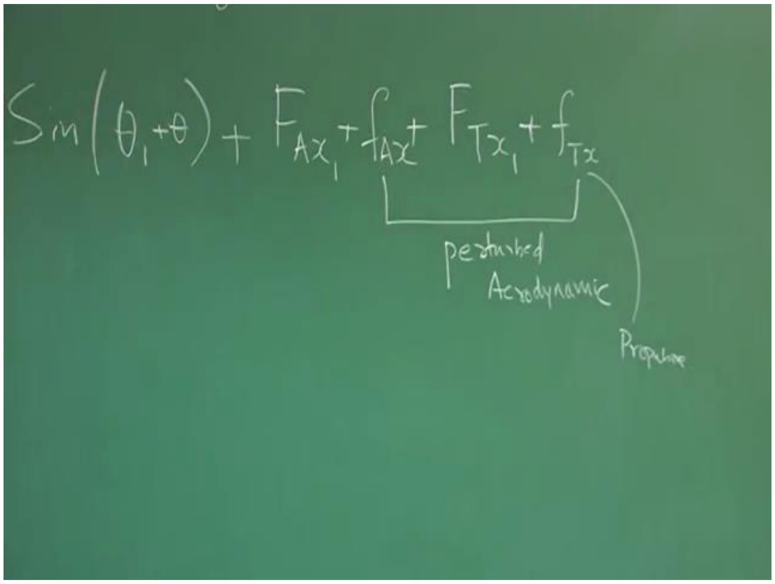


We say total V is equal to V at steady-state + small V, W is equal to W at steady-state + small W. Similarly P is equal to P1 + p. Q, Pitch rate is equal to Q1 steady-state + small Q. R is equal to R at steady-state + small R. Similarly, we will extend small perturbation understanding. We will say Theta is equal to Theta at steady-state + perturbed Theta.

Psi is equal to Psi at steady-state + perturbed Psi. Phi is equal to phi at steady-state + small phi. Now what do we do? We use this and substitute this in this equation. Let us see what happens. So what will happen?

(Refer Slide Time: 04:50)

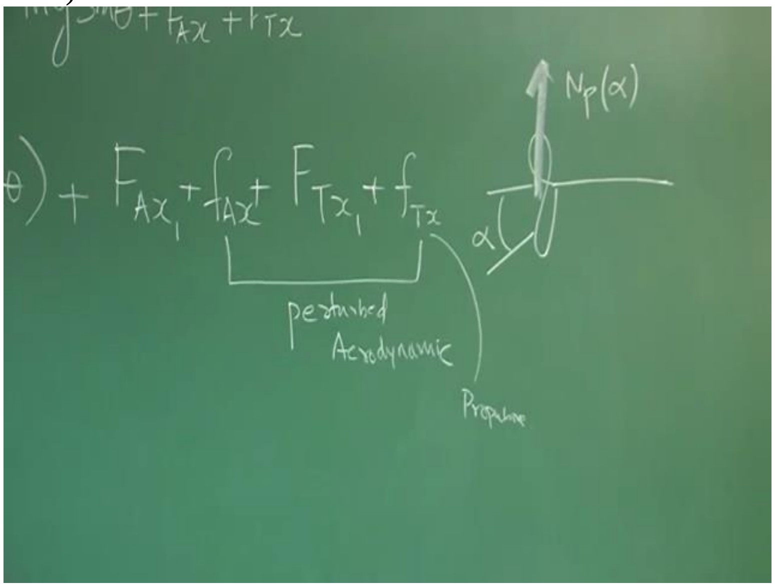




I write m, also I can write now, U1 + small U - V1 + small V into R1 + small R + W1 + small W into Q1 + small Q. This will be equal to - MG sin of Theta one + small Theta + FAX1 + fAX + FTX1 + fTX. What are these FAX and FTX? These are the perturbed aerodynamic and propulsive forces. So the question becomes, how this perturbed aerodynamic force will come?

If you understand, if I am flying at a cruise, then suddenly I give some disturbance, its angular retard will change. The angular retard will cause change in the lift, the drag, because the aerodynamic forces will change. Similarly for the propulsive side. If it is a propeller driven engine, as they change the angle of attack, there will be a change in the thrust. It impulses the direction.

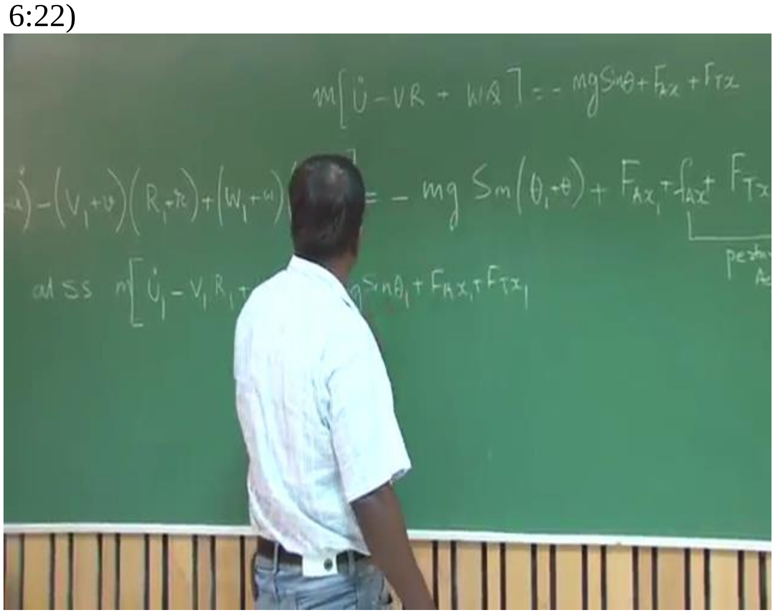
(Refer Slide Time: 5:34)

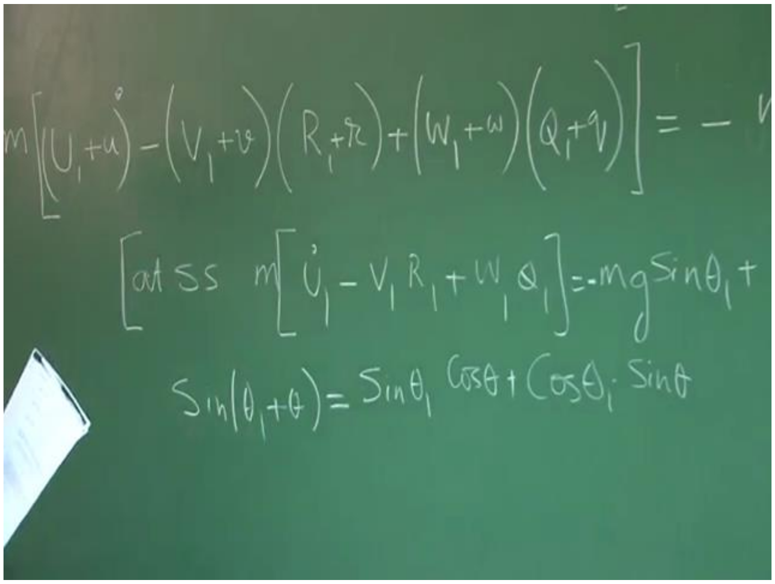


For example if I see, this is the propeller and suddenly there is a change in the angle of attack, alpha, you could expect a normal force will come, generated in the propeller which will have affected force as well as the work. So these are all perturbed. That means these are in addition to whatever you are experiencing at steady-state. Whatever you are experiencing at steady-state is FTX1 and this is because of alpha, because of change in alpha, change in Theta.

All those things are related to it when we change the speed also. You need to understand what is the physical meaning of this. After this, it is just a mechanical thing.

(Refer Slide Time: 6:22)

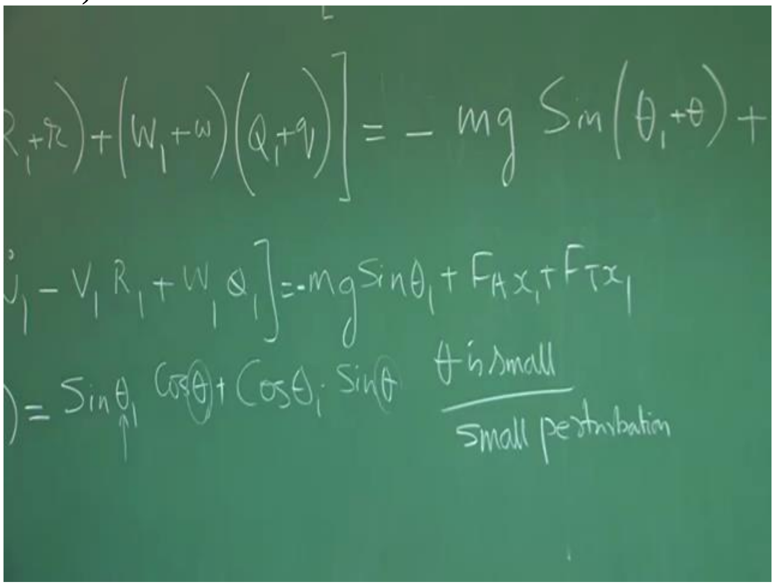


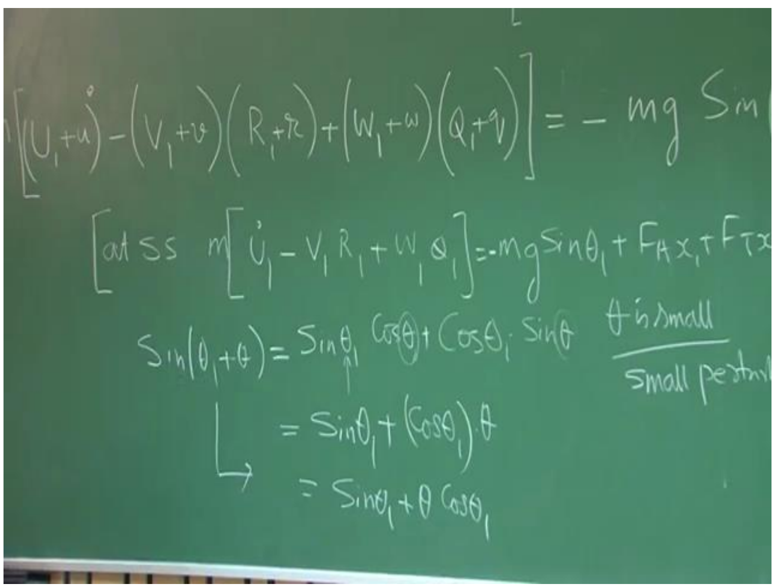


And you also need to understand that at steady-state, this is also identically true that MU1 dot - V1 R1 + W1 Q1 is equal to MG sin Theta1 + FAX dot + FTX dot. This is the relation. At steady-state, this is W1 dot - V1 R1 + W1 is equal to - MG sin Theta1 + FAX1 + FTX1. Now you this equation as well as use this trigonometric relationship, sin Theta 1 + Theta which is equal to sin Theta 1 Cos Theta + Cos Theta 1 sin Theta. I did sin A + B.

We are talking about small perturbation. Let us see here. Which one is the perturbed quantity here? Is Theta 1 the perturbed quantity? No, Theta 1 is the steady-state quantity. This Theta is the perturbed quantity. Our assumption is Theta is small because we are talking about small perturbation.

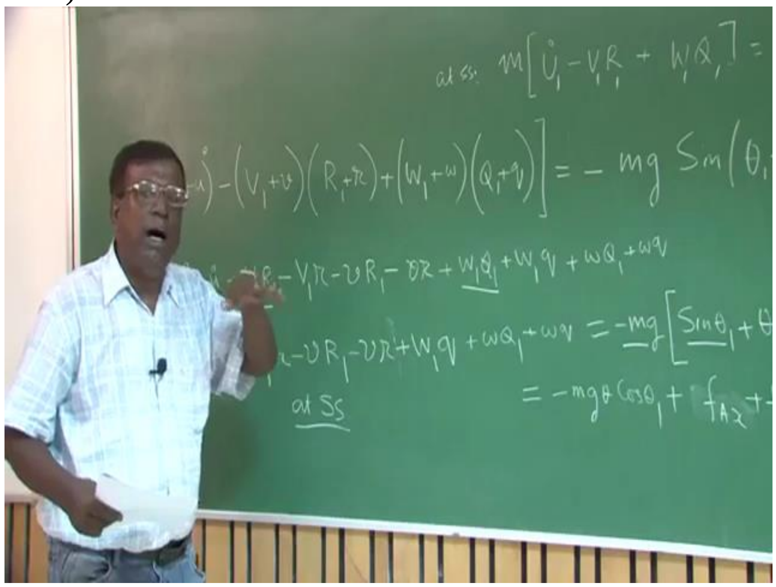
(Refer Slide Time: 08:10)

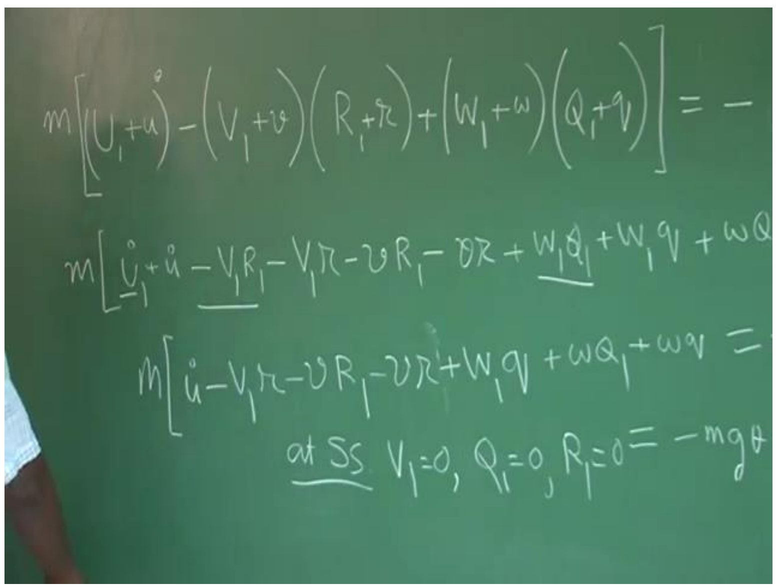




Theta is small. This comes from the fact that we are talking about small perturbation. Once I use this information, I can easily write it as it is equal to sin Theta 1 + Cos Theta 1 into Theta. If Theta is small, sin Theta is equal to Theta and cos Theta is equal to 1. So we are writing this as sin Theta 1 + Theta1 Cos Theta1. This is the simplification of this expression. If this is our, and by using this, a fraction of this (())(8:50) what do I do?

(Refer Slide Time: 9:00)





Look here. At steady-state, you know this means so now, see this equation. Here, if I write M V1 dot + U dot - V1 R1 - V1R - VR1 - VR. Now this particular V1R1, V1R, VR1 - VR. So here I get W1Q1 + W1Q + WQ1 + WQ is equal to - MG. Now sin Theta 1 + Theta, we could write as sin Theta 1 + Theta Cos Theta 1.

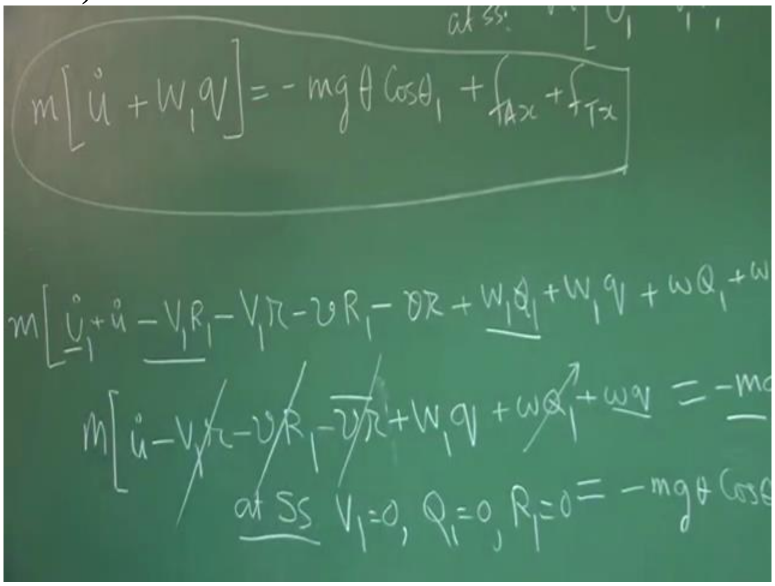
And then FAX1 + fAX + capital FTX1 + fTX. Yes, look at it. Now see here. If I see this, MU1 dot - V1R1 - W1 Q1 is identically equal to - MG sin Theta 1 FAX1 + FTX1. So I can take out the strums. So I will be left with M U dot - V1R - VR1 - VR. So this Theta comes. Then + W1Q

+ W Q1 + WQ. This will be equal to, this thing will cancel. Then - MG Theta Cos Theta 1 + fAX + fTX.

Here, I am using this. Also if I take a case where I write that at steady-state, what was our steady-state? A level cruise like this. What are the values of R1? Was there in yaw rate? No. Was there any pitch rate? No. So was there any value of R1? No. Was there any value of Q1? No. Because it is going like this, cruise. That means at steady-state.

So at steady-state, I can write V1 is equal to 0 because I am flying like this. There is no sign. So I am writing Q one is equal to 0, R1 is equal to 0. If I put this condition in this equation, what do I get?

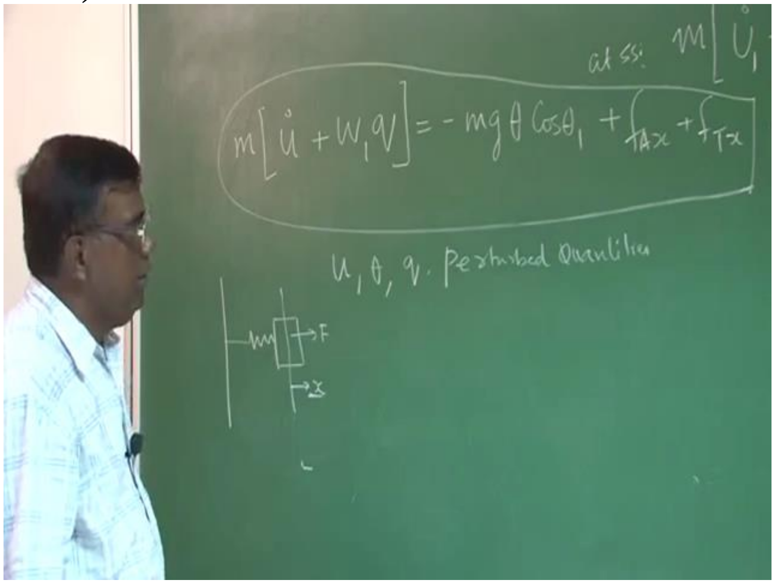
(Refer Slide Time: 12:55)



I will get from here, M U dot, V1 here goes, R1 here goes, we into R is a product of 2 small quantities, we are talking about small perturbation. So this man also vanishes. So I am left with + W1Q and then Q one is 0, so this also goes. And WQ is a product of 2 perturbed quantities. So that also vanishes. This is equal to - MG Theta Cos Theta 1 + fAX + fTX.

This is such a wonderful simplified equation. We can erase this which is taking a lot of space unnecessarily. (())(13:55) but to understand what we are doing.

(Refer Slide Time: 14:09)



If you see that if I do a small perturbation, then my equation along X direction, body fixed X direction is simplified to this but we should not forget, what are this U, Theta and Q? These are perturbed quantities. (())(14:26). Now because velocity along X direction, perturbed velocity along X direction is changing, the perturbed Theta, that is Theta with reference to the steady-state Theta, the perturbed Theta is changing or Q is changing.

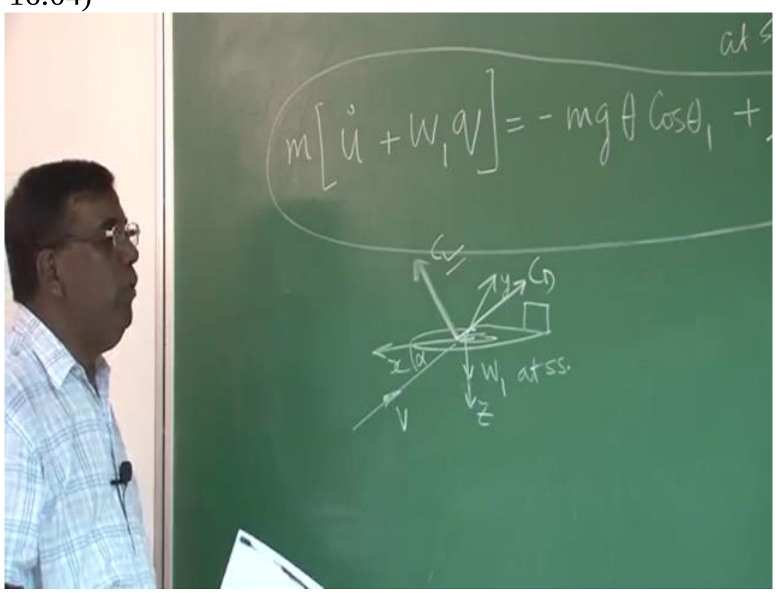
It is something like, if you go back to spring mass system, when we did spring mass system, if this is the reference, from here I am getting X. This X was perturbed quantity. So this X relevance to U, Theta and Q (())(15:04). Right? That it has 1 degree of freedom, this has got degrees of freedom. So we will get all those variables as perturbed quantities. We are talking about only one equation along X direction. So we have U, Theta and Q as perturbed quantities.

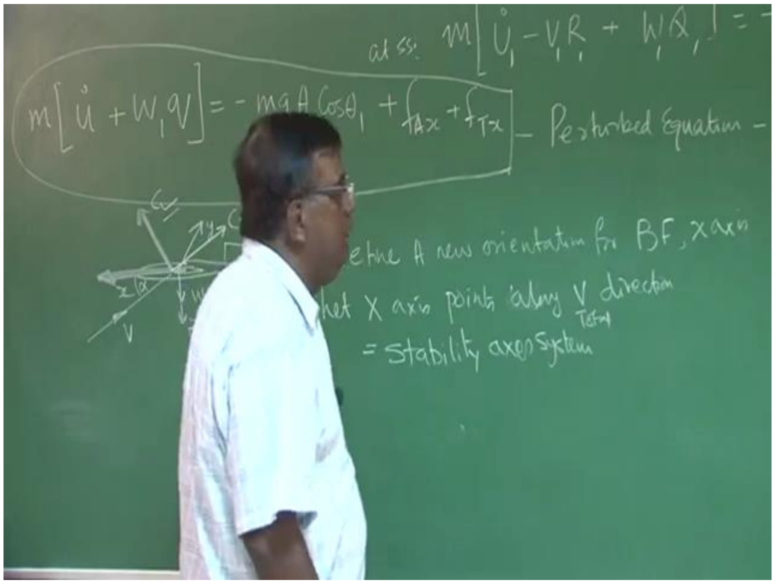
And we need to track this U, Theta and Q and see how they are changing. If they come to 0, then this system has come back to equilibrium. So it is dynamically stable. That is the motivation. (())(15:36) solving this perturbed equation.

(Refer Slide Time: 15:41)

For the first time, we know that once you know perturbed equation of course now X axis and you know very well you are operating in a body fixed X axis system. Let us have a global look to this.

(Refer Slide Time: 16:04)





When you draw this equation, we use, this is X body fixed X, body fixed Y, body fixed Z. What is the meaning of W1? W1 is the component of velocity along Z direction. At steady-state. That is very important. Let us say, the airplane is flying at 2 degrees of freedom. You know that what is the (())(16:35). For example, suppose this is the total velocity V and restricted to vertical plane.

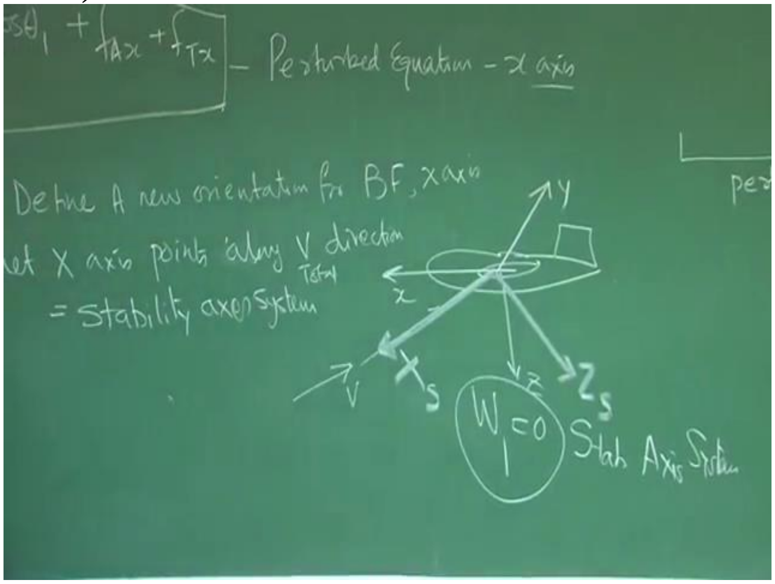
What will become W1? This is alpha. W1 will be a component of V along W1 direction or Z direction. So it has 2 components. One is V sin alpha. That component will be W1. If alpha is 0, W1 is 0. So you can see that as axis moves, the W1 component goes on changing. There is a smarter way of handling this.

For a simple reason, you know that the lift and drag, they will be always perpendicular, lift will be perpendicular to velocity and drag will be opposing it. So if I choose an axis system such that the X axis is pointed into the velocity vector in the vertical plane, then W1 automatically becomes 0 and it is very easy aerodynamics as far as mathematical manipulation is concerned. The lift will be perpendicular to the velocity vector along the local X axis steady-state.

Your W1 will always become 0 at steady-state. What is the meaning of that? We say, we define a new orientation for body fixed X axis. So far what was the assumption? The x-axis was aligned with the (inaudible 18:20) line. We were not very particular. Now we are putting a constraint. We are saying, let X axis point towards or point along V direction, V total.

And we are restricting our discussion in vertical plane. And such axis system is also known as stability axis system.

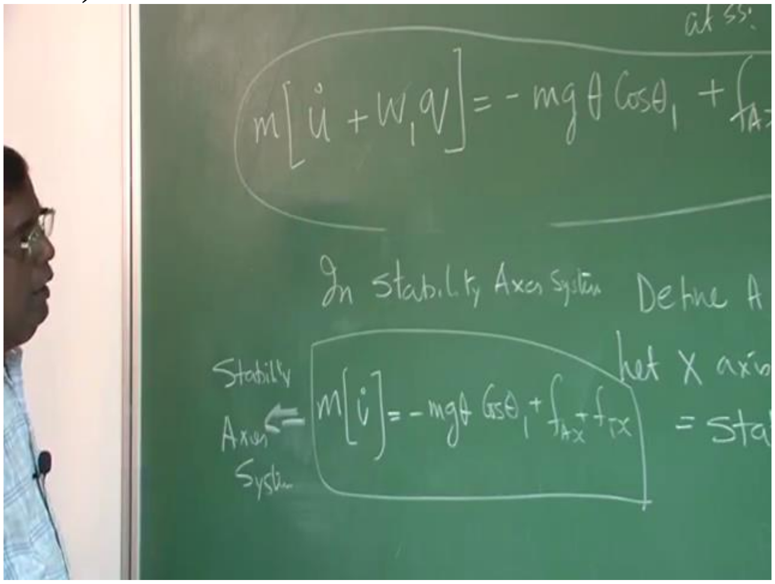
(Refer Slide Time: 19:02)



If we (())(19:03) means if this is the aeroplane, this is your X, this is your Y and this is your Z so at steady-state, what I am saying? The velocity vector will come like this. I am actually finding my X at steady-state, XS. And this is also my ZS. This axis at Z is the stability axis system. What is the advantage of that? The moment I am using stability axis, axis at ZS, we immediately see that W1 will become 0.

Because the velocity vector is along this direction it cannot have any component that belongs to XS. So in stability axis system, W1 equal to 0. What are the 2 advantages that we get as per aerodynamics computation on manipulation are concerned? We know our lift will be perpendicular to this XS. Y will be like this. Similarly other advantage we get is that W1 becomes 0.

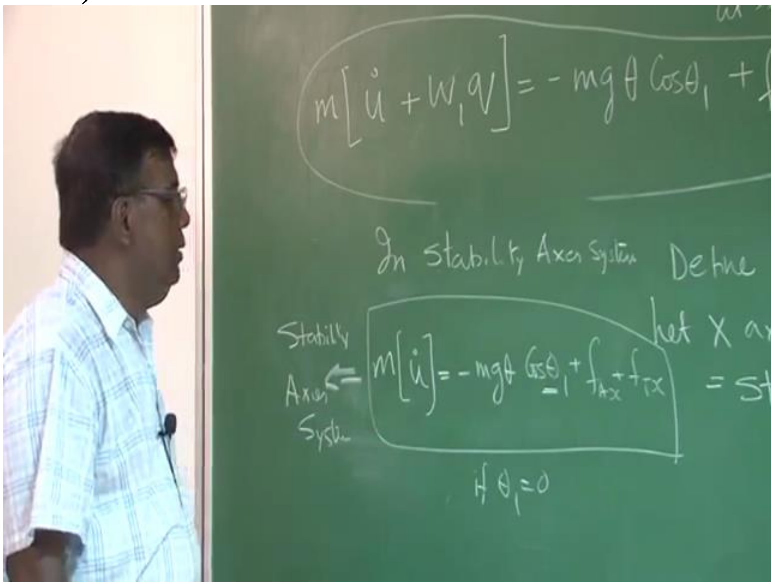
(Refer Slide Time: 20:25)



So this equation now, in stability axis system, this in stability axis system, this will become M U dot, this man will become 0. This will be equal to - MG Theta Cos Theta 1 + fAX + fTX. Further simplified. Please note that this is our stability axis system. You could very well understand that the moment I am talking about axis as ZS, the moment of inertia about axis at ZS will not be same as moment of inertia about X and Y and Z.

Small computation you have to prove (())(21:23). If the angle is very small, computationally, it will not make much of a difference but normal. So now you could see that this is the perturbed equation of motion and what all things we know? We know Theta 1. Because Theta 1 is the value of pitch angle at steady-state.

(Refer Slide Time: 21:50)

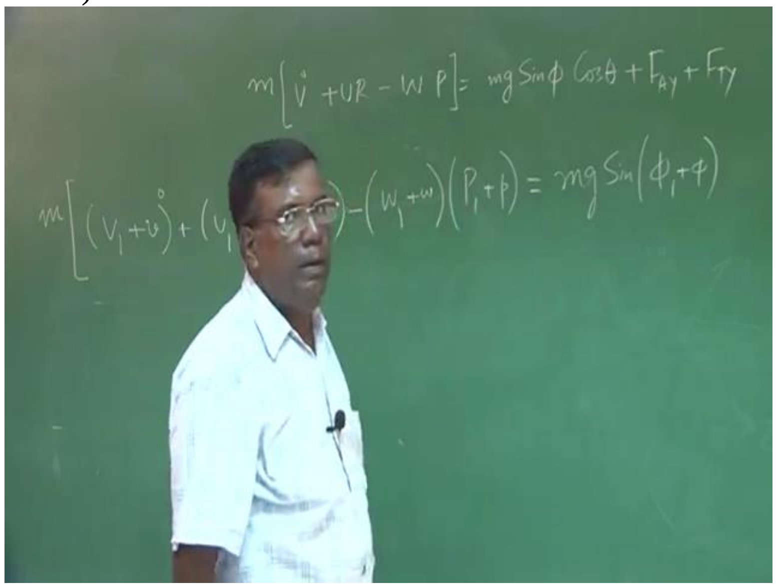


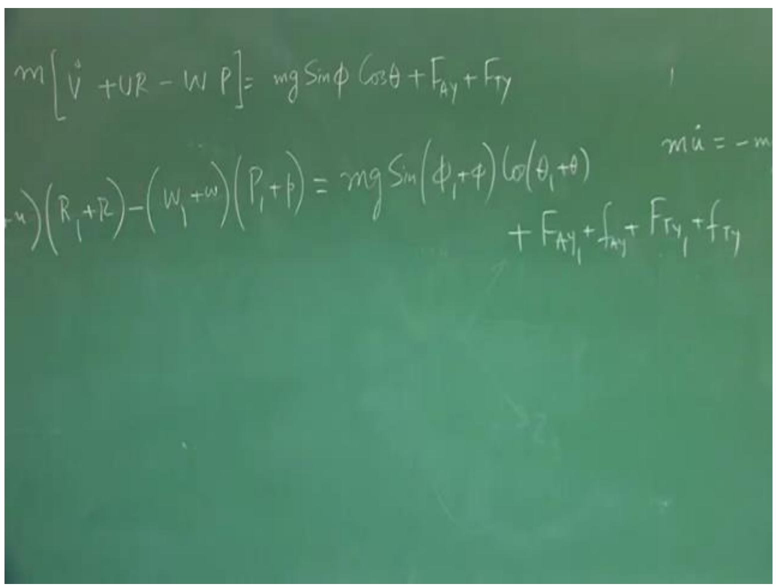
This is 0 if. If Theta will is 0, it gets further simplified to capital MU dot is equal to - MG Theta + fAX + fTX. Now, we need to know what is fAX, we need to know what is fTX? This is a perturbed aerodynamic force. This is a perturbed repulsive force. I need to know how do they change with alpha, how do they change with alpha dot, how do they change with Q and so many other motion variables.

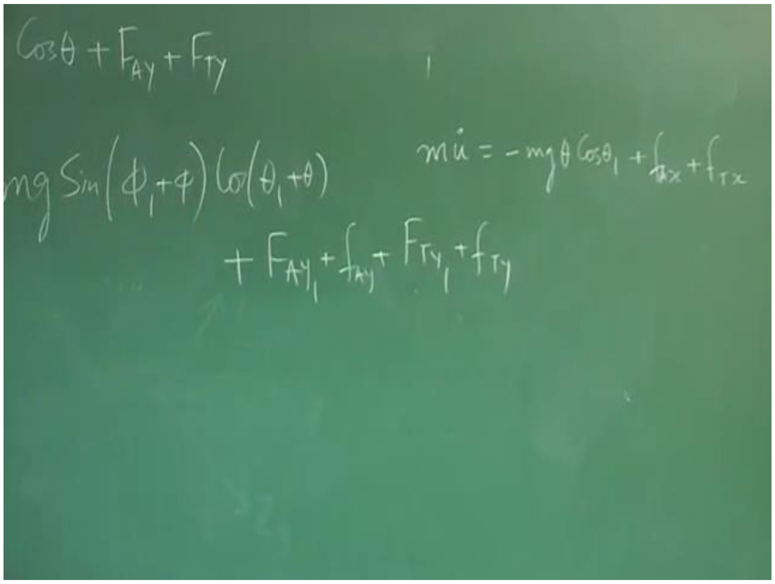
That will come later. As of now, we have simplified this general equation of motion and we have developed first equation in stability axis system. The first equation, capital MU dot - MG Theta Cos Theta 1 + fAX + fTX. This is nothing but perturbed equation along X direction. Our aim was to get perturbed equation. Successfully, we have developed this and we have tried to understand the small small understanding so that once we use this equation, we can (())(23:01)

We have demonstrated how to develop perturbed equation of motion. We have taken an example of U dot. I will quickly go through the other 2 or 3 equation. Since you have understood the first one, I am sure you will want to follow it and do some communications (())(23:27). So what we will do? We will see how much you have understood.

(Refer Slide Time: 23:33)







What I can do for V? For V, I should write V1 + small V dot. For U, I will write U1 + small U. For R, I will write R1 + small R. For W, I will write W1 + small W. For P, I will write P1 + small P. This is equal to MG Sin phi 1 + phi. What is phi1? Phi is the value of bank angle at steady-state. Cos of Theta 1 + Theta + FAI1 + FAY + FTY1 + FT1. (())(24:28) specially FAY1.

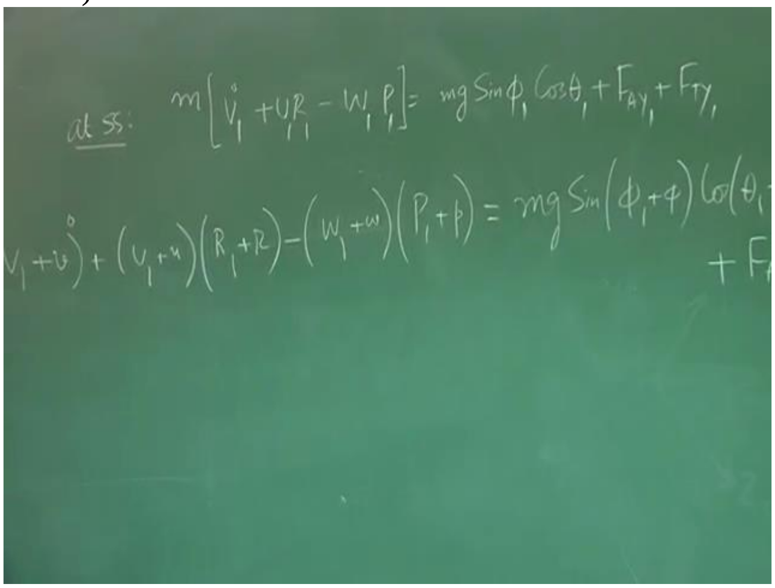
It is the aerodynamic force acting along Y direction. You can understand. If the aeroplane is moving like this, cruising just like this and if there is no side wind or side component of velocity, there should not be any force along Z direction. But the perturbation and the type of wind that the machine is flying in, we can have forces depending on what sort of perturbations we are working on.

For example, if because the reflection, the airplane started doing like this then we will have some forceful vibrations as well in the vertical plane. As the aeroplane moves like this, the vertical plane pushes here towards left and I get a force along the Y direction. So all these minor points we will see.

Those will be incorporated in FAY perturbed quantity. As far as steady-state is concerned, we have exclusive understanding of what is the adult force acting along Y direction. If there is something or not that we see depending upon the situation. There could be a component along Y direction.

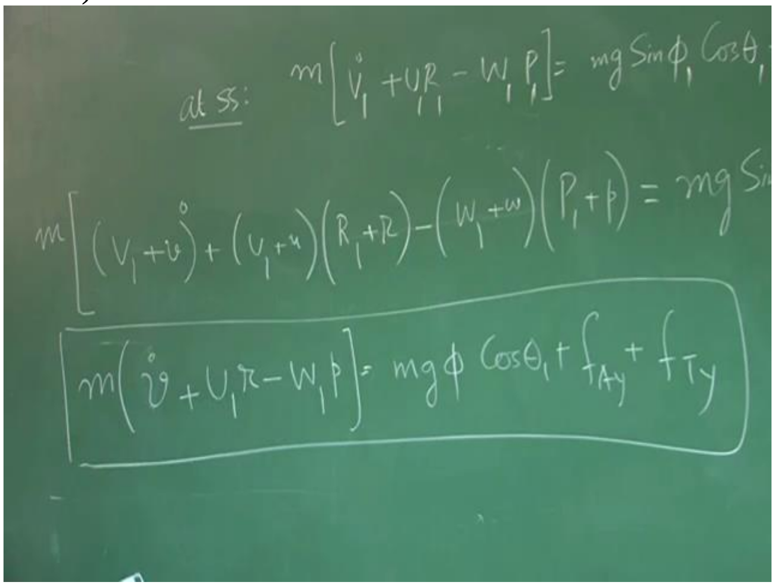
That is why we have FTY1. That is negligible but we never know depending on what is the orientation of the aeroplane. I feel, I will not like to have this. But sometimes in different air planes, you find some values that do not get disturbed.

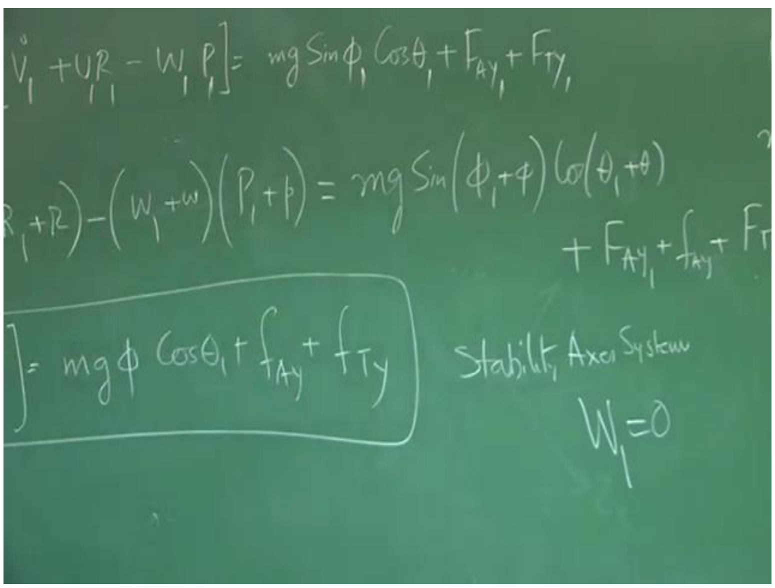
(Refer Slide Time: 26:15)



Also, we need to use, at steady-state, I need to know that this W1P1, phi 1, Theta 1, 1, 1, (())(26:27) these are steady-state conditions. Expand this, take out this and then expand Sin phi 1 + phi Cos Theta 1 + Theta, and use this small angular perturbation, Sin Theta is equal to Theta and Cos Theta is 1. Then the product of 2 perturbed quantities is neglected. When I do that, I get an equation of this form.

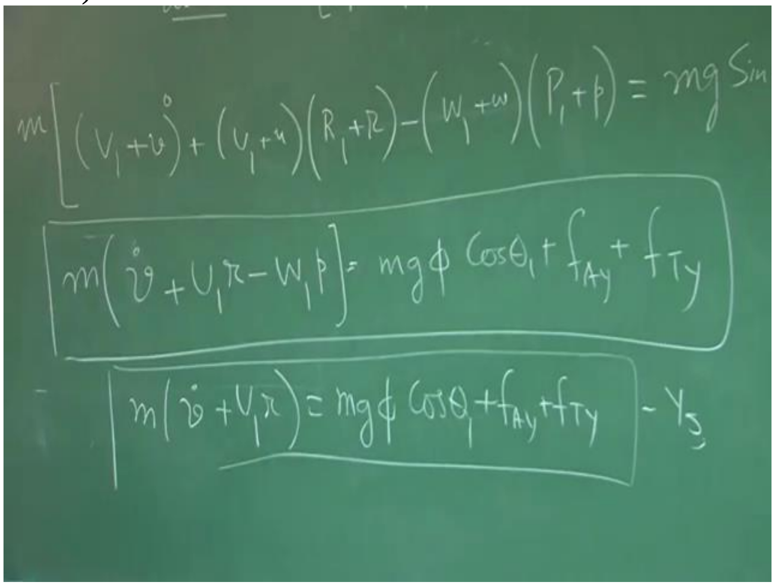
(Refer Slide Time: 26:57)





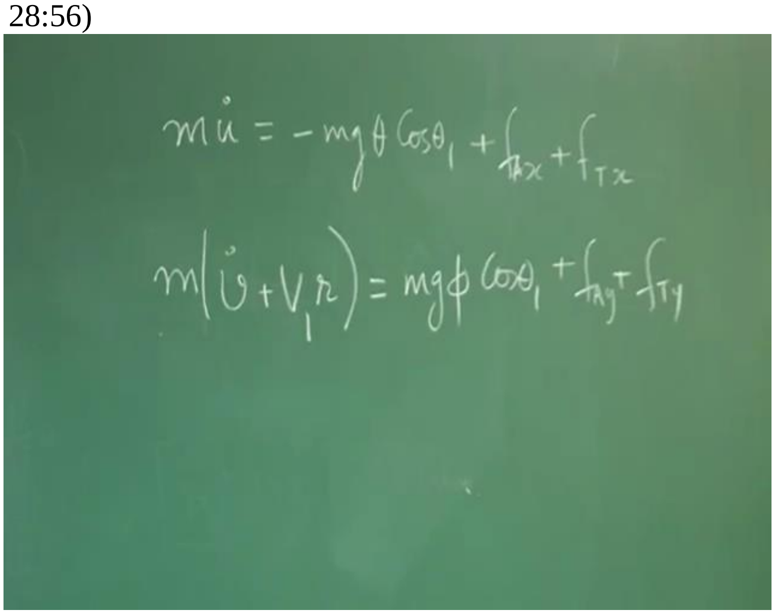
MV dot. Please understand, these are perturbed quantities. V dot + U1R - W1P equal to MG phi Cos Theta 1 + fAy + fTy. These are perturbed quantities. This is the second equation we get. What is the value of W1? We have not stated that we are using a stability axis system after this point for this very question. The moment we say unicast system, we will be using stability axis system. That means W1 is equal to 0.

(Refer Slide Time: 28:05)



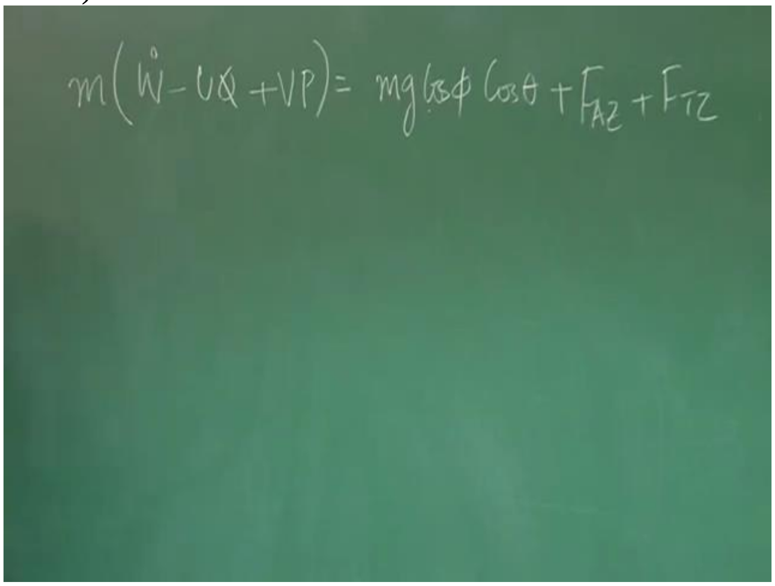
This equation is further simplified to MV dot + U1R equal to MG phi Cos Theta 1 + fAy + fTy. This is second equation which is along Y axis but we say YS, stability axis. Then, what is phi here? Phi is a perturbed bank angle. Because phi 1 was 0. Because Omega is the disturbance, it may start banking like this. So that means, that change from the steady-state of perturbed quantity. This is our second equation. So right now, I write this equation here.

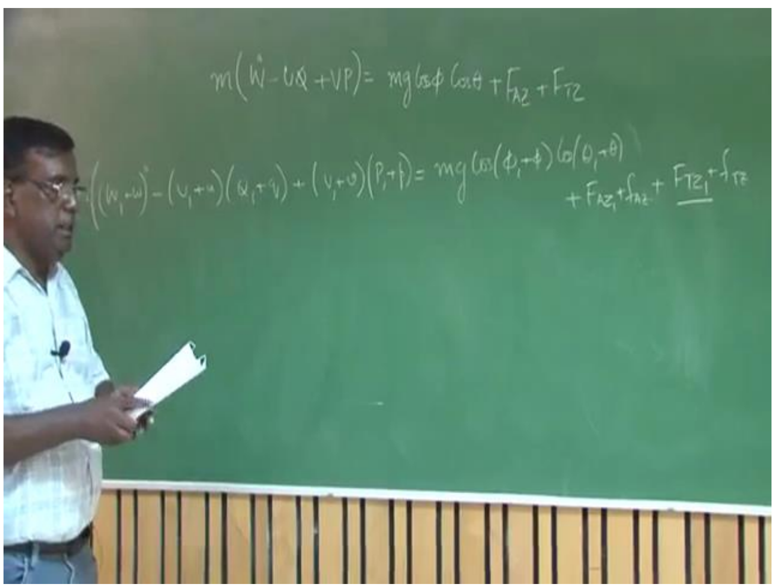
(Refer Slide Time: 28:56)



So I write MV dot + V1R equal to MG phi Cos Theta 1 + fAy + fTy. Similarly, we take the 3rd equation. We are trying to handle the third equation. We are trying to develop perturbed equation of motion.

(Refer Slide Time: 29:40)

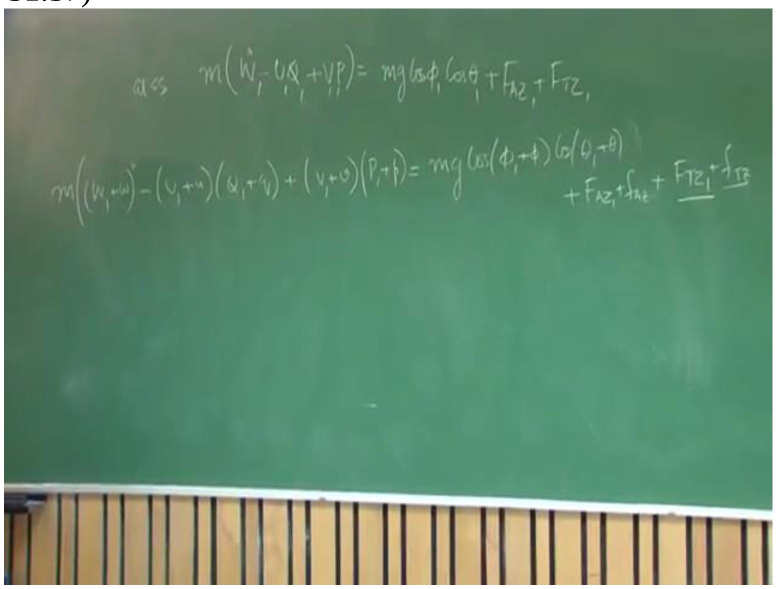




So please understand, this is general equation of motion. So again we follow the same trick, MW1 + W dot - U1 + a small U into Q1 + small Q + V1 + small V into P1 + small P equal to MG Cos phi 1 + phi Cos Theta 1 + Theta + FAZ1 + FAZ + FTZ1 + fTZ. Let us understand what is this fTZ here? fTZ means it is for propulsion, I am talking about the thrust. Many times you will find the engine setting is not exactly parallel to the reference axis.

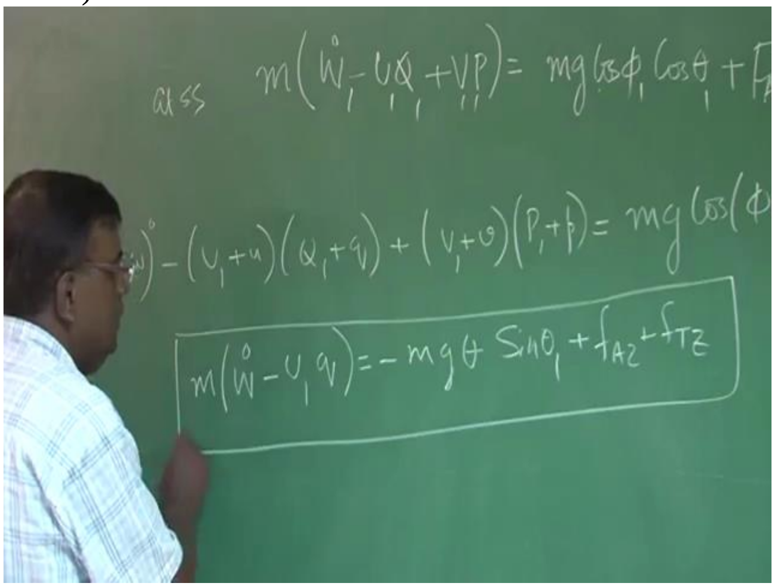
There will be some angle towards left. That is why, there will be a component of thrust coming from the engine itself at steady-state. That is FTZ1. And we talk about perturbed fTZ which means as this angle of attack changes, as this Theta changes, as there is a change in U, how this thrust is also changing along the local X direction.

That X could be stability axis depending upon how we have developed the equation. Finally we want to see, we are converting every equation into stability axis system.
(Refer Slide Time: 31:37)



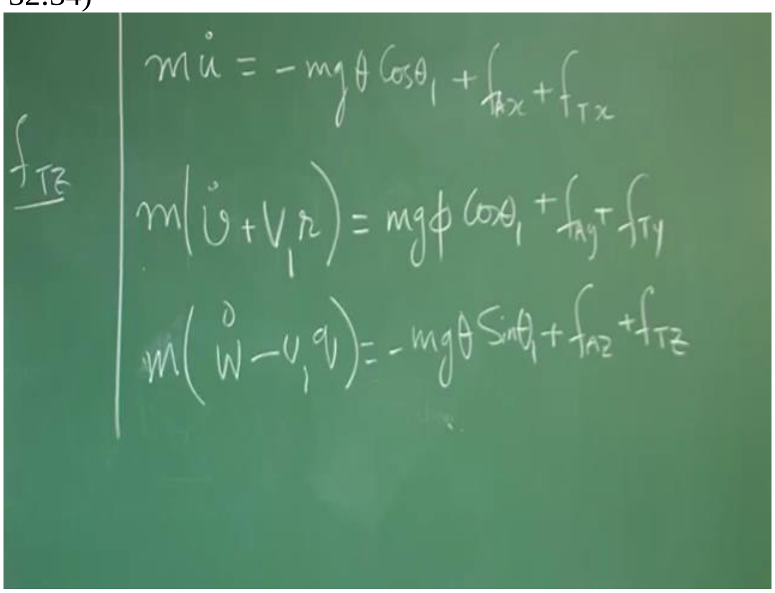
Again we follow the same technique and then before we do that, we have to first ensure that at steady-state, this is Q. (())(31:47). Take out these from the equation. Apply a small perturbation, product of 2 perturbed quantities is negligible and then phi and Theta are small. So we take sin and Cos approximation. Finally we will get an equation of this form.

(Refer Slide Time: 32:05)



MW dot - U1Q is equal to - MG Theta Sin Theta 1 + fAZ + fTZ. This becomes my third equation.

(Refer Slide Time: 32:34)



So I write MW dot, perturbed W - U1Q is equal to - MG Theta Sin Theta 1 + fAZ + fTZ. So these are the 3 force equations, 3 perturbed equations of motion. Similarly we will develop perturbed equations of motion which denote the angular motion and we will do that in the next lecture.

I strongly recommend that please derived these equations yourself and ask yourself the question, what are these variables? Why we are doing all this? That is extremely important because we want to extract information out of this. Thank you very much.